Calculus 30: Chapter 5 Exam Review

1. Determine f'(x) and f''(x) for the following functions

   a) $y = 6x^4 + 5x^3 - 8x$

   $\frac{dy}{dx} = 24x^3 + 15x^2 - 8$

   $\frac{d^2y}{dx^2} = 72x^2 + 30x$

   b) $f(x) = \sqrt[3]{3x^2 - 4} = (3x^2-4)^{\frac{1}{3}}$

   $f'(x) = \frac{1}{3}(3x^2-4)^{-\frac{2}{3}}(6x)$

   $= 2x(3x^2-4)^{-\frac{2}{3}}$

   $= \frac{2x}{(3x^2-4)^{\frac{2}{3}}}$

   $f''(x) = \frac{2(3x^2-4)^{\frac{2}{3}} - \frac{2}{3}(3x^2-4)^{-\frac{1}{3}}(6x)(2x)}{(3x^2-4)^{\frac{4}{3}}}$

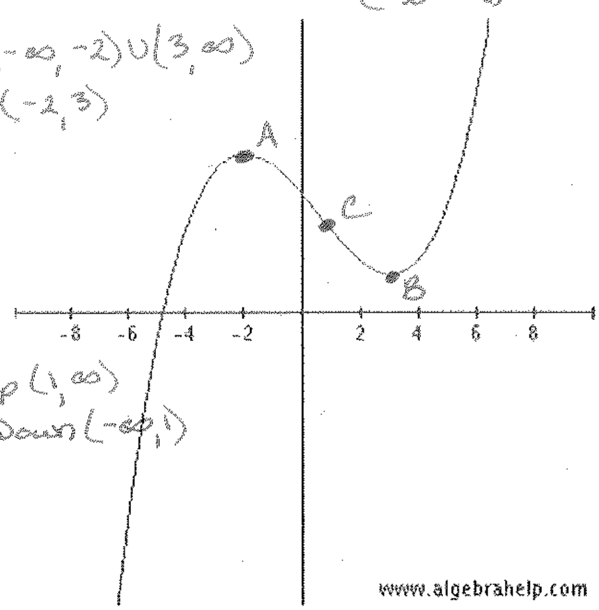
   $= \frac{2(3x^2-4)^{-\frac{1}{3}}[3x^2-4-4x^2]}{(3x^2-4)^{\frac{4}{3}}}$

   $= \frac{2(-x^2-4)}{(3x^2-4)^{\frac{5}{3}}}$

   $= \frac{-2(x^2+4)}{(3x^2-4)^{\frac{5}{3}}}$

2. For the given graph:

   - State the intervals of Increase and decrease. Increase $(-\infty,-2)\cup(3,\infty)$ Decrease $(-2,3)$
   - Put a dot on the relative Maximum point and label it with a letter A
   - Put a dot on the relative Minimum point and label it with a letter B
   - State the intervals where the function is concave up and concave down. Concave Up $(1,\infty)$ Concave Down $(-\infty,1)$
   - Put a dot on the point of Inflection and label it with a letter C



3. Without graphing, determine the absolute extrema of the following function in the given interval
$f(x) = 4x^3 - 12x - 5\ ; x \in [-3,3]$

$f'(x) = 12x^2 - 12 = 0$

$12(x^2-1) = 0$

$x = 1 \quad x = -1$

$f(-1) = 3$

$f(1) = -13$

$f(-3) = -108+36-5 = -77$

$f(3) = 108-36-5 = 67$

Absolute Max $(3, 67)$

Absolute Min $(-3, -77)$

4. For the following function determine
   a) The critical numbers
   b) The intervals in which the function is increasing/decreasing
   c) The intervals in which the function is concave up/down
   d) The coordinates of any relative extrema

