written exercises 7.1

1\. a) $\log_9 81 = x$
$9^x = 81$
$x = 2$

b) $\log_3 81 = x$
$3^x = 81$
$x = 4$

c) $\log 1000 = x$
$10^x = 1000$
$x = 3$

d) $\log_2\left(\frac{1}{4}\right) = x$
$2^x = \frac{1}{4}$
$2^x = 4^{-1}$
$2^x = 2^{-2}$
$x = -2$

e) $\log_7 7 = x$
$7^x = 7$
$x = 1$

f) $\log_6 1 = x$
$6^x = 1$
$x = 0$

g) $\log_8 8^{13} = x$
$8^x = 8^{13}$
$x = 13$

h) $\log_{11} 11^{-20} = x$
$11^x = 11^{-20}$
$x = -20$

i) $\ln 1 = x$
$e^x = 1$
$x = 0$

j) $\ln e = x$
$e^x = e$
$x = 1$

k) $\ln e^5 = x$
$e^x = e^5$
$x = 5$

l) $\ln e^{-7} = x$
$e^x = e^{-7}$
$x = -7$

m) $\ln\left(\frac{1}{e}\right) = x$
$e^x = \frac{1}{e} = e^{-1}$
$x = -1$

n) $\ln\left(\frac{1}{e^3}\right) = x$
$e^x = \frac{1}{e^3} = e^{-3}$
$x = -3$

2. a) $\log_3 12 + \log_3 10 = \log_3 (12)(10) = \log_3 120$

c) $2\log_3 11 = \log_3 11^2 = \log_3 121$

e) $\log_9 6 + \log_9 7 - \log_9 21 = \log_9 \left(\frac{(6)(7)}{(21)}\right) = \log_9 2$

g) $3\log_5 4 - 4\log_5 2 = \log_5 4^3 - \log_5 2^4$

$= \log_5 \left(\frac{64}{16}\right) = \log_5 4$

i) $\ln 12 - \ln 6 - \ln 2 - \ln 3$

$\ln\left(\frac{12}{6}\right) - \ln 2 - \ln 3$

$\ln 2 - \ln 2 - \ln 3 = -\ln 3 = \ln 3^{-1} = \ln\left(\frac{1}{3}\right)$

k) $-2\ln t + 5\ln w - \ln r + 3\ln s$

$\ln t^{-2} + \ln w^5 - \ln r + \ln s^3$

$\ln\left(\frac{w^5 t^{-2}}{r}\right) + \ln s^3 = \ln\left(\frac{w^5 t^{-2} s^3}{r}\right)$

$= \ln\left(\frac{s^3 w^5}{r t^2}\right)$

4. see textbook

7. a) $y = \log_4(5x+6)$

$\frac{dy}{dx} = \frac{1}{5x+6} \log_4 e (5) = \frac{5}{5x+6} \log_4 e$

b) $y = 8\log_6(x^2-5x)$

$$\frac{dy}{dx} = 8\left(\frac{1}{x^2-5x}\right)\log_6 e\,(2x-5) = \frac{8(2x-5)}{x^2-5x}\log_6 e$$

c) $y = \log\left(\frac{x-1}{x+1}\right) = \log(x-1) - \log(x+1)$

$$\frac{dy}{dx} = \frac{1}{x-1}\log e - \frac{1}{x+1}\log e$$

$$= \frac{\log e(x+1) - \log e(x-1)}{(x-1)(x+1)} = \frac{x\log e + \log e - x\log e + \log e}{(x-1)(x+1)}$$

$$= \frac{2\log e}{(x-1)(x+1)}$$

d) $y = \log_{12}(2x^3+5)^{10} = 10\log_{12}(2x^3+5)$

$$\frac{dy}{dx} = \frac{10}{2x^3+5}\log_{12} e\,(6x^2) = \frac{60x^2}{2x^3+5}\log_{12} e$$

8 a) $f(x) = \ln(5x)$

$$f'(x) = \frac{1}{5x}(5) = \frac{1}{x}$$

c) $f(x) = 6\ln(4x)$

$$f'(x) = 6\left(\frac{1}{4x}\right)(4) = \frac{6}{x}$$

e) $f(x) = \frac{3}{4}\ln\left(\frac{2x}{3}\right)$

$$f'(x) = \frac{3}{4}\left(\frac{1}{\frac{2x}{3}}\right)\left(\frac{2}{3}\right)$$

