Calculus 30
Chapter 6 – Derivative Applications Practice Assessment

Name: ________________

1. A particle moves along the x-axis so that its position in metres, after t seconds, is given by the function: $s = 3t^2 + 4t + 21$

a) Find the average velocity between 3 seconds and 5 seconds.

$v_{average} = \frac{\Delta s}{\Delta t} = \frac{s(5) - s(3)}{5-3} = \frac{116 - 60}{2} = \frac{56}{2} = 28 m/s$

b) Find the instantaneous velocity at 4 seconds.

$s' = v = 6t + 4$

$v(4) = 6(4) + 4$

$= 24 + 4$

$= 28 m/s$

2. A particle moves along the x-axis so that its position in metres, after t seconds, is given by the function: $s = -5t^3 + 2t^2 - 8t$

a) Find the average velocity between 2 seconds and 4 seconds.

$v_{average} = \frac{\Delta s}{\Delta t} = \frac{s(4) - s(2)}{4-2} = \frac{(-320) - (-48)}{2} = \frac{-272}{2} = -136 m/s$

b) Find the instantaneous velocity at 3 seconds.

$s' = v = -15t^2 + 4t - 8$

$v(3) = -15(3)^2 + 4(3) - 8$

$= -131 m/s$

3. The motion of a particle is described by the position function:

$$s = t^3 - 9t^2 + 24t, \quad t \leq 0$$

a) Find the velocity and acceleration at time t.

$s' = v = 3t^2 - 18t + 24$

$s'' = v' = a = 6t - 18$

b) Find the velocity and acceleration at 2 seconds.

$v(2) = 3(2)^2 - 18(2) + 24$

$= 0\ m/s$

$a(2) = 6(2) - 18$

$= -6\ m/s^2$

c) Find the position of the particle when its velocity is 60 m/s.

$v = 3t^2 - 18t + 24$

$60 = 3t^2 - 18t + 24$

$0 = 3t^2 - 18t - 36$

$0 = 3(t^2 - 6t - 12)$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-12)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 + 48}}{2}$

$x = 7.58$ or $x = \cancel{-1.58}$

7.58 seconds

$s = (7.58)^3 - 9(7.58)^2 + 24(7.58)$

$s = 100.33m$

d) When is the particle at rest?

$0 = 3t^2 - 18t + 24$

$0 = 3(t^2 - 6t + 8)$

$0 = 3(t-2)(t-4)$

$t - 2 = 0 \qquad t - 4 = 0$

$t = 2 \qquad t = 4$

particle is at rest at 2sec & 4sec

4. If a ball is thrown upward from a 150m high cliff, then its height after t seconds, before it hits the ground, is:

$$h = 150 + 12t - 4.9t^2$$

a) Find the initial velocity of the ball.

$v = 12 - 9.8t$

$v(0) = 12 - 9.8(0)$

$= 12 m/s$

b) When does the ball hit the ground?

$$x = \frac{-12 \pm \sqrt{12^2 - 4(-4.9)(150)}}{2(-4.9)} = \frac{-12 \pm \sqrt{3084}}{-9.8}$$

$6.89 sec$

c) What is the velocity when it hits the ground?

$v(6.89) = 12 - 9.8(6.89)$

$= -55.53 m/s$

5. Two numbers have a product of 16. Find these numbers if they sum of their squares is to be a minimum.

$xy = 16$

$y = \frac{16}{x}$

$x^2 + y^2 = min$

$x^2 + \left(\frac{16}{x}\right)^2 = min$

$\frac{x^4 + 256}{x^2} = min$

$\frac{(4x^3)(x^2) - (2x)(x^4 + 256)}{x^4} = m'$

$\frac{4x^5 - 2x^5 - 516x}{x^4} = m'$

$\frac{2x^5 - 516x}{x^4} = m'$

$\frac{2x^4 - 516}{x^3} = m'$

critical $x = 4$

$x = -4$

$x = 0$

+ + 0 - - - - - - 0 + + $\quad 2x^4 - 516$

- - - - 0 + + + + + $\quad x^3$

-4 $\quad$ 0 $\quad$ 4

m'

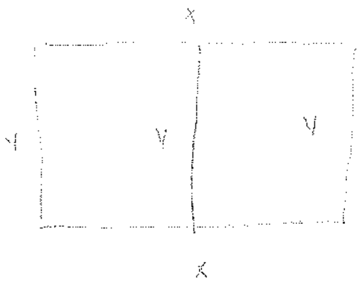
- 0 + + 0 0 - - - 0 + +

$x = -4$ & $y = -4$

or

$x = 4$ & $y = 4$

6. A farmer has 600 metres of fencing with which to make two adjacent rectangular pens. What is the maximum total area that can be enclosed? Be sure to list all known and unknown values at the start of the question.