i) $y = x^3 + 6x^2 - 15x + 90$

$\frac{dy}{dx} = 3x^2 + 12x - 15 = 0$

$3(x^2+4x-5) = 0$

$3(x+5)(x-1) = 0$

a) Critical Values $x = -5 \quad x = 1$

x	$(-\infty,-5)$	-5	$(-5,1)$	1	$(1,\infty)$
$f'(x)$	$+$	0	$-$	0	$+$
	$\nearrow$	$-$	$\searrow$	$-$	$\nearrow$

d) local max $(-5, 190)$

local min $(1, 82)$

b) Increasing $(-\infty,-5)\cup(1,\infty)$

Decreasing $(-5, 1)$

$\frac{d^2y}{dx^2} = 6x + 12 = 0$

$x = -2$

x	$(-\infty,-2)$	-2	$(-2,\infty)$
$f''(x)$	$-$	0	$+$

c) Concave up $(-2,\infty)$

Concave down $(-\infty, -2)$

ii) $y = 2x^3 - x^4$

$$\frac{dy}{dx} = 6x^2 - 4x^3 = 0$$

$$2x^2(3 - 2x) = 0$$

a) Critical Values $x = 0$ $x = \frac{3}{2}$

x	$(-\infty, 0)$	0	$(0, \frac{3}{2})$	$\frac{3}{2}$	$(\frac{3}{2}, \infty)$
$f'(x)$	$+$	0	$+$	0	$-$

b) Increasing $(-\infty, 0) \cup (0, \frac{3}{2})$

Decreasing $(\frac{3}{2}, \infty)$

d) local max $(\frac{3}{2}, +\frac{27}{16})$

$$f''(x) = 12x - 12x^2 = 0$$

$$12x(1 - x) = 0$$

$x = 0$ $x = 1$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f''(x)$	$-$	0	$+$	0	$-$

c) Concave up $(0, 1)$

Concave down $(-\infty, 0) \cup (1, \infty)$

iii) $f(x) = \frac{x^2}{x+2}$

$$f'(x) = \frac{2x(x+2) - x^2}{(x+2)^2}$$

$$= \frac{2x^2 + 4x - x^2}{(x+2)^2}$$

$$= \frac{x^2 + 4x}{(x+2)^2}$$

$$= \frac{x(x+4)}{(x+2)^2} = 0$$

a) Critical Values $x = 0$ $x = -4$

Asymptote $x = -2$

x	$(-\infty, -4)$	-4	$(-4, -2)$	-2	$(-2, 0)$	0	$(0, \infty)$
$f'(x)$	$+$	0	$-$	$\vdots$	$-$	0	$+$

b) Increasing $(-\infty, -4) \cup (0, \infty)$

Decreasing $(-4, -2) \cup (-2, 0)$

$$f''(x) = \frac{\overset{2(x+2)}{(2x+4)}(x+2)^2 - 2(x+2)(x^2+4x)}{(x+2)^4}$$

$$= \frac{2(x+2)[(x+2)(x+2) - (x^2+4x)]}{(x+2)^4}$$

$$= \frac{2\cancel{(x+2)}(x^2 + 4x + 4 - x^2 - 4x)}{(x+2)^{\cancel{4}\,3}}$$

$$= \frac{2(4)}{(x+2)^3} = \frac{8}{(x+2)^3}$$ no critical values

x	$(-\infty, -2)$	-2	$(-2, \infty)$
$f''(x)$	$-$	$\vdots$	$+$

c) Concave up $(-2, \infty)$

Concave down $(-\infty, -2)$

d) local max $(-4, -8)$

local min $(0, 0)$

5. For the following function, find the following.
   a) a sign analysis of $f'(x)$
   b) the intervals on which $f(x)$ is increasing and/or decreasing
   c) the critical numbers
   d) the relative extrema
   e) a sign analysis of $f''(x)$
   f) the intervals on which $f(x)$ is concave up and concave down
   g) the coordinates of any inflection points
   h) the $x$and y intercepts
   i) the equations of any vertical and horizontal asymptotes
   j) a careful sketch of the function

