Calculus
Practice Final Exam #1

Total____
80

Name: ____

1. Determine the following limits, if they exist. **(12 marks)**

a) $\lim_{x \to 1} \frac{x-4}{x^2-6x+8}$ $\frac{1-4}{1^2-6(1)+8} = \frac{-3}{3} = -1$

b) $\lim_{x \to 3} \frac{2x^2-6x}{x-3}$ $\frac{2x\cancel{(x-3)}}{\cancel{x-3}} = 2x = 2(3) = 6$

c) $\lim_{x \to 10^+} \frac{x}{x-10}$

+ + + + + + + + + x
− − − 0 + + + + x−10
10
− − − 0 + + + + +

∞

d) $\lim_{x \to 0} \frac{\sin^3 2x}{\sin^3 3x}$ $\frac{\frac{(2x)^3}{(2x)^3}}{\frac{(3x)^3}{(3x)^3}} \cdot \frac{\frac{\sin^3 2x}{1}}{\frac{\sin^3 3x}{1}} = \frac{\frac{(2x)^3}{1}}{\frac{(3x)^3}{1}} \cdot \frac{\frac{\sin^3 2x}{(2x)^3}}{\frac{\sin^3 3x}{(3x)^3}} = \frac{(2x)^3}{(3x)^3} \cdot 1$

$= \frac{8}{27}$

e) $\lim_{x \to -2} \frac{x+2}{\sqrt{2-x}-\sqrt{-2x}}$ $\left(\frac{\sqrt{2-x}+\sqrt{-2x}}{\sqrt{2-x}+\sqrt{-2x}}\right) = \frac{(x+2)(\sqrt{2-x}+\sqrt{-2x})}{(2-x)-(-2x)} = \frac{\cancel{(x+2)}(\sqrt{2-x}+\sqrt{-2x})}{\cancel{2+x}}$

$= \sqrt{2-(-2)} + \sqrt{-2(-2)}$

$= \sqrt{4} + \sqrt{4}$

$= 4$

$f)\ \lim_{x \to \infty} \frac{2x+1}{4x-3}$

$\frac{2}{4} = \frac{1}{2}$

2. Determine the derivative of the following functions. **<u>Do Not</u>** simplify your answers. **(20 marks)**

a) $f(x) = (2x^3 - 1)^7$

$7(2x^3-1)^6(6x^2)$

$42x^2(2x^3-1)^6$

b) $y = x^6 - 2x^3 + \sqrt{x} - 3$

$6x^5 - 6x^2 + \frac{1}{2}x^{-\frac{1}{2}}$

c) $f(x) = \sqrt{x^2 - x - 6}$

$\frac{1}{2}(x^2-x-6)^{-\frac{1}{2}}(2x-1)$

d) $y = \frac{x}{(x-1)^2}$

$\frac{(1)(x-1)^2 - 2(x-1)(1)(x)}{(x-1)^4}$

e) $f(x) = (x^4+1)^3(1-2x)$

$3(x^4+1)^2(4x^3)(1-2x) + (-2)(x^4+1)^3$

f) $y = \cos(-4x)$

$-\sin(-4x)(-4)$

$4\sin(-4x)$

g) $f(x) = 2e^x$

$2e^x \ln e (1)$

h) $y = x\,lnx - x$

$(1)\ln x + \frac{1}{x}\ln e(1)(x) - 1$

$\ln x + \ln e - 1$

i) $f(x) = \sqrt[3]{x}(1+\cos x)^{10}$

$\left(\frac{1}{3}x^{-\frac{2}{3}}\right)(1+\cos x)^{10} + 10(1+\cos x)^9(-\sin x)(x^{\frac{1}{3}})$

j) $y = ln(2 - \cos x)$

$\frac{1}{2-\cos x} \ln e \sin x$

3. Determine $\frac{dy}{dx}$ for $4x^3 - 6y^2 + 2y^2 - 6x = 0$ **(3 marks)**

$12x^2 - 12yy' + 4yy' - 6 = 0$

$-12yy' + 4yy' = 6 - 12x^2$

$y'(-12y + 4y) = 6 - 12x^2$

$y' = \frac{6-12x^2}{-12y+4y}$

$y' = \frac{3-6x^2}{-4y}$

4. Using the first derivative test, find the open intervals on which $f(x)$ is increasing or decreasing. Find the coordinates of any local extrema.
$f(x) = x^3 - 3x + 2$ **(6 marks)**

