Calculus
Practice Final Exam #4

Total____
80

Name: ____

1. Determine the following limits, if they exist. (**12 marks**)

a) $\lim\limits_{x\to 3}\dfrac{x^3-x^2-4}{x-2}$ $\quad \dfrac{3^3-3^2-4}{3-2}=\dfrac{27-9-4}{1}=16$

b) $\lim\limits_{x\to -5}\dfrac{x^3+125}{x^3+2x^2-13x+10}$ $\quad \dfrac{\cancel{(x+5)}(x^2-5x+25)}{\cancel{(x+5)}(x^2-3x+2)}=\dfrac{(-5)^2-5(-5)+25}{(-5)^2-3(-5)+2}$

$=\dfrac{75}{42}=\dfrac{25}{14}$

c) $\lim\limits_{x\to 10^-}\dfrac{x}{(x-10)^2}$

- - 0 + + + + + x

+ + + + + 0 + + $(x-10)^2$

0   10

f

- - 0 + + + ∞ + +

∞

d) $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx}$ $\quad \dfrac{\frac{ax}{ax}}{\frac{bx}{bx}}\cdot\dfrac{\frac{\sin ax}{1}}{\frac{\sin bx}{1}}=\dfrac{\frac{ax}{1}}{\frac{bx}{1}}\cdot\dfrac{\frac{\sin ax}{ax}}{\frac{\sin bx}{bx}}=\dfrac{ax}{bx}=\dfrac{a}{b}$

e) $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{1-x}-1}$ $\quad \left(\dfrac{\sqrt{1-x}+1}{\sqrt{1-x}+1}\right)=\dfrac{x(\sqrt{1-x}+1)}{1-x-1}=\dfrac{x(\sqrt{1-x}+1)}{-x}$

$=-(\sqrt{1-0}+1)$

$=-(1+1)$

$=-2$

f) $\lim_{x \to -\infty} \frac{2x^2 + 5x + 2}{x^2 + x}$

$\frac{2}{1} = 2$

2. Determine the derivative of the following functions. **<u>Do Not</u>** simplify your answers. **(20 marks)**

a) $f(x) = (3 - 2x)^2$

$f'(x) = 2(3-2x)(-2)$

b) $y = 15x^3 - 3x^2 + 7x - 10$

$f'(x) = 45x^2 - 6x + 7$

c) $f(x) = \sqrt{x^4 - 2x + 1}^7$

$f'(x) = \frac{7}{2}(x^4 - 2x + 1)^{5/2}(4x^3 - 2)$

d) $y = \frac{2x-1}{x^2-5}$

$f'(x) = \frac{(2)(x^2-5) - (2x)(2x-1)}{(x^2-5)^2}$

e) $f(x) = (2x^3 + x)^4(4 - x)$

$f'(x) = 4(2x^3+x)^3(6x^2+1)(4-x) + (-1)(2x^3+x)^4$

f) $y = -\cos x^2$

$f'(x) = -\sin x^2 \, (2x)$

g) $f(x) = e^{\frac{-x}{4}}$

$f'(x) = e^{-x/4} \left(-\frac{1}{4}\right)$

h) $y = ln(x^2)$

$f'(x) = \frac{1}{x^2} (2x)$

i) $f(x) = \sin x - x \cos x$

$f'(x) = \cos x - (1)(\cos x) + (-\sin x)(x)$

$\cos x \quad - \cos x - x \sin x$

j) $y = e^{tan^{-1}x}$

$f'(x) = e^{tan^{-1}x} \left(\frac{-\sec^2 x}{\tan^2 x}\right)$

3. Determine $\frac{dy}{dx}$ for $3xy = x^3 + y^3$ **(3 marks)**

$(3)(y) + (y')(3x) = 3x^2 + 3y^2 y'$

$3xy' - 3y^2 y' = 3x^2 - 3y$

$y' = \frac{3x^2 - 3y}{3x - 3y^2}$

4. Using the first derivative test, find the open intervals on which $f(x)$ is increasing or decreasing. Find the coordinates of any local extrema.
$f(x) = 2x^3 - 3x^2 - 36x + 62$ **(6 marks)**