i) $y = \frac{x}{x^2-1}$

$$\frac{dy}{dx} = \frac{(x^2-1) - 2x(x)}{(x^2-1)^2}$$

$$= \frac{x^2-1-2x^2}{(x^2-1)^2}$$

$$= \frac{-x^2-1}{(x^2-1)^2}$$

$$= \frac{-(x^2+1)}{(x^2-1)^2} = 0$$

1c) No critical values

Asymptotes $x=1$ $x=-1$

a)

x	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,\infty)$
$f'(x)$	$-$	$\vdots$	$-$	$\vdots$	$-$

b) Increasing never
Decreasing $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

d) No local max or min

$$\frac{d^2y}{dx^2} = \frac{-2x(x^2-1)^2 - 2(x^2-1)(2x)(-x^2-1)}{(x^2-1)^4}$$

$$= \frac{-2x(x^2-1)[x^2-1+2(-x^2-1)]}{(x^2-1)^4}$$

$$= \frac{-2x(x^2-1)(x^2-1-2x^2-2)}{(x^2-1)^4}$$

$$= \frac{-2x\cancel{(x^2-1)}(-x^2-3)}{(x^2-1)^{\cancel{4}3}}$$

$$= \frac{2x(x^2+3)}{(x^2-1)^3} = 0 \quad x=0$$

e)

x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
$f''(x)$	$-$	$\vdots$	$+$	0	$-$	$\vdots$	$+$

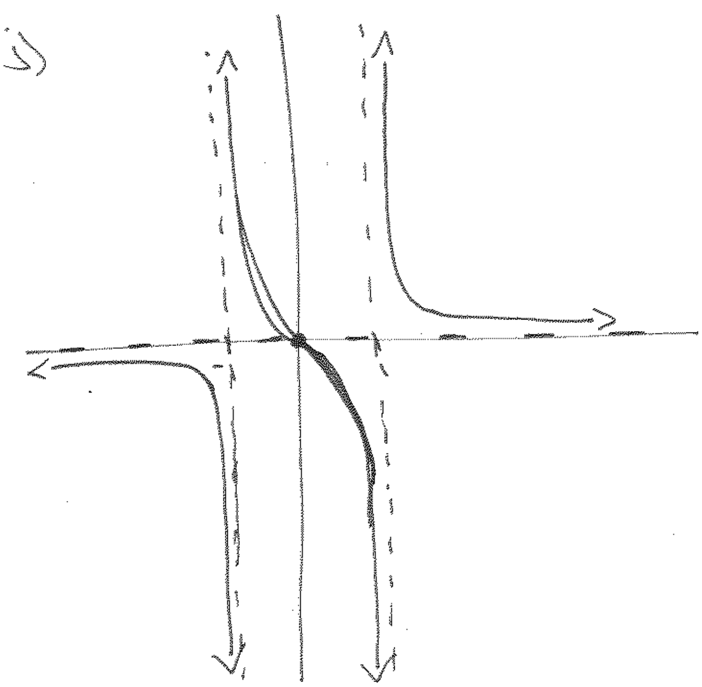
f) Concave up $(-1,0)\cup(1,\infty)$
Concave down $(-\infty,-1)\cup(0,1)$

g) Inflection Point $(0,0)$

h) x-int: $x=0$ y-int: $y=0$

i) V.A @ $x=-1$ $x=1$
H.A @ $y=0$

j)



a) a sign analysis of $f'(x)$
b) the intervals on which $f(x)$ is increasing and/or decreasing
c) the critical numbers
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g) the coordinates of any inflection points
h) the x and y intercepts
i) the equations of any vertical and horizontal asymptotes
j) a careful sketch of the function

ii) $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$(x-2)(x+1) = 0$

c) $x = 2 \quad x = -1$

a)

x	$(-\infty,-1)$	-1	$(-1,2)$	2	$(2,\infty)$
$f'(x)$	+	0	−	0	+

b) Increasing $(-\infty,-1) \cup (2,\infty)$

Decreasing $(-1,2)$

d) local max $(-1, 7)$

local min $(2, -20)$

$f''(x) = 12x - 6 = 0$

$x = \frac{1}{2}$

e)