$$= \frac{1}{2}\left(\frac{3}{2x}\right) = \frac{3}{4x}$$

g) $f(x) = -2\ln(6-5x)$

$$f'(x) = -2\left(\frac{1}{6-5x}\right)(-5)$$

$$= \frac{+10}{6-5x}$$

i) $f(x) = \frac{1}{3}\ln(x^{-3})$

$= \frac{1}{3}(-3)\ln x$

$= -\ln x$

$f'(x) = -\frac{1}{x}$

k) $f(x) = -2\ln\sqrt{x+3}$

$= -2\ln(x+3)^{1/2}$

$= -\ln(x+3)$

$f'(x) = -\frac{1}{x+3}$

m) $f(x) = \ln(6x-1)^{2/3}$

$= \frac{2}{3}\ln(6x-1)$

$f'(x) = \frac{2}{3}\left(\frac{1}{6x-1}\right)(6)$

$= \frac{4}{6x-1}$

o) $f(x) = \ln\left[(x-2)(3x+1)\right]$

$= \ln(x-2) + \ln(3x+1)$

$f'(x) = \frac{1}{x-2} + \frac{1}{3x+1}(3)$

$= \frac{3x+1 + 3(x-2)}{(x-2)(3x+1)}$

$= \frac{6x-5}{(x-2)(3x+1)}$

q) $f(x) = \ln\left[\frac{3x^3}{x^2+4}\right] = \ln 3x^3 - \ln(x^2+4)$

$f'(x) = \frac{1}{3x^3}\cdot 9x^2 - \frac{1}{x^2+4}\cdot 2x = \frac{3}{x} - \frac{2x}{x^2+4}$

$= \frac{3(x^2+4) - 2x(x)}{x(x^2+4)} = \frac{x^2+12}{x(x^2+4)}$

9 a) $f(x) = x^3 \ln x$

$$f'(x) = 3x^2 \ln x + \frac{1}{x}(x^3) = 3x^2 \ln x + x^2$$
$$= x^2(3\ln x + 1)$$

c) $f(x) = \dfrac{2x}{\ln x}$

$$f'(x) = \frac{2(\ln x) - \frac{1}{x}(2x)}{(\ln x)^2} = \frac{2\ln x - 2}{(\ln x)^2}$$
$$= \frac{2(\ln x - 1)}{(\ln x)^2}$$

e) $f(x) = 2x^3 \ln(1-x^3)$

$$f'(x) = 6x^2 \ln(1-x^3) + \frac{1}{1-x^3}(-3x^2)(2x^3)$$
$$= 6x^2\left[\ln(1-x^3) - \frac{x^3}{1-x^3}\right] = 6x^2\left[\frac{(1-x^3)\ln(1-x^3) - x^3}{1-x^3}\right]$$

g) $f(x) = 2\left[\ln(3x^2+8)\right]^6$

$$f'(x) = 12\left[\ln(3x^2+8)\right]^5\left[\frac{1}{3x^2+8}\right](6x)$$
$$= \frac{72x\left[\ln(3x^2+8)\right]^5}{3x^2+8}$$

i) $f(x) = \ln 6$

$f'(x) = 0$ since $\ln 6$ is a constant

k) $f(x) = 2\ln(\ln x)$

$f'(x) = 2\left[\frac{1}{\ln x}\right]\left[\frac{1}{x}\right] = \frac{2}{x\ln x}$

12. $f(x) = 2x + \ln x \quad x = 1$

$f'(x) = 2 + \frac{1}{x} \quad f'(1) = 2 + \frac{1}{1} = 3$

at $x = 1 \quad f(1) = 2(1) + \ln 1$

$= 2 + 0 = 2$

$m = 3 \quad (1, 2) \qquad \frac{3}{1} = \frac{y-2}{x-1} \qquad 3x - 3 = y - 2$

$y = 3x - 1$

14a) $f(x) = \ln x - \frac{1}{2}x$

$f'(x) = \frac{1}{x} - \frac{1}{2} = \frac{2-x}{2x}$

$\frac{2-x}{2x} = 0 \qquad x = 2$

x	$(0,2)$	2	$(2,\infty)$
$f'(x)$	$+$	0	$-$

increasing $(0,2)$ max$(2, \ln 2 - 1)$

decreasing $(2,\infty)$

17 $M(t) = 6\ln(t+1) + 1$

a) $M(30) = 6\ln(31) + 1 = 22$ words

b) $M'(t) = 6\left(\frac{1}{t+1}\right) = \frac{6}{t+1}$

$M'(11) = \frac{6}{12} = \frac{1}{2}$ words/minute

Written exercises 7.2

2\. a) $e^{\ln 23} = 23$

b) $23^{\log_{23} 4} = 4$

c) $\log_8 8^{-7} = -7$

d) $\ln e^{\frac{1}{4}} = \frac{1}{4}$

e) $5^{-2\log_5 3}$

$5^{\log_5 3^{-2}}$

$5^{\log_5(\frac{1}{9})} = \frac{1}{9}$

f) $e^{4\ln 3}$

$e^{\ln 3^4} = e^{\ln 81}$

$= 81$

g) $4\ln e^{-3}$

$4(-3) = -12$

h) $-2\log_3 3^{-1}$

$-2(-1) = 2$

5\. a) $y = 7^x$

$\frac{dy}{dx} = 7^x \ln 7$

c) $y = \left(\frac{2}{3}\right)^{x^3-4x}$

$\frac{dy}{dx} = \left(\frac{2}{3}\right)^{x^3-4x}(3x^2-4)\left(\ln\frac{2}{3}\right)$