$f'(x) = 6x^2 - 6x - 36$

$6x^2 - 6x - 36 = 0$

$6(x^2 - x - 6) = 0$

$6(x-3)(x+2) = 0$

$x = 3 \qquad x = -2$

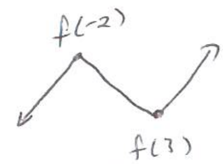
+ + + + + + + 6

− − − − − 0 + + $x-3$

− − 0 + + + + + + $x+2$

−2 3

+ + 0 − − − 0 + + $f'(x)$



Inc $(-\infty, -2) \cup (3, \infty)$

dec $(-2, 3)$

max @ $(-2, 106)$

min @ $(3, -19)$

5. Find the open intervals on which $f(x)$ is concave up or concave down. Find the coordinates of any inflection points
$f(x) = x^4 - 2x^3 + x - 2$ **(5 marks)**

− − 0 + + + + + $12x$

− − − − 0 + + $x-1$

0 1

+ + 0 − − 0 + + $f''(x)$

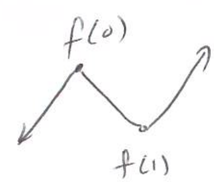
$f'(x) = 4x^3 - 6x^2 + 1$

$f''(x) = 12x^2 - 12x$

$12x^2 - 12x = 0$

$12x(x-1) = 0$

$x = 0 \qquad x = 1$



CU $(-\infty, 0) \cup (1, \infty)$

CD $(0, 1)$

P.O.I. @ f(0, −2)

f(1, −2

6. Determine the equations of all vertical and horizontal asymptotes of

$f(x) = \frac{2x^2}{x^2+3x-4}$ **(3 marks)**

$(x+4)(x-1)$

V.A. @ $x = -4$

$x = 1$

H.A. @ $y = 2$

7. Solve any **<u>three</u>** of the following five problems **(15 marks)**

   a) A ball is thrown upward from the upper deck of the CN Tower, 450m above the ground. The distance, in metres, of the ball above the ground level after t seconds is:

   $h = 450 + 10t - 5t^2, t \geq 0.$

   i) Find the initial velocity of the ball.

   $v = 10 - 10t$

   $v(0) = 10m/s$

   $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

   $x = \frac{-10 \pm \sqrt{10^2-4(-5)(450)}}{2(-5)}$

   $x = \frac{-10 \pm \sqrt{9100}}{-10}$

   ii)Find how long it takes for the stone to hit the ground.

   $0 = -5t^2 + 10t + 450$

   $x = 10.54$ sec

   b) A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of $25cm/sec$. Find the rate at which the area of the circle is increasesing after 4 seconds.

   $r' = 25$

   $t = 4$

   $r = 100$

   $A' = ?$

   $A = \pi r^2$

   $A' = 2\pi r r'$

   $A' = 2\pi(100)(25)$

   $A' = 5000\pi$

   $15708 cm^2/sec$

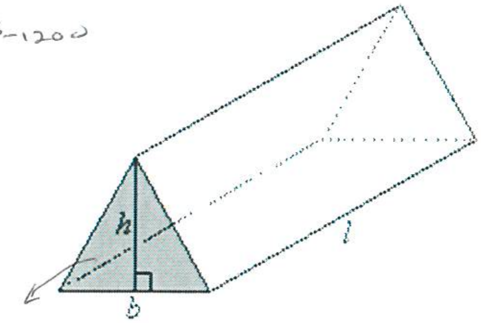
c) A chocolate manufacturer uses an equilateral triangular prism package. What dimensions of the package will use the minimum amount of materials to contain 300cm^3 of chocolate?

$+ + + + + + \sqrt{3}b^{-2}$

$- - - 0 + + + b^3 - 1200$

10.63

$- - - 0 + + + +$



$A = \frac{\sqrt{3}}{4}b^2$

$b = 10.63$ cm

$l = 6.13$ cm

$V = \frac{1}{2}bhl$

$V = \frac{\sqrt{3}}{4}b^2 l$

$300 = \frac{\sqrt{3}}{4}b^2 l$

$1200 = \sqrt{3}b^2 l$

$\frac{1200}{\sqrt{3}b^2} = l$

$A = 2\frac{\sqrt{3}}{4}b^2 + 3bl$

$A = \frac{\sqrt{3}}{2}b^2 + \frac{3600}{\sqrt{3}b}$

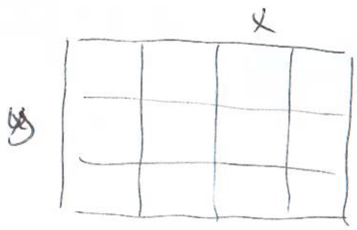
$A = \frac{\sqrt{3}}{2}b^2 + 1200\sqrt{3}b^{-1}$