x	$(-\infty,\frac{1}{2})$	$\frac{1}{2}$	$(\frac{1}{2},\infty)$
$f''(x)$	−	0	+

f) Concave up $(\frac{1}{2},\infty)$

Concave down $(-\infty,\frac{1}{2})$

g) Inflection Point $(\frac{1}{2}, -\frac{13}{2})$

h) x-int

$x(2x^2 - 3x - 12) = 0$

$x = 0 \quad x = 3.3$

$x = -1.8$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-12)}}{2(2)}$

$= \frac{3 \pm \sqrt{9 + 96}}{4}$

$= \frac{3 \pm \sqrt{105}}{4}$

$x = \frac{3 + \sqrt{105}}{4} \quad x = \frac{3 - \sqrt{105}}{4}$

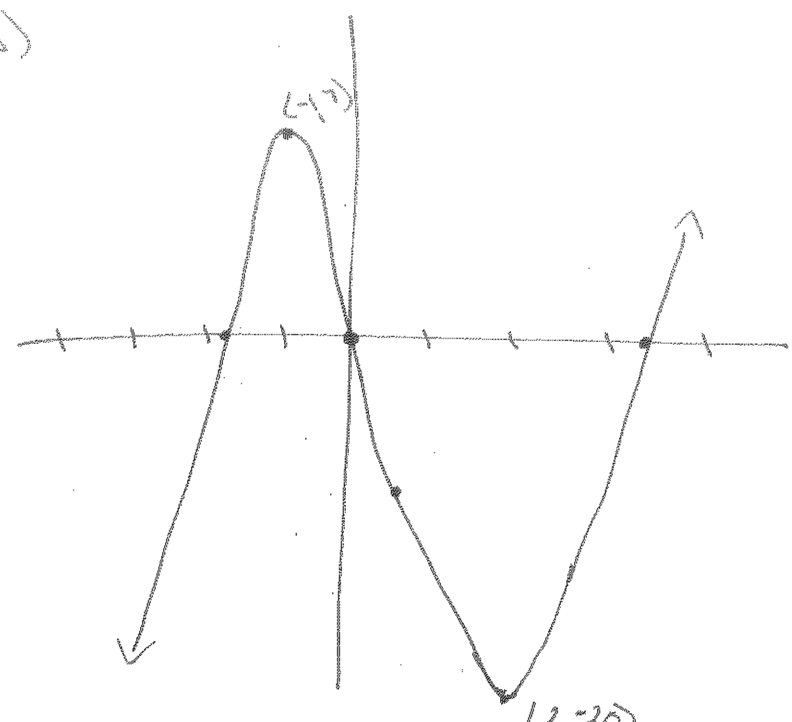
$x = 3.3\ldots \quad x = -1.8\ldots$

y-int

$y = 0$

i) No asymptotes

j)



a) a sign analysis of $f'(x)$
b) the intervals on which $f(x)$ is increasing and/or decreasing
c) the critical numbers
d) the relative extrema
e) a sign analysis of $f''(x)$
f) the intervals on which $f(x)$ is concave up and concave down
g) the coordinates of any inflection points
h) the $x$and y intercepts
i) the equations of any vertical and horizontal asymptotes
j) a careful sketch of the function

iii) $f(x) = \frac{2x^2}{x^2+12}$

$$f'(x) = \frac{4x(x^2+12) - 2x(2x^2)}{(x^2+12)^2}$$

$$= \frac{4x^3 + 48x - 4x^3}{(x^2+12)^2}$$

$$= \frac{48x}{(x^2+12)^2} = 0$$

c) $x = 0$

a)

