Calculus
Practice Final Exam #3

Total____
80

Name: ____

1. Determine the following limits, if they exist. **(12 marks)**

a) $\lim\limits_{x\to 3}\dfrac{-\sqrt{x+3}}{x+2}$

$\dfrac{-\sqrt{3+3}}{3+2} = \dfrac{-\sqrt{6}}{5}$

b) $\lim\limits_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}$

$\dfrac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{\sqrt{x}-2}} = \sqrt{4}+2 = 4$

c) $\lim\limits_{x\to 6^-}\dfrac{3x}{x^2-6x}$

critical $x=0$

$x(x-6) \to x=0$, $x=6$

$- - - 0 + + + + + + + \quad 3x$

$- - - 0 + + + + + + + \quad x$

$- - - - - - - - 0 + + + \quad x-6$

0   6

$- - - \varnothing - - - - - \infty + + + \quad f(x)$

$-\infty$

d) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}$

$\dfrac{\frac{1}{1}}{\frac{3}{3}} \cdot \dfrac{\frac{\sin 3x}{1}}{\frac{x}{1}} = \dfrac{1}{\frac{1}{3}} \cdot \dfrac{\sin 3x}{3x} = \dfrac{1}{\frac{1}{3}} = 3$

e) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}$

$\left(\dfrac{\sqrt{x}+1}{\sqrt{x}+1}\right) = \dfrac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)} = \dfrac{1}{\sqrt{x}+1} = \dfrac{1}{\sqrt{1}+1} = \dfrac{1}{2}$

$f)\ \lim_{x\to\infty} \frac{12x^2}{2x^3 - 6x}$ 0

2. Determine the derivative of the following functions. **<u>Do Not</u>** simplify your answers. **(20 marks)**

a) $f(x) = (2x+1)^3$

$f'(x) = 3(2x+1)^2(2)$

b) $y = x^2 - 7x + 4$

$f'(x) = 2x - 7$

c) $f(x) = \sqrt{3x+1}$

$f'(x) = \frac{1}{2}(3x+1)^{-\frac{1}{2}}(3)$

d) $y = \frac{x^2+3}{2x-1}$ $f'(x) = \frac{(2x)(2x-1) - (2)(x^2+3)}{(2x-1)^2}$

e) $f(x) = (x^2-1)^4(2x+1)^3$

$f'(x) = 4(x^2-1)^3(2x)(2x+1)^3 + 3(2x+1)^2(2)(x^2-1)^4$

f) $y = sin(3x + 2\pi)$

$f'(x) = \cos(3x + 2\pi)(3)$

g) $f(x) = e^{\frac{2x}{3}}$

$f'(x) = e^{\frac{2x}{3}} \cdot \left(\frac{2}{3}\right)$

h) $y = ln(2x + 2)$

$f'(x) = \frac{1}{2x+2}(2)$

i) $f(x) = 2\sin x \cos x$

$f'(x) = 2(\cos x)(\cos x) + (-\sin x)(2\sin x)$

j) $y = ln(\sin x)$

$\frac{1}{\sin x}(\cos x)$

3. Determine $\frac{dy}{dx}$ for $2x^2y^2 = x^3 + y^3$ **(3 marks)**

$(4x)(y^2) + (2yy')(2x^2) = 3x^2 + 3y^2y'$

$4xy^2 + 4x^2yy' = 3x^2 + 3y^2y'$

$4x^2yy' - 3y^2y' = 3x^2 - 4xy^2$

$y' = \frac{3x^2 - 4xy^2}{4x^2y - 3y^2}$

4. Using the first derivative test, find the open intervals on which $f(x)$ is increasing or decreasing. Find the coordinates of any local extrema.
$f(x) = 2x^3 - 3x^2$ **(6 marks)**

$f'(x) = 6x^2 - 6x$

$6x^2 - 6x = 0$

$6x(x-1) = 0$

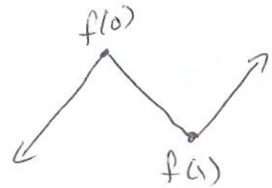
$x = 0 \qquad x = 1$

$- \; - \; 0 \; + \; + \; + \; + \; + \quad 6x$

$- \; - \; - \; - \; - \; 0 \; + \; + \quad x-1$

$0 \qquad 1 \qquad f'(x)$

$+ \; + \; 0 \; - \; - \; 0 \; + \; +$



inc $(-\infty, 0) \cup (1, \infty)$

dec $(0, 1)$

max @ $(0, 0)$

min @ $(1, -1)$

5. Find the open intervals on which $f(x)$ is concave up or concave down. Find the coordinates of any inflection points
$f(x) = 16 + 4x + x^2 - x^3$ **(5 marks)**

