

Calculus
Practice Final Exam #3

Total _____
80

Name: _____

1. Determine the following limits, if they exist. (12 marks)

$$a) \lim_{x \rightarrow 3} \frac{-\sqrt{x+3}}{x+2}$$

$$b) \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$$

$$c) \lim_{x \rightarrow 6^-} \frac{3x}{x^2 - 6x}$$

$$d) \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

$$e) \lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$$

$$f) \lim_{x \rightarrow \infty} \frac{12x^2}{2x^3 - 6x}$$

2. Determine the derivative of the following functions. **Do Not** simplify your answers. (20 marks)

a) $f(x) = (2x + 1)^3$

b) $y = x^2 - 7x + 4$

c) $f(x) = \sqrt{3x + 1}$

d) $y = \frac{x^2 + 3}{2x - 1}$

e) $f(x) = (x^2 - 1)^4(2x + 1)^3$

f) $y = \sin(3x + 2\pi)$

g) $f(x) = e^{\frac{2x}{3}}$

h) $y = \ln(2x + 2)$

i) $f(x) = 2 \sin x \cos x$

j) $y = \ln(\sin x)$

3. Determine $\frac{dy}{dx}$ for $2x^2y^2 = x^3 + y^3$ (3 marks)

4. Using the first derivative test, find the open intervals on which $f(x)$ is increasing or decreasing. Find the coordinates of any local extrema.

$$f(x) = 2x^3 - 3x^2 \quad \text{(6 marks)}$$

5. Find the open intervals on which $f(x)$ is concave up or concave down. Find the coordinates of any inflection points

$$f(x) = 16 + 4x + x^2 - x^3 \quad \text{(5 marks)}$$

6. Determine the equations of all vertical and horizontal asymptotes of

$$f(x) = \frac{4x+5}{3-2x} \quad (3 \text{ marks})$$

7. Solve any **three** of the following five problems (15 marks)

a) A stone is thrown downward from a cliff that is 80m high. Its height in metres after t seconds is:

$$h = 80 - 15t - 4.9t^2, t \geq 0.$$

i) Find the initial velocity of the ball.

ii) Find how long it takes for the stone to hit the ground.

b) A snowball melts so that its surface area decreases at a rate of $0.5 \text{ cm}^2/\text{min}$. Find the rate at which the radius decreases when the radius is 4cm.

c) A cracker box in the shape of a rectangular prism is to be constructed with a square base. The total capacity of the package must be 1000cm^3 . What dimensions provide the minimum surface area?

d) A typical automotive battery has six cells divided into six rectangles side by side in a one by 6 pattern. What dimensions will give the smallest total wall length with a total area of 390 cm^2 .

e) A 10ft ladder leans against a vertical wall. If the bottom of the ladder is pulled away from the wall at a rate of 2ft/sec, at what rate is the top of the ladder sliding down the wall when the top is 8ft from the ground?

8. Determine the following indefinite integrals by sight **(6 marks)**

a) $\int \left(2x - \frac{1}{2x} \right) dx$

b) $\int (\sin 2x + 2 \cos x) dx$

c) $\int (\sqrt{x} + \sqrt[3]{x}) dx$

9. Determine the following indefinite integral by u substitution. **(3 marks)**

$$\int x(x^2 - 6)^{11} dx$$

10. Evaluate the following definite integrals. **(4 marks)**

a) $\int_0^1 \sqrt[3]{x^2} dx$

b) $\int_0^{\frac{3\pi}{2}} \cos \frac{x}{3} dx$

11. Find the area bounded by the x -axis below, $f(x)$ above, and the given pair of vertical lines. **(3 marks)**

$$f(x) = 2x, x = 0, x = \frac{\pi}{4}$$