x	$(-\infty,0)$	0	$(0,\infty)$
$f'(x)$	$-$	0	$+$

b) Increasing $(0,\infty)$
Decreasing $(-\infty,0)$

d) local min $(0,0)$

$$f''(x) = \frac{48(x^2+12)^2 - 2(x^2+12)(2x)(48x)}{(x^2+12)^4}$$

$$= \frac{48(x^2+12)[x^2+12-4x^2]}{(x^2+12)^{\cancel{4}\,3}}$$

$$= \frac{48(-3x^2+12)}{(x^2+12)^3}$$

$$= \frac{-144(x^2-4)}{(x^2+12)^3} = 0$$

$x = 2 \quad x = -2$

e)

x	$(-\infty,-2)$	-2	$(-2,2)$	2	$(2,\infty)$
$f''(x)$	$-$	0	$+$	0	$-$

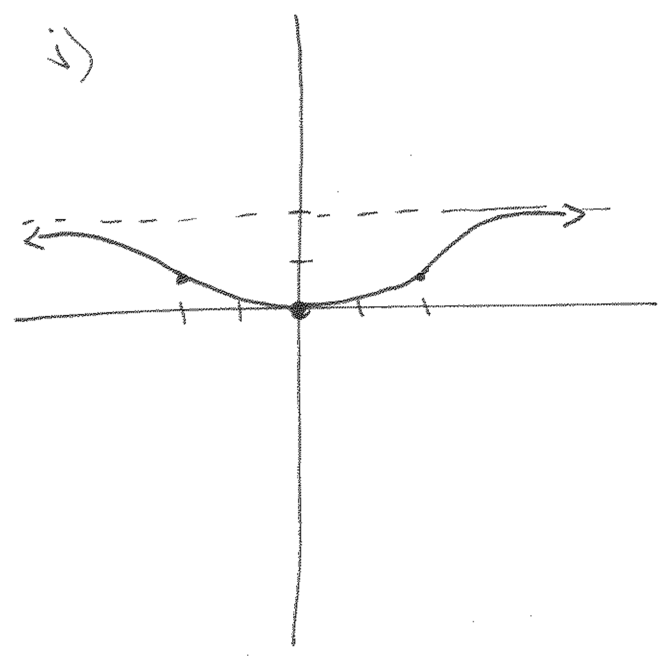
f) Concave up $(-2,2)$
Concave down $(-\infty,-2) \cup (2,\infty)$

g) Inflection Pts $(-2, \frac{1}{2})$
$(2, \frac{1}{2})$

h) x-int $\quad$ y-int
$x = 0 \quad y = 0$

i) V.A @ none
H.A @ $y = 2$

j)



a) a sign analysis of $f'(x)$
b) the intervals on which $f(x)$ is increasing and/or decreasing
c) the critical numbers
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f) the intervals on which $f(x)$ is concave up and concave down
g) the coordinates of any inflection points
h) the x and y intercepts
i) the equations of any vertical and horizontal asymptotes
j) a careful sketch of the function

iv) $y = 3x^5 - 10x^3$

$\frac{dy}{dx} = 15x^4 - 30x^2 = 0$

$15x^2(x^2 - 2) = 0$

c) $x = 0 \quad x = \pm\sqrt{2}$

a)

x	$(-\infty,-\sqrt{2})$	$-\sqrt{2}$	$(-\sqrt{2},0)$	0	$(0,\sqrt{2})$	$\sqrt{2}$	$(\sqrt{2},\infty)$
$f'(x)$	+	0	−	0	−	0	+

b) Increasing $(-\infty,-\sqrt{2}) \cup (\sqrt{2},\infty)$
Decreasing $(-\sqrt{2},0) \cup (0,\sqrt{2})$

d) local max $(-\sqrt{2}, 8\sqrt{2})$
local min $(\sqrt{2}, -8\sqrt{2})$

$\frac{d^2y}{dx^2} = 60x^3 - 60x = 0$

$60x(x^2-1) = 0$

$x = 0 \quad x = \pm 1$

e)