$A' = \sqrt{3}b - 1200\sqrt{3}b^{-2}$

$\sqrt{3}b^{-2}(b^3 - 1200)$

$b = 0$ $\qquad$ $b = 10.63$

d) The holding area of a country fair is made up of 12 identical pens in a three by four grid. If 100m of fencing is available, what dimensions of each pen will maximize the total holding area.



Fence $= 5y + 4x$

$100 = 5x + 4y$

$100 - 5x = 4y$

$25 - \frac{5}{4}x = y$

$A = xy$

$A = x(25 - \frac{5}{4}x)$

$A = 25x - \frac{5}{4}x^2$

$A' = 25 - \frac{5}{2}x = 0$

$25 = \frac{5}{2}x$

$10 = x$

$X = 10$

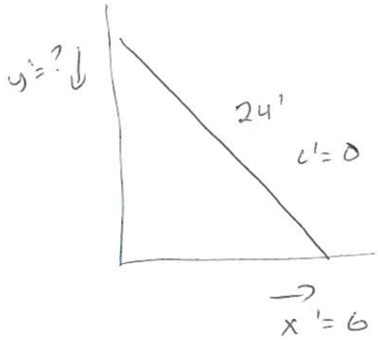
$y = 10$

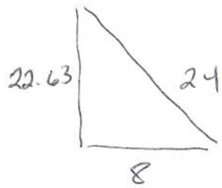
$+ + + + 0 - - -$ $\quad 25 - \frac{5}{2}x$

10

$+ + + 0 - - -$ $\quad A'$

e) A 24ft ladder leans against a high wall. If the foot of the ladder is pulled away from the base of the wall at a rate of 6ft/sec. How fast is the top moving when the foot is 8ft from the base of the wall?





$a^2 + b^2 = c^2$

$2aa' + 2bb' = 2cc'$

$2(8)(6) + 2(22.63)b' = 2(24)(0)$

$45.26\, b' = -96$

$b' = -2.12$

falling down at 2.12 ft/sec

8. Determine the following indefinite integrals by sight **(6 marks)**

a) $\int (4x^3 - 11)dx$

$x^4 - 11x + C$

b) $\int \left(\frac{-3}{x} + \frac{5}{x^2}\right) dx$

$-3\ln|x| - 5x^{-1} + C$

c) $\int (-3\cos 5x + 8\sin x)dx$

$-\frac{3}{5}\sin 5x - 8\cos x + C$

9. Determine the following indefinite integral by u substitution. **(3 marks)**

$\int x(x^2+5)^5 dx$

$u = x^2+5$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int u^5 \cdot \frac{1}{2}du$

$\frac{1}{2}\int u^5\,du$

$\frac{1}{2}\cdot\frac{1}{6}u^6 + C$

$\frac{1}{12}(x^2+5)^6 + C$

10. Evaluate the following definite integrals. **(4 marks)**

a) $\int_{-4}^{-2} \frac{1}{x^2} dx$

$F(x) = -x^{-1} + C$

$F(-2) - F(-4)$

$\left[-(-2)^{-1}\right] - \left[-(-4)^{-1}\right]$

$\frac{1}{2} - \frac{1}{4}$

$\frac{1}{4}$

b) $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x\, dx$

$F(x) = -\frac{1}{4}\cos 4x + C$

$F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{8}\right)$

$\left[-\frac{1}{4}\cos 4\left(\frac{\pi}{4}\right) + C\right] - \left[-\frac{1}{4}\cos 4\left(\frac{\pi}{8}\right) + C\right]$

$\left[-\frac{1}{4}(-1) + C\right] - \left[C\right]$

$\frac{1}{4}$

11. Find the area bounded by the x-axis below, $f(x)$ above, and the given pair of vertical lines. **(3 marks)**

$f(x) = x\sqrt{25-x^2}, x = 0, x = 5$

$F(x) = \frac{-3}{4}(25-x^2)^{2/3} + C$

$F(5) - F(0)$

$0 - (-6.41)$

6.41