Calculus Total____

Practice Final Exam #2 80

Name: ____

1. Determine the following limits, if they exist. **(12 marks)**

a) $\lim\limits_{x \to 2} \dfrac{-x-3}{x^2+x+1}$ $\dfrac{-2-3}{2^2+2+1} = \dfrac{-5}{7}$

b) $\lim\limits_{x \to -1} \dfrac{x^2+5x+4}{x^3+1}$ $\dfrac{\cancel{(x+1)}(x+4)}{\cancel{(x+1)}(x^2-x+1)} = \dfrac{-1+4}{(-1)^2-(-1)+1} = \dfrac{3}{3} = 1$

c) $\lim\limits_{x \to 3^+} \dfrac{x+3}{2^x-8}$

+ + + + + + + $x+3$

- - - - 0 + + + + 2^x-8

3

- - - - ∞ + + + +

∞

d) $\lim\limits_{x \to 0} \dfrac{\sin 2x}{\sin 3x}$ $\dfrac{\frac{2x}{2x}}{\frac{3x}{3x}} \cdot \dfrac{\frac{\sin 2x}{1}}{\frac{\sin 3x}{1}} = \dfrac{\frac{2x}{1}}{\frac{3x}{1}} \cdot \dfrac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}} = \dfrac{2x}{3x} = \dfrac{2}{3}$

e) $\lim\limits_{x \to 0} \dfrac{\sqrt{4-x}-2}{x}$ $\left(\dfrac{\sqrt{4-x}+2}{\sqrt{4-x}+2}\right) = \dfrac{4-x-4}{x(\sqrt{4-x}+2)} = \dfrac{-x}{x(\sqrt{4-x}+2)}$

$= \dfrac{-1}{\sqrt{4-x}+2} = \dfrac{-1}{\sqrt{4-0}+2} = \dfrac{-1}{\sqrt{4}+2} = \dfrac{-1}{4}$

$f)\ \lim_{x\to\infty} \frac{3-2x}{3+2x}$

$\frac{-2}{2} = -1$

2. Determine the derivative of the following functions. **Do Not** simplify your answers. **(20 marks)**

a) $f(x) = (x^2-1)^4$

$4(x^2-1)^3(2x)$

b) $y = 12x^3 + 8x - 1$

$36x^2+8$

c) $f(x) = (x^4+2x^2+1)^{\frac{2}{3}}$

$\frac{2}{3}(x^4+2x^2+1)^{-\frac{1}{3}}(4x^3+4x)$

d) $y = \frac{x^2+x}{x^4+1}$

$\frac{(2x+1)(x^4+1)-(4x^3)(x^2+x)}{(x^4+1)^2}$

e) $f(x) = (x^2+x)\sqrt{1-x^2}$

$(2x+1)(1-x^2)^{\frac{1}{2}} + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)(x^2+x)$

f) $y = x^3 + \sqrt{x} + \cos x$

$3x^2 + \frac{1}{2}x^{-\frac{1}{2}} - \sin x\ (1)$

g) $f(x) = e^{-5x}$

$e^{-5x}\ (-5)$

h) $y = ln(x^2 + 1)$

$\frac{1}{x^2+1}\ \ln e\ (2x)$

i) $f(x) = x\ \sin x$

$(1)(\sin x) + (\cos x)(1)(x)$

j) $y = log_{10}e^x$ $= x \log_{10} e$

$(1)(\log_{10} e) + \frac{1}{e} \log_{10} e\ (0)(x)$

3. Determine $\frac{dy}{dx}$ for $x^2 - x^2y + y^2 = 1$ **(3 marks)**

$2x - [(2x)(y) + (y')(x^2)] + 2yy' = 0$

$2x - 2xy - x^2y' + 2yy' = 0$

$-x^2y' + 2yy' = -2x + 2xy$

$y' = \frac{-2x+2xy}{-x^2+2y}$

4. Using the first derivative test, find the open intervals on which $f(x)$ is increasing or decreasing. Find the coordinates of any local extrema.
$f(x) = x^4 - 4x^3 - 8x^2 - 1$ **(6 marks)**