x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
$f''(x)$	−	0	+	0	−	0	+

f) Concave up $(-1,0) \cup (1,\infty)$
Concave down $(-\infty,-1) \cup (0,1)$

g) Inflection Pts $(-1,7)$
$(0,0)$
$(1,-7)$

h) x-int

$3x^5 - 10x^3 = 0$

$x^3(3x^2 - 10) = 0$

$x = 0 \quad 3x^2 = 10$

$x^2 = \frac{10}{3}$

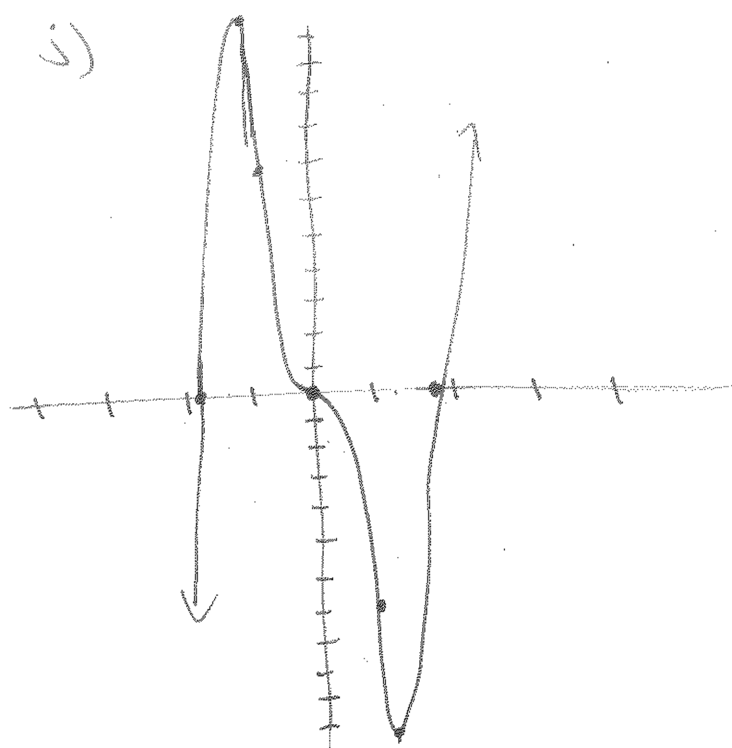
$x = \pm\sqrt{\frac{10}{3}}$

y-int

$y = 0$

i) No asymptotes

j)



a) a sign analysis of $f'(x)$
b) the intervals on which $f(x)$ is increasing and/or decreasing
c) the critical numbers
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e) a sign analysis of $f''(x)$
f) the intervals on which $f(x)$ is concave up and concave down
g) the coordinates of any inflection points
h) the $x$and y intercepts
i) the equations of any vertical and horizontal asymptotes
j) a careful sketch of the function

v) $y = \frac{3x}{(x+1)^2}$

$$\frac{dy}{dx} = \frac{3(x+1)^2 - 2(x+1)(3x)}{(x+1)^4}$$

$$= \frac{3(x+1)(x+1-2x)}{(x+1)^4}$$

$$= \frac{3\cancel{(x+1)}(-x+1)}{(x+1)^{\cancel{4}3}}$$

$$= \frac{-3(x-1)}{(x+1)^3} = 0$$

c) $x = 1$

a)

x	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,\infty)$
$f'(x)$	$-$		$+$	0	$-$

b) Increasing $(-1,1)$
Decreasing $(-\infty,-1) \cup (1,\infty)$

d) local max $(1, \frac{3}{4})$

$$\frac{d^2y}{dx^2} = \frac{-3(x+1)^3 - 3(x+1)^2(-3)(x-1)}{(x+1)^6}$$

$$= \frac{-3(x+1)^2(x+1-3x+3)}{(x+1)^6}$$

$$= \frac{-3(-2x+4)}{(x+1)^4}$$

$$= \frac{6(x-2)}{(x+1)^4} \qquad x = 2$$

e)