e) $y = 6(5^{-3x})$

$\frac{dy}{dx} = 6(5^{-3x})(-3)\ln 5$

$= -18(5^{-3x})\ln 5$

g) $y = \frac{3^x}{x^2}$

$\frac{dy}{dx} = \frac{(3^x \ln 3)(x^2) - 2x(3^x)}{(x^2)^2}$

$= \frac{(x)3^x\left[x\ln 3 - 2\right]}{x^4}$

$= \frac{3^x(x\ln 3 - 2)}{x^3}$

7. $y = e^{-4x}$

$$\frac{dy}{dx} = e^{-4x}(-4) = -4e^{-4x}$$

9. $y = -2e^{-4x}$

$$\frac{dy}{dx} = -2e^{-4x}(-4) = 8e^{-4x}$$

11. $y = e^{x^2-5x+6}$

$$\frac{dy}{dx} = e^{x^2-5x+6}(2x-5) = (2x-5)e^{x^2-5x+6}$$

13. $y = 10e^{4x-2x^2}$

$$\frac{dy}{dx} = 10e^{4x-2x^2}(4-4x) = 40(1-x)e^{4x-2x^2}$$

15. $y = -5e^{-(3x+1)^2}$

$$\frac{dy}{dx} = -5e^{-(3x+1)^2}\left[-2(3x+1)(3)\right] = 30(3x+1)e^{-(3x+1)^2}$$

17. $y = 11e^{-\frac{4}{x^2}} = 11e^{-4x^{-2}}$

$$\frac{dy}{dx} = 11e^{-4x^{-2}}(8x^{-3}) = \frac{88e^{-4x^{-2}}}{x^3}$$

19. $y = 9e^{\frac{-2}{x^3+4}} = 9e^{-2(x^3+4)^{-1}}$

$$\frac{dy}{dx} = 9e^{-2(x^3+4)^{-1}}\left[2(x^3+4)^{-2}(3x^2)\right] = \frac{54x^2e^{-\frac{2}{x^3+4}}}{(x^3+4)^2}$$

21. $y = e^{\ln(2x^3-x^2)} = 2x^3-x^2$

$\frac{dy}{dx} = 6x^2-2x = 2x(3x-1)$

23 $y = e^{-3\ln(2x-1)}$

$= e^{\ln(2x-1)^{-3}} = (2x-1)^{-3}$

$\frac{dy}{dx} = -3(2x-1)^{-4}(2) = \frac{-6}{(2x-1)^4}$

25. $y = 2x^2e^x$

$\frac{dy}{dx} = 4xe^x + e^x(2x^2)$

$= 2xe^x(2+x)$

27. $y = (x+2)\,e^{(x+2)^2}$

$\frac{dy}{dx} = (1)\,e^{(x+2)^2} + e^{(x+2)^2}(2)(x+2)(x+2)$

$= e^{(x+2)^2}\left[1 + 2(x^2+4x+4)\right]$

$= e^{(x+2)^2}(2x^2+8x+9)$

29. $y = \frac{x}{e^x}$ $\quad \frac{dy}{dx} = \frac{(1)(e^x) - xe^x}{(e^x)^2} = \frac{e^x(1-x)}{e^{2x}}$

$= \frac{1-x}{e^x}$

31. $$y = \frac{e^{4x+3}}{4x+3}$$

$$\frac{dy}{dx} = \frac{(e^{4x+3})(4)(4x+3) - 4(e^{4x+3})}{(4x+3)^2}$$

$$= \frac{4e^{4x+3}[4x+3-1]}{(4x+3)^2} = \frac{4e^{4x+3}(4x+2)}{(4x+3)^2}$$

$$= \frac{8e^{4x+3}(2x+1)}{(4x+3)^2}$$

33. $$y = -4e^{\sqrt[4]{x^3}} = -4e^{x^{3/4}}$$

$$\frac{dy}{dx} = -4e^{x^{3/4}}\left(\tfrac{3}{4}x^{-1/4}\right) = -\frac{3e^{x^{3/4}}}{x^{1/4}} = \frac{-3e^{\sqrt[4]{x^3}}}{\sqrt[4]{x}}$$