$f'(x) = 4x^3 - 12x^2 - 16x = 0$

$4x(x^2 - 3x - 4) = 0$

$4x(x-4)(x+1) = 0$

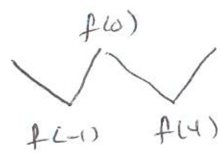
$- - - - - - 0 + + + \quad 4x$

$- - - - - - - - 0 + + \quad (x-4)$

$- - 0 + + + + + + + + \quad (x+1)$

$-1 \qquad 0 \qquad 4$

$- - 0 + + + 0 - - - 0 + + \quad f'(x)$



Local min@ $(-1, -4)$
and
$(4, -129)$

Local max@ $(0, -1)$

5. Find the open intervals on which $f(x)$ is concave up or concave down. Find the coordinates of any inflection points
$f(x) = x^4 - 24x^2 + x - 1$ **(5 marks)**

$f'(x) = 4x^3 - 48x + 1$

$f''(x) = 12x^2 - 48$

$12x^2 - 48 = 0$

$12(x^2 - 4) = 0$

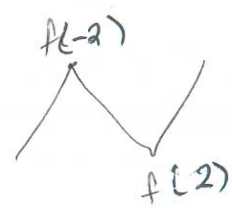
$x = \pm 2$

$+ + + + + + + + + + \quad 16$

$+ + 0 - - - - 0 + + \quad x^2 - 3$

$-2 \qquad 2$

$+ + 0 - - - - - 0 + + + \quad f''(x)$



Cup $(-\infty, -2) \cup (2, \infty)$

C-down $(-2, 2)$

POI@ $-2, -91$

$2, -87$

6. Determine the equations of all vertical and horizontal asymptotes of $f(x) = \frac{2x-3}{5-4x}$ **(3 marks)**

V.A. @ $5-4x=0$

$-4x=-5$

$x=\frac{5}{4}$

H.A.@ $\frac{2}{-4} = -\frac{1}{2}$

7. Solve any **three** of the following five problems (**15 marks**)
   a) A ball is thrown upward with an initial velocity of 24.5 m/s, then its height after t seconds in metres is:

   $h = 24.5t - 4.9t^2, t \geq 0.$

   i) Find the initial velocity of the ball.

   $v = 24.5 - 9.8t$

   $v(0) = 24.5 - 9.8(0)$

   $= 24.5 m/s$

   ii)Find the maximum height of the ball.

   $24.5 - 9.8t = 0$

   $-9.8t = -24.5$

   $t = 2.5 sec$

   $h(2.5) = 24.5(2.5) - 4.9(2.5)^2$

   $= 30.625 m$

   b) A spherical balloon is being inflated so that the volume is increasing at a rate of $144m^3/min$. How fast is the radius of the balloon increasing when the diameter is 2m?

   $V' = 144$

   $r' = ?$

   $r = 2$

   $V = \frac{4}{3}\pi r^3$

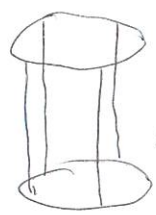
   $V' = 4\pi r^2 r'$

   $144 = 4\pi(2)^2 r'$

   $\frac{144}{16\pi} = r'$

   $2.86 m/min = r'$

c) A cylinder kite frame is made out of light bendable rod. The frame is made up of two circles of rod connected by four straight rods of equal length. What dimensions create the maximum volume for the cylinder ~~?~~ if 4 m of rod is available?



$L = 4h + 2(2\pi r)$

$L = 4h + 4\pi r$

$4m = 4h + 4\pi r$

$4 - 4\pi r = 4h$

$1 - \pi r = h$

$V = \pi r^2 h$

$V = \pi r^2 (1 - \pi r)$

$V = \pi r^2 - \pi^2 r^3$

$V' = 2\pi r - 3\pi^2 r^2$

$0 = 2\pi r - 3\pi^2 r^2$

$0 = \pi r (2 - 3\pi r)$

$r = 0$ $\quad$ $2 - 3\pi r = 0$

$-3\pi r = -2$

$r = \frac{2}{3\pi} \approx 0.21$

$+ + + + + + + + + \quad \pi r$

$+ + + + 0 - - - - \quad 2 - 3\pi r$

$\frac{2}{3\pi}$

$+ + + + 0 - - - -$

max @ $\frac{2}{3\pi}$ m for radius

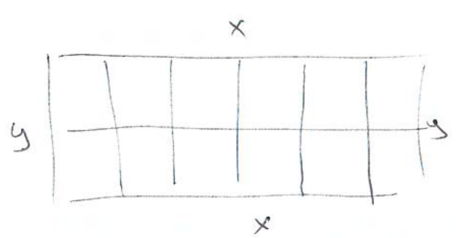
$\frac{1}{3}$m for length

$h = 1 - \pi\left(\frac{2}{3\pi}\right)$

$h = 1 - \frac{2}{3}$

$h = \frac{1}{3}$

d) The holding area at a country fair is 12 identical pens arranged in a two by six grid. If 100m of fencing is available, what dimensions of each pen will maximize the total area?