x	$(-\infty,-1)$	-1	$(-1,2)$	2	$(2,\infty)$
$f''(x)$	$-$		$-$	0	$+$

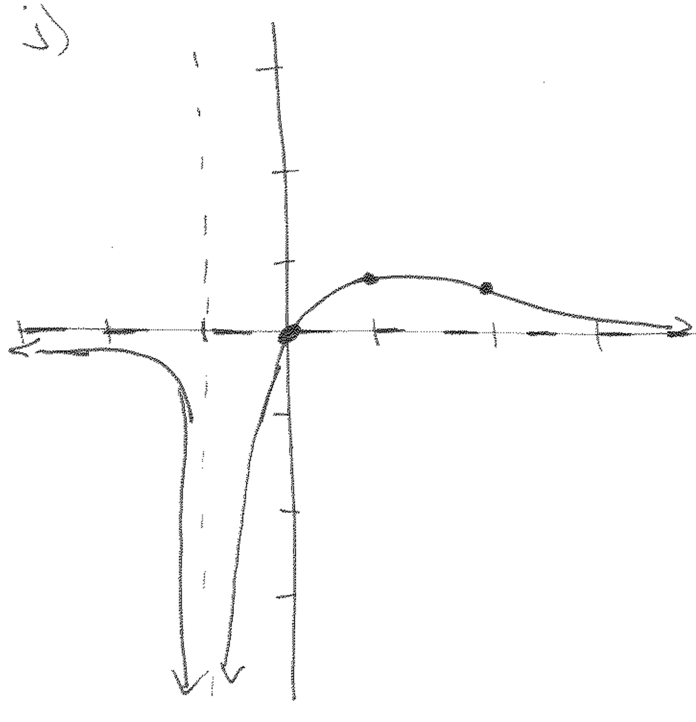
f) Concave up $(2,\infty)$
Concave down $(-\infty,-1) \cup (-1,2)$

g) Inflection Point $(2, \frac{2}{3})$

h) x-int $x = 0$ y-int $y = 0$

i) Asymptotes
V.A @ $x = -1$
H.A @ $y = 0$

j)



a) a sign analysis of $f'(x)$
b) the intervals on which $f(x)$ is increasing and/or decreasing
c) the critical numbers
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e) a sign analysis of $f''(x)$
f) the intervals on which $f(x)$ is concave up and concave down
g) the coordinates of any inflection points
h) the x and y intercepts
i) the equations of any vertical and horizontal asymptotes
j) a careful sketch of the function

vi) $f(x) = 3x^{\frac{2}{3}} - x^2$

$f'(x) = 2x^{-\frac{1}{3}} - 2x = 0$

$2x^{-\frac{1}{3}}(1 - x^{\frac{4}{3}}) = 0$

c) $x = 0 \quad x = 1 \quad x = -1$

$\dfrac{2(1 - \sqrt[3]{x^4})}{\sqrt[3]{x}} = 0$

a)

x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
$f'(x)$	+	0	−	und	+	0	−

b) Increasing $(-\infty,-1) \cup (0,1)$
Decreasing $(-1,0) \cup (1,\infty)$

d) local max $(-1,2)$
$(1,2)$
local min $(0,0)$

$f''(x) = -\frac{2}{3}x^{-\frac{4}{3}} - 2 = 0$

$\dfrac{-2}{3x^{\frac{4}{3}}} - \dfrac{2}{1} = 0$

$\dfrac{-2 - 6x^{\frac{4}{3}}}{3x^{\frac{4}{3}}} = 0$

$\dfrac{-2(1 + 3x^{\frac{4}{3}})}{3x^{\frac{4}{3}}} = 0$ — no critical value.

$x = 0$

e)

x	$(-\infty,0)$	0	$(0,\infty)$
$f''(x)$	−	und	−

f) concave down $(-\infty,0) \cup (0,\infty)$

g) no inflection pts.

h) x-int

$3x^{\frac{2}{3}} - x^2 = 0$

$x^{\frac{2}{3}}(3 - x^{\frac{4}{3}}) = 0$

$x = 0 \qquad 3 = \sqrt[3]{x^4}$

$27 = x^4$

$x \approx 2.3 \quad x \approx -2.3$

y-int

$y = 0$

i) No asymptotes

j)