Answer Key

Calculus 30
Chapter 7 – Differentiating Transcendental Functions
Practice Assessment

PART A: Logarithmic and Exponential Functions

1. Write the following as a single logarithm.

   a. $ln\,6 + ln\,4 - \ln 3$

   $\ln \frac{6 \cdot 4}{3} = \ln 8$

   b. $3ln\,a - 2ln\,b$

   $= \ln a^3 - \ln b^2$

   $= \ln \frac{a^3}{b^2}$

2. Solve the following equations. Answers must be in exact form, not decimals.

   a. $e^{2x} = 5$

   $\log_e 5 = 2x$

   $\ln 5 = 2x$

   $x = \frac{\ln 5}{2}$

   b. $\ln x + 3 = 0$

   $\ln x = -3$

   $\log_e x = -3$

   $e^{-3} = x$

3. Find the derivative of the following functions:

   a. $y = \ln(3x^2 + 5x - 2)$

   $\frac{dy}{dx} = \frac{1}{3x^2+5x-2} \cdot 6x+5$

   $= \frac{6x+5}{3x^2+5x-2}$

   b. $y = 3^{4x-3}$

   $\frac{dy}{dx} = 3^{4x-3} \cdot \ln 3\,(4)$

c. $y = x^2 \ln 3x$

$$\frac{dy}{dx} = 2x(\ln 3x) + \frac{1}{3x}(3)(x^2)$$
$$= 2x(\ln 3x) + x$$

d. $y = \frac{e^{x^2}}{x^3}$

$$\frac{dy}{dx} = \frac{e^{x^2}(2x)(x^3) - 3x^2(e^{x^2})}{(x^3)^2}$$
$$= \frac{e^{x^2}(2x^4) - 3x^2(e^{x^2})}{x^6}$$

e. $f(x) = \log_4(5x+6)$

$$f'(x) = \frac{1}{5x+6} \cdot \log_4 e \cdot 5$$

f) $f(x) = e^{(7x-2)^3}$

$$f'(x) = e^{(7x-2)^3} \cdot 3(7x-2)^2(7)$$

4. An ingot of steel is removed from a furnace. Its surface temperature, T, in degress Celsius, t minutes after removal is $T = 1000e^{\frac{-t}{12}} + 40$. Find the rate of change of the temperature after 10 minutes. Interpret this result.

$$T'(t) = 1000\,e^{-\frac{t}{12}}\left(-\frac{1}{12}\right)$$
$$= -\frac{1000}{12}\,e^{-\frac{t}{12}}$$

$$T'(10) \approx -36^\circ$$

PART B: Trigonometric Functions

5. Solve the following Limits

a. $\lim_{x \to 0} \frac{\sin 10x}{10x}$ $= 1$

b. $\lim_{x \to 0} \frac{\sin x}{3x}$ $= \lim_{x \to 0} \frac{1}{3}\left(\frac{\sin x}{x}\right)$

$= \frac{1}{3} \cdot 1$

$= \frac{1}{3}$

6. Determine the derivative of the following. Answers **do not** need to be simplified.

a. $y = \tan\left(3x + \frac{\pi}{4}\right)$

$\frac{dy}{dx} = \sec^2\left(3x + \frac{\pi}{4}\right)(3)$

b. $y = \cos^3\sqrt{x}$ $\quad (\cos x^{\frac{1}{2}})^3$

$\frac{dy}{dx} = 3(\cos x^{\frac{1}{2}})^2(-\sin x^{\frac{1}{2}})(\frac{1}{2}x^{-\frac{1}{2}})$

c. $y = (\sin x + \sec x)^2$

$\frac{dy}{dx} = 2(\sin x + \sec x)(\cos x + \sec x \tan x)$

d. $y = x^2 \cot 2x$

$\frac{dy}{dx} = 2x(\cot 2x) + (-\csc^2 2x)(2)(x^2)$

7. Determine the slope of the tangent line to

$y = sin^2 x$ at $x = \frac{\pi}{6}$.

$y = (\sin x)^2$

$\frac{dy}{dx} = 2\sin x \cdot \cos x$

$= 2\sin(\frac{\pi}{6}) \cdot \cos(\frac{\pi}{6})$

$= 2(\frac{1}{2})(\frac{\sqrt{3}}{2})$

$= \frac{\sqrt{3}}{2}$

8. Use implicit differentiation to solve for dy/dx if $x = 3y + siny$.

$\frac{d(x)}{dx} = \frac{d(3y)}{dx} + \frac{d(\sin y)}{dx}$

$1 = 3\frac{dy}{dx} + \cos y \frac{dy}{dx}$

$1 = \frac{dy}{dx}(3 + \cos y)$

$\frac{dy}{dx} = \frac{1}{3 + \cos y}$

9. Over the interval (0,π), determine the values of x for which the tangent line to the curve of $f(x) = sin2x - 2sinx$ is horizontal. $=0$ It will be helpful to use the identity $cos2x = 2cos^2 x - 1$ at some point in your solution.

$f'(x) = \cos 2x \cdot 2 - 2(\cos x)$

$2\cos 2x - 2\cos x = 0$

$\cos 2x - \cos x = 0$

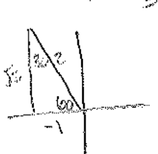
$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$ $\qquad 2x^2 - x - 1$

$(2\cos x + 1)(\cos x - 1) = 0$ $\qquad (2x + 1)(x - 1)$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$



$2\cos x + 1 = 0$ $\qquad \cos x - 1 = 0$

$2\cos x = -1$ $\qquad \cos x = 1$

$\cos x = \frac{-1}{2}$ $\qquad x = 0$

$x = \frac{2\pi}{3}$

$\pi - \frac{\pi}{3}$