$f'(x) = 3x^2 - 3$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x = 1 \quad x = -1$

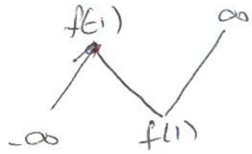
+ + + + + + + + 3

− − − − − 0 + + $(x-1)$

− − 0 + + + + + + $(x+1)$

−1   1

+ + 0 − − − − 0 + + $f'(x)$



max @ $f(-1) = 4$

min @ $f(1) = 0$

5. Find the open intervals on which $f(x)$ is concave up or concave down. Find the coordinates of any inflection points
$f(x) = 2x^3 + 24x^2 - 5x - 21$ **(5 marks)**

$f'(x) = 6x^2 + 48x - 5$

$f''(x) = 12x + 48$

$12x + 48 = 0$

$12(x+4) = 0$

$x = -4$

+ + + + + + + 12

− − − 0 + + $x+4$

−4

− − − 0 + + $f''(x)$

concave down $(-\infty, -4)$

concave up $(-4, \infty)$

IP @ $(-4, 255)$

6. Determine the equations of all vertical and horizontal asymptotes of $f(x) = \frac{x^2}{(x+2)^2}$ **(3 marks)**

V.A. @ $x = -2$

H.A @ $y = 1$

7. Solve any **three** of the following five problems **(15 marks)**

a) If a ball is dropped from the top of the CN Tower, 550 m above the ground, then its height in metres after *t* seconds is

$h = 550 - 5t^2, t \geq 0.$

$550 - 5t^2 = 0$
$-5t^2 = -550$
$t^2 = 110$
$t = 10.49$ sec

i)When does the ball hit the ground?

10.49 sec

ii)Find the velocity when the ball hits the ground.

$V = -10t$

$v(10.49) = -10(10.49)$

$= -104.9$ m/s

b) How fast is the edge length of a cube increasing when the volume of the cube is increasing at a rate of $144cm^3/sec$ and the edge length is 4cm?

$V = s^3$

$V' = 3s^2 s'$

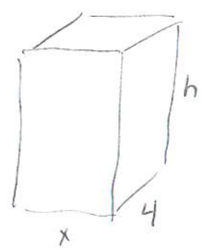
$144 = 3(4)^2 s'$

$144 = 48s'$

$3 = s'$

It is increasing at a rate of 3cm/sec

c) A juice company is studying the most economical size for a 355ml juicebox. If a child's hand can only hold a box that is 4cm wide, what height and length of box uses the least material to make?



$V = l \cdot w \cdot h$

$V = 4xh$

$355 = 4xh$

$\frac{355}{4x} = h$

$SA = 2(4x) + 2(4h) + 2(xh)$

$SA = 8x + 8\left(\frac{355}{4x}\right) + 2x\left(\frac{355}{4x}\right)$

$SA = 8x + \frac{710}{x} + \frac{355x}{2x}$

$SA = \frac{16x^2}{2x} + \frac{1420}{2x} + \frac{355x}{2x}$

$SA = \frac{16x^2 + 355x + 1420}{2x}$

$SA' = \frac{(32x + 355)(2x) - (2)(16x^2 + 355x + 1420)}{4x^2}$

$SA' = \frac{32x^2 - 2840}{4x^2} \rightarrow 32x^2 - 2840 = 0$

$x = 0$

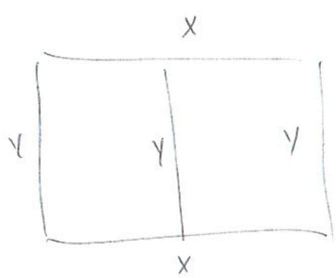
$32x^2 = 2840$

$x^2 = 88.75$

$x = 9.42$

length is 9.42cm height is 9.42cm

d) A farmer wants to fence two pens. If he has 300m of fence to use, what is the largest total pen size possible?