$f'(x) = 4 + 2x - 3x^2$

$f''(x) = 2 - 6x$

$2 - 6x = 0$

$2(1 - 3x) = 0$

$1 - 3x = 0$

$1 = 3x$

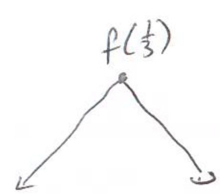
$\frac{1}{3} = x$

$+ \; + \; + \; + \; + \; + \; + \quad 2$

$+ \; + \; + \; + \; 0 \; - \; - \; - \quad 1-3x$

$\frac{1}{3} \qquad f''(x)$

$+ \; + \; + \; + \; 0 \; - \; - \; -$



C.U. $(-\infty, \frac{1}{3})$

C.D. $(\frac{1}{3}, \infty)$

P.O.I. @ $(\frac{1}{3}, \frac{470}{27})$

6. Determine the equations of all vertical and horizontal asymptotes of $f(x) = \frac{4x+5}{3-2x}$ **(3 marks)**

V. A. @ $x = \frac{3}{2}$

H. A. @ $\frac{4}{-2} = -2$

7. Solve any **three** of the following five problems **(15 marks)**

a) A stone is thrown downward from a cliff that is 80m high. Its height in metres after t seconds is:

$$h = 80 - 15t - 4.9t^2, t \geq 0.$$

i) Find the initial velocity of the ball.

$V = -15 - 9.8t$

$V(0) = -15$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{15 \pm \sqrt{(-15)^2 - 4(-4.9)(80)}}{2(-4.9)}$

$x = \frac{15 \pm \sqrt{1793}}{-9.8}$

ii)Find how long it takes for the stone to hit the ground.

$0 = 80 - 15t - 4.9t^2$

$x = 2.79$ sec

b) A snowball melts so that its surface area decreases at a rate of $0.5cm^3/min$. Find the rate at which the radius decreases when the radius is 4cm.

$A' = 0.5$

$r' = ?$

$r = 4$

$A = 4\pi r^2$

$A' = 8\pi r r'$

$0.5 = 8\pi(4) r'$

$\frac{0.5}{32\pi} = r'$

$0.005 cm/min = r'$ or $\frac{1}{64\pi} = r'$

c) A cracker box in the shape of a rectangular prism is to be constructed with a square base. The total capacity of the package must be $1000cm^3$. What dimensions provide the minimum surface area?

$V = x^2 h$

$1000 = x^2 h$

$\frac{1000}{x^2} = h$

$SA = 4xh + 2x^2$

$SA = 4x\left(\frac{1000}{x^2}\right) + 2x^2$

$SA = \frac{4000}{x} + 2x^2$

$SA' = -4000x^{-2} + 4x$

$-4000x^{-2} + 4x = 0$

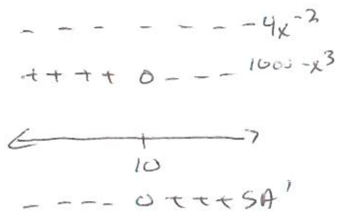
$-4x^{-2}(1000 - x^3) = 0$

$x = 0$

$1000 - x^3 = 0$

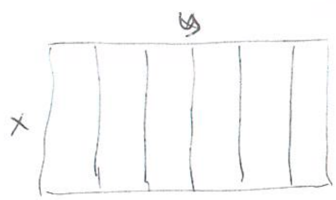
$1000 = x^3$

$10 = x$



minimum @ $x = 10$
$h = 10$

d) A typical automotive battery has six cells divided into six rectangles side by side in a one by 6 pattern. What dimensions will give the smallest total wall length with a total area of 390 cm^3.



