Calculus 30
Chapter 4 – Differentiation Practice Test

1. Find the derivative of the following using the Definition of a Derivative Formula.

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

a) $f(x) = 3x^2 - 4x - 2$

$= \lim_{h \to 0} \frac{(3(x+h)^2 - 4(x+h) - 2) - (3x^2 - 4x - 2)}{h}$

$= \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 4x - 4h - 2 - 3x^2 + 4x + 2}{h}$

$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6hx + 3h^2 \cancel{-4x} - 4h \cancel{-2} \cancel{-3x^2} \cancel{+4x} \cancel{+2}}{h}$

$= \lim_{h \to 0} \frac{3h^2 + 6hx - 4h}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(3h + 6x - 4)}{\cancel{h}}$ $= 3(0) + 6x - 4$ $= \boxed{6x - 4}$

b) $f(x) = 4\sqrt{x}$

$= \lim_{h \to 0} \frac{4\sqrt{x+h} - 4\sqrt{x}}{h} \cdot \frac{4\sqrt{x+h} + 4\sqrt{x}}{4\sqrt{x+h} + 4\sqrt{x}}$

$= \lim_{h \to 0} \frac{16(x+h) \cancel{+ 16\sqrt{x}\sqrt{x+h}} \cancel{- 16\sqrt{x}\sqrt{x+h}} - 16x}{h(4\sqrt{x+h} + 4\sqrt{x})}$

$= \lim_{h \to 0} \frac{\cancel{16x} + 16\cancel{h} \cancel{-16x}}{\cancel{h}(4\sqrt{x+h} + 4\sqrt{x})}$

$= \lim_{h \to 0} \frac{16}{4(\sqrt{x+h} + \sqrt{x})}$

$= \frac{4}{\sqrt{x+0} + \sqrt{x}}$

$= \frac{4}{2\sqrt{x}}$

$= \frac{2}{\sqrt{x}}$

Find the Derivative for each using the Power Rule, Product Rule, Quotient Rule or Chain rule. DO NOT SIMPLIFY!!

1. $f(x) = -4x^3 + 6x^2 - 3x + 2$

$f'(x) = -12x^2 + 12x - 3$

2. $f(x) = x^4 - 6x^2 - 8$

$f'(x) = 4x^3 - 12x$

3. $f(x) = (-3x^{-2})(2x^5 + 4x^3)$

$f'(x) = 6x^{-3}(2x^5 + 4x^3) + (10x^4 + 12x^2)(-3x^{-2})$

OR

$f(x) = -6x^3 - 12x$

$f'(x) = -18x^2 - 12$

4. $f(x) = (\sqrt{x})(x^2 - 6x + 5)$

$f(x) = (x^{\frac{1}{2}})(x^2 - 6x + 5)$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}(x^2 - 6x + 5) + (2x - 6)(x^{\frac{1}{2}})$

OR

$f(x) = x^{\frac{5}{2}} - 6x^{\frac{3}{2}} + 5x^{\frac{1}{2}}$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + \frac{5}{2}x^{-\frac{1}{2}}$

5. $f(x) = \frac{x+4}{x-4}$

$f'(x) = \frac{(1)(x-4) - 1(x+4)}{(x-4)^2}$

OR

$f(x) = (x+4)(x-4)^{-1}$

$f'(x) = (1)(x-4)^{-1} + (-1)(x-4)^{-2}(1)(x+4)$

6. $f(x) = \frac{\sqrt{x}+2}{\sqrt{x}-2}$ $\quad \frac{x^{\frac{1}{2}}+2}{x^{\frac{1}{2}}-2}$

$$f'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}}(x^{\frac{1}{2}}-2) - \frac{1}{2}x^{-\frac{1}{2}}(x^{\frac{1}{2}}+2)}{(x^{\frac{1}{2}}-2)^2}$$

OR

$$f(x) = (x^{\frac{1}{2}}+2)(x^{\frac{1}{2}}-2)^{-1}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}(x^{\frac{1}{2}}-2)^{-1} + (-1)(x^{\frac{1}{2}}-2)^{-2}(\tfrac{1}{2}x^{-\frac{1}{2}})(x^{\frac{1}{2}}+2)$$

7. $f(x) = (x^2+6x-3)^4$

$$f'(x) = 4(x^2+6x-3)^3(2x+6)$$

8. $(x) = \frac{\sqrt{3-x}}{x^4}$ $= \frac{(3-x)^{\frac{1}{2}}}{x^4}$

$$f'(x) = \frac{\frac{1}{2}(3-x)^{-\frac{1}{2}}(-1)(x^4) - 4x^3(3-x)^{\frac{1}{2}}}{x^8}$$

OR

$$f(x) = x^{-4}(3-x)^{\frac{1}{2}}$$

$$f'(x) = -4x^{-5}(3-x)^{\frac{1}{2}} + \frac{1}{2}(3-x)^{-\frac{1}{2}}(-1)(x^{-4})$$

9. $f(x) = (x-2)\sqrt{x^2-3x-1}$

$$f(x) = (x-2)(x^2-3x-1)^{\frac{1}{2}}$$

$$f'(x) = (1)(x^2-3x-1)^{\frac{1}{2}} + \frac{1}{2}(x^2-3x-1)^{-\frac{1}{2}}(2x-3)(x-2)$$

