Calculus 30
Chapters 8/9 – Integration Practice Assessment

Name: ________________

1. Find the differential (antiderivative) for the following:

   a. $y = 20x^4$

   $20\left(\frac{1}{4+1}x^{4+1}\right)$

   $4x^5 + C$

   b. $y = 4x^3 - 11$

   $4\left(\frac{1}{3+1}x^{3+1}\right) - 11x$

   $x^4 - 11x + C$

   c. $y = sinx - e^x + cosx$

   $-\cos x - e^x + \sin x + C$

2. Use U substitution to find the differential for the following:

   a. $\int cos4x\, dx$ $\longrightarrow$ $u = 4x$

   $\int \cos u \left(\frac{1}{4}du\right)$ $du = 4\,dx$

   $\frac{1}{4}\int \cos u\, du$ $\frac{1}{4}du = dx$

   $\frac{1}{4}\int \sin u + C$

   $\frac{1}{4}\sin(4x) + C$

   b. $\int (6x - 11)^8 dx$ $\longrightarrow$ $u = 6x - 11$

   $\int u^8\left(\frac{1}{6}du\right)$ $du = 6\,dx$

   $\frac{1}{6}du = dx$

   $\frac{1}{6}\int u^8\, du$

   $\frac{1}{6}\int \frac{1}{9}u^9 + C$

   $\frac{1}{54}(6x-11)^9 + C$

3. Find the Integral (area below the curve).

$$\int_{-4}^{-2} \frac{1}{x^2}\,dx$$

$F(-2) = \frac{-1}{-2} + C$

$= \frac{1}{2} + C$

$F(-4) = \frac{-1}{-4} + C$

$= \frac{1}{4} + C$

$\int \frac{1}{x^2}\,dx$

$\int x^{-2}\,dx$

$\int -1x^{-1} + C$

$\frac{-1}{x} + C$

$F(-2) - F(-4)$

$\left(\frac{1}{2} + c\right) - \left(\frac{1}{4} + c\right)$

$\frac{1}{4}$

4. Find the area above the curve.

$$\int_{1}^{3} (x^2 - 9)\,dx$$

$F(3) = \frac{1}{3}(3)^3 - 9(3) + C$

$= 9 - 27 + C$

$= -18 + C$

$F(1) = \frac{1}{3}(1)^3 - 9(1) + C$

$= \frac{1}{3} - 9 + C$

$= \frac{-8}{9} + C$

$\int x^2 - 9$

$\int \frac{1}{3}x^3 - 9x + C$

$F(3) - F(1)$

$(-18 + c) - \left(-\frac{8}{9} + c\right)$

$-17\frac{1}{9} \rightarrow 17\frac{1}{9}$

5. Find the area between the curves bounded by the given vertical lines.

$f(x) = x$

$g(x) = x^2$

$x = 1$

$x = 2$

$$(f-g)(x) = x^2 - x$$

$$F(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + C$$

$$F(2) - F(1)$$

$$\left(\frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 + c\right) - \left(\frac{1}{3}(1)^3 - \frac{1}{2}(1)^2 + c\right)$$

$$\left(\frac{8}{3} - 2 + c\right) - \left(\frac{1}{3} - \frac{1}{2} + c\right)$$

$$\frac{5}{6}$$

6. Find the area bounded by $y = x^3$, the tangent line to $y = x^3$ drawn at the point (2,8) and the x-axis.

$$y' = 3x^2$$

$$y' = 3(2)^2$$

$$y' = 12$$

$$m = \frac{y - y_1}{x - x_1}$$

$$12 = \frac{y-8}{x-2}$$

$$12x - 24 = y - 8$$

$$12x - 16 = y$$

$$\int_0^1 x^3\,dx + \int_1^2 (x^3 - 12x + 16)\,dx$$

$$\frac{1}{4}x^4 + C \qquad \frac{1}{4}x^4 - 6x^2 + 16x + C$$

$$F(1) - F(0) \qquad F(2) - F(1)$$

$$\left[\frac{1}{4}(1)^4 + c\right] - \left[\frac{1}{4}(0)^4 + c\right] \qquad \left[\frac{1}{4}(2)^4 - 6(2)^2 + 16(2) + c\right] - \left[\frac{1}{4}(1)^4 - 6(1)^2 + 16(1) + c\right]$$

$$\left(\frac{1}{4} + c\right) - (c) \qquad (4 - 24 + 32 + c) - \left(\frac{1}{4} - 6 + 16 + c\right)$$

$$\frac{1}{4} \qquad (12 + c) - \left(\frac{17}{4} + c\right)$$

$$16\frac{1}{4}$$

$16\frac{1}{2}$ units^2