$Fence = 7y + 3x$

$100 = 7y + 3x$

$100 - 7y = 3x$

$\frac{100 - 7y}{3} = x$

$xy = A$

$\left(\frac{100 - 7y}{3}\right) y = A$

$\frac{100}{3}y - \frac{7}{3}y^2 = A$

$\frac{100}{3} - \frac{14}{3}y = A'$

$0 = \frac{100}{3} - \frac{14}{3}y$

$\frac{-100}{3} = -\frac{14}{3}y$

$\frac{50}{7} = y \approx 7.14$

$\frac{100 - 7\left(\frac{50}{7}\right)}{3} = x$

$\frac{100 - 50}{3} = x$

$\frac{50}{3} = x \approx 16.67$

$+ + + + 0 - - - - \quad \frac{100}{3} - \frac{14}{3}y$

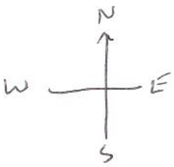
$\frac{50}{7}$

$+ + + + 0 - -$

max @ $y = \frac{50}{7}$

$x = \frac{50}{3}$

e) One ship leaves port and steams due north at 10 knots. Three hours later another ship leaves the same part and steams due west at 30 knots. How fast is the distance between them increasing when the first ship has been out for 5 hours?



$a^2 + b^2 = c^2$

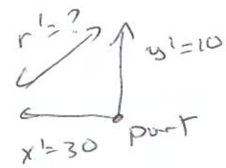
$2aa' + 2bb' = 2cc'$

$2(50)(10) + 2(60)(30) = 2(78.10)r'$

$1000 + 3600 = 156.2\, r'$

$29.45 = r'$

29.45 knots



8. Determine the following indefinite integrals by sight **(6 marks)**

a) $\int \left(\frac{2}{x^3} + \frac{3}{x^2} + 5\right) dx$ $\quad 2x^{-3} + 3x^{-2} + 5$

$-x^{-2} - 3x^{-1} + 5x + C$

b) $\int (e^x + e^{-x}) dx$

$e^x - e^{-x} + C$

c) $\int \left(\sqrt{x^3} + 7x\right) dx$ $\quad x^{\frac{3}{2}} + 7x$

$\frac{2}{5} x^{\frac{5}{2}} + \frac{7}{2} x^2 + C$

9. Determine the following indefinite integral by u substitution. **(3 marks)**

$\int(6x - 11)^8 dx \longrightarrow u = 6x - 11$

$du = 6dx$

$\int u^8 \cdot \frac{1}{6} du$

$\frac{1}{6}du = dx$

$\frac{1}{6}\int u^8 du$

$\frac{1}{6} \cdot \frac{1}{9}u^9 + C$

$\frac{1}{54}(6x-11)^9 + C$

10. Evaluate the following definite integrals. **(4 marks)**

a) $\int_{-5}^{1}(x^2 + 4x - 5)dx$

$F(1) - F(-5)$

$F(x) = \frac{1}{3}x^3 + 2x^2 - 5x + C$

$\left[\frac{1}{3}(1)^3 + 2(1)^2 - 5(1) + C\right] - \left[\frac{1}{3}(-5)^3 + 2(-5)^2 - 5(-5) + C\right]$

$\left(\frac{1}{3} + 2 - 5 + C\right) - \left(-\frac{125}{3} + 50 + 25 + C\right)$

$\left(-\frac{8}{3} + C\right) - \left(\frac{100}{3} + C\right)$

-36

b) $\int_{0}^{\frac{3\pi}{4}} \sin x \, dx$

$F(x) = -\cos x + C$

$F\left(\frac{3\pi}{4}\right) - F(0)$

$-\cos\left(\frac{3\pi}{4}\right) + \cos(0)$

$\frac{\sqrt{2}}{2} + 1 \rightarrow \frac{2+\sqrt{2}}{2}$

11. Find the area bounded by the x-axis below, $f(x)$ above, and the given pair of vertical lines. **(3 marks)**

$$f(x) = x^{-3}, x = \frac{1}{2}, x = 1$$

$F(x) = -\frac{1}{2}x^{-2} + C$

$F(1) - F(\frac{1}{2})$

$\left[-\frac{1}{2}(1)^{-2}\right] - \left[-\frac{1}{2}(\frac{1}{2})^{-2}\right]$

$\left(-\frac{1}{2}\right) - (-2) \rightarrow 1.5$