$L = 7x + 2y$

$L = 7x + 2\left(\frac{390}{x}\right)$

$L = 7x + \frac{780}{x}$

$L' = 7 - 780x^{-2}$

$7 - 780x^{-2} = 0$

$x^{-2}(7x^2 - 780) = 0$

$x = 0$

$7x^2 - 780 = 0$

$7x^2 = 780$

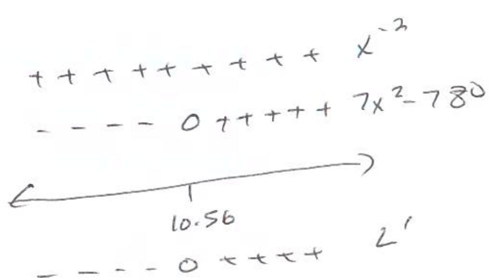
$x^2 = \frac{780}{7}$

$x = 10.56$

$A = xy$

$390 = xy$

$\frac{390}{x} = y$

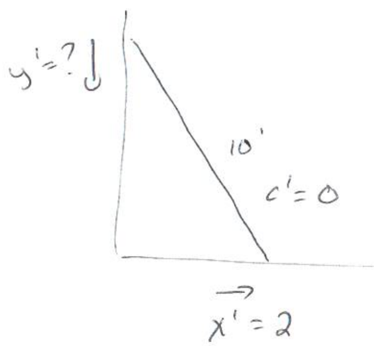


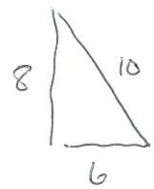
$\frac{390}{10.56} = y$

$36.93 = y$

10.56cm by
36.93cm

e) A 10ft ladder leans against a vertical wall. If the bottom of the ladder is pulled away from the wall at a rate of 2ft/sec, at what rate is the top of the ladder sliding down the wall when the top if 8ft from the ground?





$a^2 + b^2 = c^2$

$2aa' + 2bb' = 2cc'$

$2(8)a' + 2(6)(2) = 2(10)(0)$

$16a' + 24 = 0$

$16a' = -24$

$a' = -1.5$

down at 1.5ft/sec

8. Determine the following indefinite integrals by sight **(6 marks)**

a) $\int \left(2x - \frac{1}{2x}\right) dx$

$x^2 - \frac{1}{2}\ln|x| + C$

b) $\int (\sin 2x + 2\cos x) dx$

$-\frac{1}{2}\cos 2x + 2\sin x + C$

c) $\int (\sqrt{x} + \sqrt[3]{x}) dx$

$\frac{2}{3}x^{\frac{3}{2}} + \frac{3}{4}x^{\frac{4}{3}} + C$

9. Determine the following indefinite integral by u substitution. **(3 marks)**

$\int x(x^2-6)^{11}dx$ $\rightarrow$ $u = x^2 - 6$

$du = 2x\, dx$

$\int u^{11} \cdot \frac{1}{2} du$ $\leftarrow$ $\frac{1}{2} du = x\, dx$

$\frac{1}{2} \int u^{11} du$

$\frac{1}{2} \cdot \frac{1}{12} u^{12} + C$

$\frac{1}{24} u^{12} + C$

10. Evaluate the following definite integrals. **(4 marks)**

a) $\int_0^1 \sqrt[3]{x^2}dx$

$F(x) = \frac{3}{5} x^{\frac{5}{3}} + C$

$F(1) - F(0)$

$\left(\frac{3}{5}(1)^{5/3} + C\right) - \left(\frac{3}{5}(0)^{5/3} + C\right)$

$\frac{3}{5} + C - C$

$\frac{3}{5}$

b) $\int_0^{\frac{3\pi}{2}} \cos\frac{x}{3}dx$

$F(x) = 3\sin\frac{x}{3} + C$

$F\left(\frac{3\pi}{2}\right) - F(0)$

$\left(3\sin\frac{\frac{3\pi}{2}}{3} + C\right) - \left(3\sin\frac{0}{3} + C\right)$

$3 + C \quad - C$

3

11. Find the area bounded by the x-axis below, $f(x)$ above, and the given pair of vertical lines. **(3 marks)**

$F(x) = x^2$

$$f(x) = 2x, x = 0, x = \frac{\pi}{4}$$

$F\left(\frac{\pi}{4}\right) - F(0)$

$\left(\frac{\pi}{4}\right)^2 - (0)^2$ $\rightarrow$ $\frac{\pi^2}{16}$