$Fence = 2x + 3y$

$300 = 2x + 3y$

$300 - 2x = 3y$

$\frac{300 - 2x}{3} = y$

$A = xy$

$A = x\left(\frac{300 - 2x}{3}\right)$

$A = 100x - \frac{2x^2}{3}$

$A' = 100 - \frac{4}{3}x$

$100 - \frac{4}{3}x = 0$

$100 = \frac{4}{3}x$

$75 = x$

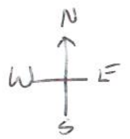
$50 = y$

largest pen is $\frac{1}{2}$ of (50×75)

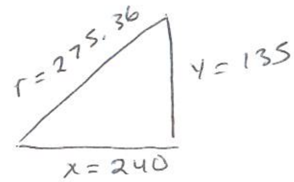
or

$1875m^2$

e) Two automobiles start from point A at the same time. One travels west at 80 miles/hour and the other travels north at 45 miles/hour. How fast is the distance between them increasing 3 hours later?



at 3hours



$$x^2 + y^2 = r^2$$

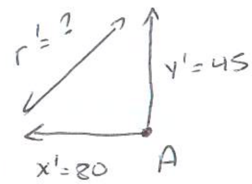
$$2xx' + 2yy' = 2rr'$$

$$2(240)(80) + 2(135)(45) = 2(275.36)r'$$

$$38400 + 12150 = 550.72\,r'$$

$$\frac{50550}{550.72} = r'$$

$$91.79 \text{ km/h} = r'$$



8. Determine the following indefinite integrals by sight **(6 marks)**

a) $\int (3 - 3x - x^2)dx$

$$3x - \frac{3}{2}x^2 - \frac{1}{3}x^3 + C$$

b) $\int (3\sin x + 4\cos x)dx$

$$-3\cos x + 4\sin x + C$$

c) $\int \sqrt[3]{x^4}dx$ $= \int x^{\frac{4}{3}}dx$

$$\frac{3}{7}x^{\frac{7}{3}} + C$$

9. Determine the following indefinite integral by u substitution. **(3 marks)**

$$\int \frac{1}{3x+8}\,dx \longrightarrow \quad u = 3x+8$$

$du = 3\,dx$

$\int \frac{1}{u} \cdot \frac{1}{3}\,du \qquad \frac{1}{3}du = dx$

$\frac{1}{3}\int \frac{1}{u}\,du$

$\frac{1}{3}\ln|u| + C$

$\frac{1}{3}\ln|3x+8| + C$

10. Evaluate the following definite integrals. **(4 marks)**

a) $\int_{-3}^{3}(x-1)^2\,dx$

$F(x) = \frac{1}{3}x^3 - x^2 + x + C$

$F(3) - F(-3)$

$\left[\frac{1}{3}(3)^3 - (3)^2 + (3) + C\right] - \left[\frac{1}{3}(-3)^3 - (-3)^2 + (-3) + C\right]$

$(9 - 9 + 3 + C) - (-9 - 9 - 3 + C)$

$(3 + C) - (-21 + C)$

24

b) $\int_{0}^{\frac{\pi}{2}} \cos x\,dx$

$F(x) = \sin x + C$

$F\left(\frac{\pi}{2}\right) - F(0)$

$\left[\sin\left(\frac{\pi}{2}\right) + C\right] - \left[\sin(0) + C\right]$

$(1 + C) - (0 + C)$

1

11. Find the area bounded by the x-axis below, $f(x)$ above, and the given pair of vertical lines. **(3 marks)**

$$f(x) = e^x, x = -2, x = 2$$

$F(x) = e^x$

$F(2) - F(-2)$

$e^2 - e^{-2}$

$7.3891 - 0.1353$

7.2537