$2x + 3y = 600$

$3y = 600 - 2x$

$y = 200 - \frac{2}{3}x$

$xy =$ Area

$x\left(200 - \frac{2}{3}x\right) =$ Area

$200x - \frac{2}{3}x^2 =$ Area

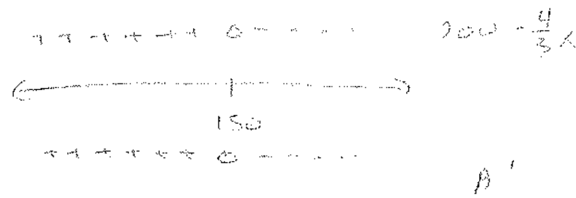
$200 - \frac{4}{3}x = A'$

$0'$

$200 - \frac{4}{3}x = 0$

$200 = \frac{4}{3}x$

$150 = x$ critical #



max

$2(150) + 3y = 600$

$300 + 3y = 600$

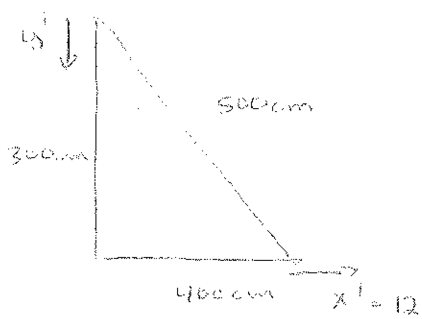
$3y = 300$

$y = 100$

Maximum Area is (150×100)

$= 15000\,m^2$

7. The base of a 5m ladder slides along the ground away from a building at a rate of 12cm/s. At what rate is the top of the ladder coming down the wall when the base of the ladder is 4m from the wall? Be sure to list all known and unknown values at the start of the question.



$$a^2 + b^2 + c^2$$

$$2aa' + 2bb' = 2cc'$$

$$2(400)(12) + 2(300)b' = 2(500)(0)$$

$$9600 + 600b' = 0$$

$$600b' = -9600$$

$$b' = -16$$

The top of the ladder is coming down at 16cm/sec.

8. A storage box with square ends and no open sides is to be built to have a volume of 50 m^3. If the material for the square ends is \$80/m^2 and the material for the rectangular sides is \$200/m^2, find the dimensions to minimize its cost.

$V = 50m^3$

$V = lwh$

$50 = lwh$

$50 = x \cdot x \cdot y$

$50 = x^2 y$

$y = \frac{50}{x^2}$

$S.A = 2x^2(80) + 4xy(200)$

$S.A = 2x^2(80) + 4x\left(\frac{50}{x^2}\right)(200)$

$S.A = 160x^2 + \frac{40000}{x}$

$S.A = 160x^2 + 40000x^{-1}$

$SA' = 320x - 40000x^{-2}$

$SA' = 320x^{-2}(x^3 - 125) \rightarrow$ difference of cubes

$0 = 320x^{-2}(x-5)(x^2+5x+25)$

$x = 5$

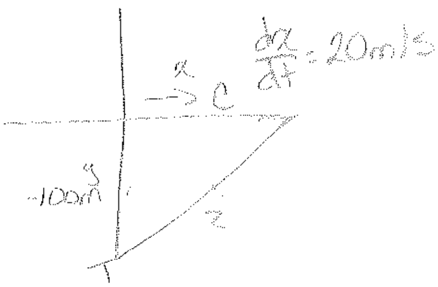
$y = \frac{50}{5^2}$

$y = \frac{50}{25}$

$y = 2$

5m by 5m by 2m

9. A truck is parked 100 m directly south of an intersection. A car is travelling east at 20 m/s. At what rate is the distance between the car and the truck increasing 12 seconds after the car has passed through the intersection?



Find $\left.\frac{dz}{dt}\right|_{t=12s}$

$x = 12 \cdot 20$

$x = 240m$

$x^2 + y^2 = z^2$

$240^2 + (-100)^2 = z^2$

$600 + 10\,000 = z^2$

$67600 = z^2$

$z = 260$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

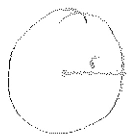
$240(20) + (-100)(0) = 260\frac{dz}{dt}$

$4800 = 260\frac{dz}{dt}$

$\frac{dz}{dt} = 18.46 m/s$

10. A metal ball bearing is being heated up. That causes the radius to grow 2 mm/min. At what rate is the volume of the ball increasing when the radius is 15 mm? ($V = \frac{4}{3}\pi r^3$)

Find $\left.\frac{dV}{dt}\right|_{r=15}$



$\frac{dr}{dt} = 2$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (15)^2 (2)$

$\frac{dV}{dt} = 1800\pi$

$\frac{dV}{dt} = 5654.87 \text{ mm}^3/\text{min}$