

**Calculus**  
**Practice Final Exam #4**

Total \_\_\_\_\_  
80

Name: \_\_\_\_\_

1. Determine the following limits, if they exist. **(12 marks)**

a)  $\lim_{x \rightarrow 3} \frac{x^3 - x^2 - 4}{x - 2}$

b)  $\lim_{x \rightarrow -5} \frac{x^3 + 125}{x^3 + 2x^2 - 13x + 10}$

c)  $\lim_{x \rightarrow 10^-} \frac{x}{(x - 10)^2}$

d)  $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$

e)  $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1-x} - 1}$

$$f) \lim_{x \rightarrow -\infty} \frac{2x^2 + 5x + 2}{x^2 + x}$$

2. Determine the derivative of the following functions. **Do Not** simplify your answers. (20 marks)

a)  $f(x) = (3 - 2x)^2$

b)  $y = 15x^3 - 3x^2 + 7x - 10$

c)  $f(x) = \sqrt{x^4 - 2x + 1}^7$

d)  $y = \frac{2x-1}{x^2-5}$

e)  $f(x) = (2x^3 + x)^4(4 - x)$

f)  $y = -\cos x^2$

g)  $f(x) = e^{\frac{-x}{4}}$

h)  $y = \ln(x^2)$

i)  $f(x) = \sin x - x \cos x$

j)  $y = e^{\tan^{-1}x}$

3. Determine  $\frac{dy}{dx}$  for  $3xy = x^3 + y^3$  (3 marks)

4. Using the first derivative test, find the open intervals on which  $f(x)$  is increasing or decreasing. Find the coordinates of any local extrema.

$$f(x) = 2x^3 - 3x^2 - 36x + 62 \quad \text{(6 marks)}$$

5. Find the open intervals on which  $f(x)$  is concave up or concave down. Find the coordinates of any inflection points

$$f(x) = x^4 - 2x^3 + x - 2 \quad \text{(5 marks)}$$

6. Determine the equations of all vertical and horizontal asymptotes of

$$f(x) = \frac{2x^2}{x^2+3x-4} \quad (3 \text{ marks})$$

7. Solve any **three** of the following five problems (15 marks)

a) A ball is thrown upward from the upper deck of the CN Tower, 450m above the ground. The distance, in metres, of the ball above the ground level after  $t$  seconds is:

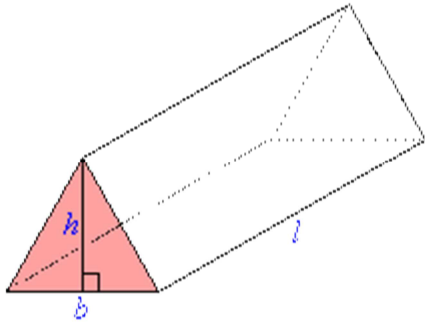
$$h = 450 + 10t - 5t^2, t \geq 0.$$

i) Find the initial velocity of the ball.

ii) Find how long it takes for the stone to hit the ground.

b) A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of  $25 \text{ cm/sec}$ . Find the rate at which the area of the circle is increasing after 4 seconds.

- c) A chocolate manufacturer uses an equilateral triangular prism package. What dimensions of the package will use the minimum amount of materials to contain  $300\text{cm}^3$  of chocolate?



- d) The holding area of a country fair is made up of 12 identical pens in a three by four grid. If 100m of fencing is available, what dimensions of each pen will maximize the total holding area.

e) A 24ft ladder leans against a high wall. If the foot of the ladder is pulled away from the base of the wall at a rate of 6ft/sec. How fast is the top moving when the foot is 8ft from the base of the wall?

8. Determine the following indefinite integrals by sight **(6 marks)**

a)  $\int (4x^3 - 11) dx$

b)  $\int \left( \frac{-3}{x} + \frac{5}{x^2} \right) dx$

c)  $\int (-3 \cos 5x + 8 \sin x) dx$

9. Determine the following indefinite integral by  $u$  substitution. **(3 marks)**

$$\int x(x^2 + 5)^5 dx$$

10. Evaluate the following definite integrals. **(4 marks)**

a)  $\int_{-4}^{-2} \frac{1}{x^2} dx$

b)  $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x dx$

11. Find the area bounded by the  $x$ -axis below,  $f(x)$  above, and the given pair of vertical lines. **(3 marks)**

$$f(x) = x\sqrt{25 - x^2}, x = 0, x = 5$$