Calculus Total____

Practice Final Exam #1 80

Name: _____

1. Determine the following limits, if they exist. (**12 marks**)

a) $\lim_{x\to 1} \dfrac{x-4}{x^2-6x+8}$

$b)\ \lim_{x\to 3} \dfrac{2x^2-6x}{x-3}$

c) $\lim_{x\to 10^+} \dfrac{x}{x-10}$

$d)\ \lim_{x\to 0} \dfrac{\sin^3 2x}{\sin^3 3x}$

$e)\ \lim_{x\to -2} \dfrac{x+2}{\sqrt{2-x}-\sqrt{-2x}}$

f) $\lim_{x\to\infty} \frac{2x+1}{4x-3}$

2. Determine the derivative of the following functions. **Do Not** simplify your answers. (**20 marks**)

a) $f(x) = (2x^3 - 1)^7$

b) $y = x^6 - 2x^3 + \sqrt{x} - 3$

c) $f(x) = \sqrt{x^2 - x - 6}$

d) $y = \frac{x}{(x-1)^2}$

e) $f(x) = (x^4 + 1)^3(1 - 2x)$

f) $y = \cos(-4x)$

g) $f(x) = 2e^x$

h) $y = x\ lnx - x$

i) $f(x) = \sqrt[3]{x}(1 + \cos x)^{10}$

j) $y = ln(2 - \cos x)$

3. Determine $\frac{dy}{dx}$ for $4x^3 - 6y^2 + 2y^2 - 6x = 0$ (**3 marks**)

4. Using the first derivative test, find the open intervals on which $f(x)$ is increasing or decreasing. Find the coordinates of any local extrema.
   $f(x) = x^3 - 3x + 2$ (**6 marks**)

5. Find the open intervals on which $f(x)$ is concave up or concave down. Find the coordinates of any inflection points
   $f(x) = 2x^3 + 24x^2 - 5x - 21$ (**5 marks**)

6. Determine the equations of all vertical and horizontal asymptotes of $f(x) = \frac{x^2}{(x+2)^2}$ (**3 marks**)

7. Solve any **<u>three</u>** of the following five problems (**15 marks**)
   a) If a ball is dropped from the top of the CN Tower, 550 m above the ground, then its height in metres after *t* seconds is
      $h = 550 - 5t^2, t \geq 0.$
      i)When does the ball hit the ground?

      ii)Find the velocity when the ball hits the ground.

   b) How fast is the edge length of a cube increasing when the volume of the cube is increasing at a rate of $144cm^3/sec$ and the edge length is 4cm?

c) A juice company is studying the most economical size for a 355ml juicebox. If a child's hand can only hold a box that is 4cm wide, what height and length of box uses the least material to make?

d) A farmer wants to fence two pens. If he has 300m of fence to use, what is the largest total pen size possible?

e) Two automobiles start from point A at the same time. One travels west at 80 miles/hour and the other travels north at 45 miles/hour. How fast is the distance between them increasing 3 hours later?

8. Determine the following indefinite integrals by sight **(6 marks)**

a) $\int (3 - 3x - x^2) dx$

b) $\int (3 \sin x + 4 \cos x) dx$

c) $\int \sqrt[3]{x^4} dx$

9. Determine the following indefinite integral by u substitution. **(3 marks)**

$\int \frac{1}{3x+8} dx$

10. Evaluate the following definite integrals. **(4 marks)**

a) $\int_{-3}^{3} (x-1)^2 dx$

b) $\int_{0}^{\frac{\pi}{2}} \cos x \, dx$

11. Find the area bounded by the x-axis below, $f(x)$ above, and the given pair of vertical lines. **(3 marks)**

$$f(x) = e^x, x = -2, x = 2$$