10. Find an equation of the tangent line to the curve at the given point.

$f(x) = \frac{1}{6}x^3 - \frac{3}{4}x^2 + 2x - 7 \quad @\ x = 4$

$f'(x) = \frac{1}{2}x^2 - \frac{3}{2}x + 2$

$f'(4) = \frac{1}{2}(4)^2 - \frac{3}{2}(4) + 2$

$= 8 - 6 + 2$

$= 4$

$f(4) = \frac{1}{6}(4)^3 - \frac{3}{4}(4)^2 + 2(4) - 7$

$= \frac{64}{6} - 12 + 8 - 7$

$= -\frac{1}{3}$

$(4, -\frac{1}{3})$

$y + \frac{1}{3} = 4(x-4)$

$y + \frac{1}{3} = 4x - 16$

$y = 4x - 16\frac{1}{3}$

11. Find $\frac{dy}{dx}$

$\frac{x}{y} = 12$

$(x)(y^{-1}) = 12$

$(1)(y^{-1}) + (-y^{-2})\frac{dy}{dx}(x) = 0$

$y^{-1} = xy^{-2}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y^{-1}}{xy^{-2}}$

$\frac{dy}{dx} = \frac{y}{x}$

b) $y^2 - 3xy = -2$

$\frac{d(y^2)}{dx} - \frac{d(3xy)}{dx} = \frac{d(-2)}{dx}$

$2y\frac{dy}{dx} - [(3)(y) + (1)\cdot\frac{dy}{dx}(3x)] = 0$

$2y\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y$

$\frac{dy}{dx}(2y - 3x) = 3y$

$\frac{dy}{dx} = \frac{3y}{2y-3x}$

c) $x^2 - xy^2 + y^2 = 4$

$\frac{d(x^2)}{dx} - \frac{d(xy^2)}{dx} + \frac{d(y^2)}{dx} = \frac{d(4)}{dx}$

$2x - [(1)(y^2) + 2y\frac{dy}{dx}(x)] + 2y\frac{dy}{dx} = 0$

$-2xy\frac{dy}{dx} + 2y\frac{dy}{dx} = y^2 - 2x$

$\frac{dy}{dx}(-2xy + 2y) = y^2 - 2x$

$\frac{dy}{dx} = \frac{y^2-2x}{-2xy+2y} = \frac{y^2-2x}{-2y(x-1)}$

12. Determine the derivative. Answer must be in **simplified form.**

a) $f(x) = (x^3 - 7x)(9x^2 + 3)$

$f'(x) = (3x^2 - 7)(9x^2+3) + (18x)(x^3 - 7x)$

$= 27x^4 + 9x^2 - 63x^2 - 21 + 18x^4 - 126x^2$

$= 45x^4 - 180x^2 - 21$

$= 3(15x^4 - 60x^2 - 7)$

b) $y = (2x+1)^3(x^2-5)^5$

$\frac{dy}{dx} = 3(2x+1)^2(2)(x^2-5)^5 + 5(x^2-5)^4(2x)(2x+1)^3$

$= 2(2x+1)^2(x^2-5)^4[3(x^2-5) + 5x(2x+1)]$

$= 2(2x+1)^2(x^2-5)^4(3x^2 - 15 + 10x^2 + 5x)$

$= 2(2x+1)^2(x^2-5)^4(13x^2 + 5x - 15)$

c) $f(x) = \frac{x^3 - 4x^2}{x+1}$

$f'(x) = \frac{(3x^2 - 8x)(x+1) - (1)(x^3 - 4x^2)}{(x+1)^2}$

$= \frac{3x^3 + 3x^2 - 8x^2 - 8x - x^3 + 4x^2}{(x+1)^2}$

$= \frac{2x^3 - x^2 - 8x}{(x+1)^2}$

$= \frac{x(2x^2 - x - 8)}{(x+1)^2}$

$(2x+1)^{\frac{1}{2}}$

d) $y = (x-1)^2(\sqrt{2x+1})$

$$\frac{dy}{dx} = 2(x-1)(1)(2x+1)^{\frac{1}{2}} + \frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)(x-1)^2$$

$$= 2(x-1)(2x+1)^{-\frac{1}{2}}[(2x+1) + \frac{1}{2}(x-1)]$$

$$= 2(x-1)(2x+1)^{-\frac{1}{2}}(2x+1 + \frac{1}{2}x - \frac{1}{2})$$

$$= 2(x-1)(2x+1)^{-\frac{1}{2}}(\frac{5}{2}x + \frac{1}{2})$$

$$= 2(x-1)(2x+1)^{-\frac{1}{2}}\frac{1}{2}(5x+1)$$

$$= (x-1)(2x+1)^{-\frac{1}{2}}(5x+1) \quad \text{OR} \quad \frac{(x-1)(5x+1)}{(2x+1)^{\frac{1}{2}}}$$

13. Find the coordinates the point(s) on the graph of $f(x) = x^3 - 3x^2 - 6x + 10$ at which the slope of the tangent line is 3.

$f'(x) = 3x^2 - 6x - 6$

$\therefore 3 = 3x^2 - 6x - 6$

$0 = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = 3(x-3)(x+1)$

$x = 3 \qquad x = -1$

$f(3) = 3^3 - 3(3)^2 - 6(3) + 10$

$= 27 - 27 - 18 + 10$

$= -8$

$f(-1) = (-1)^3 - 3(-1)^2 - 6(-1) + 10$

$= -1 - 3 + 6 + 10$

$= 12$

$(3, -8)$ and $(-1, 12)$

14. The line y = 4 is tangent to the function y = x^2 + ax + b at the point (5,4). Determine the values of a and b.

$\frac{dy}{dx} = 2x + a$

$0 = 2x + a$

$0 = 2(5) + a$

$\boxed{a = -10}$

$y = x^2 - 10x + b$

$4 = 5^2 - 10(5) + b$

$4 = 25 - 50 + b$

$4 = -25 + b$

$\boxed{29 = b}$