35. $$y = \tfrac{1}{3}x^3 e^{\ln 6x} = \tfrac{1}{3}x^3(6x) = 2x^4$$

$$\frac{dy}{dx} = 8x^3$$

37. $$y = \frac{e^{3x-1}}{2x+1}$$

$$\frac{dy}{dx} = \frac{e^{3x-1}(3)(2x+1) - 2(e^{3x-1})}{(2x+1)^2}$$

$$= \frac{e^{3x-1}[6x+3-2]}{(2x+1)^2} = \frac{e^{3x-1}(6x+1)}{(2x+1)^2}$$

39. $y = e^3 x^3$

$$\frac{dy}{dx} = e^3(3x^2)$$
$$= 3e^3x^2$$

41. $y = \left(e^{6x-5}\right)^{10}$

$$\frac{dy}{dx} = 10\left(e^{6x-5}\right)^9\left(e^{6x-5}\right)(6)$$
$$= 60\left(e^{6x-5}\right)^{10}$$

43. $y = e^x \ln x$

$$\frac{dy}{dx} = e^x \ln x + \frac{1}{x} e^x$$
$$= e^x\left(\ln x + \frac{1}{x}\right) = e^x\left(\frac{x \ln x + 1}{x}\right)$$

45. $y = x \ln(xe^x)$

$$\frac{dy}{dx} = (1)\ln(xe^x) + \frac{1}{xe^x}\left[(1)(e^x) + xe^x\right]\left[x\right]$$
$$= \ln(xe^x) + \frac{xe^x(1+x)}{xe^x}$$
$$= \ln(xe^x) + (1+x)$$
$$= \ln x + \ln e^x + 1 + x$$
$$= \ln x + x + 1 + x$$
$$= \ln x + 2x + 1$$

47. $e^{x+y} + xy = 40$

$$\frac{d\left(e^{x+y}\right)}{dx} + \frac{d(xy)}{dx} = \frac{d(40)}{dx}$$

$$e^{x+y}\left[1+\frac{dy}{dx}\right] + (1)(y) + \frac{dy}{dx}(x) = 0$$

$$e^{x+y} + \frac{dy}{dx}e^{x+y} + y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}e^{x+y} + x\frac{dy}{dx} = -e^{x+y} - y$$

$$\frac{dy}{dx}\left(e^{x+y} + x\right) = -e^{x+y} - y$$

$$\frac{dy}{dx} = \frac{-e^{x+y} - y}{e^{x+y} + x}$$

48. $f(x) = xe^x$ at $x = 1$

$f(1) = (1)e^1 = e$

$(1, e)$

$f'(x) = (1)e^x + e^x(x)$

$f'(1) = e + e = 2e$

$$\frac{2e}{1} = \frac{y-e}{x-1}$$

$$y - e = 2xe - 2e$$

$$y = 2xe - e$$

50. $f(x) = x^2 e^x$

$f'(x) = 2xe^x + e^x(x^2)$

$= (e^x)(x)[2+x] = 0$

$e^x = 0 \quad x = 0 \quad x = -2$

no sol

x	$(-\infty,-2)$	-2	$(-2,0)$	0	$(0,\infty)$
$f'(x)$	+	0	−	0	+

increasing $(-\infty,-2) \cup (0,\infty)$

decreasing $(-2,0)$

relative max $(-2, f(-2)) = (-2, \frac{4}{e^2})$

min $(0, f(0)) = (0,0)$

52. $L(t) = 1000(t-10)^2 e^{t-8} + 1$

b) $L'(t) = 1000\left[2(t-10)e^{t-8} + e^{t-8}(t-10)^2\right]$

a) $L(4) = 600 \quad L(7) = 3312 \quad L(9) = 2719$

c) $L'(t) = 0 = 1000(t-10)(e^{t-8})[2 + t - 10]$

$t - 10 = 0 \quad e^{t-8} = 0 \quad t - 8 = 0$

$t = 10 \quad$ no sol $\quad \boxed{t=8}$

d) $L(8) = 4001$

e) $L'(7) = 1104$
increasing

f) $L'(9) = -2718$
decreasing

written exercises 7.3

1. $\lim_{x\to 0} \frac{\sin(10x)}{(10x)} = 1$

3. $\lim_{x\to 0} \frac{\sin 3x}{x} \cdot \frac{3}{3}$

$3\frac{\sin(3x)}{(3x)} = 3(1)$
$= 3$

5. $\lim_{x\to 0} \frac{2\sin x}{7x} = \frac{2}{7}\left(\frac{\sin x}{x}\right)$
$= \frac{2}{7}(1) = \frac{2}{7}$

7. $\lim_{x\to 0} \frac{\tan 2x}{2x}$

$\frac{\sin 2x}{\cos 2x} \cdot \frac{1}{2x} = \frac{\sin 2x}{2x}\left(\frac{1}{\cos 2x}\right)$

$= 1\left(\frac{1}{\cos 2(0)}\right) = 1$

9. $\lim_{x\to 0} \frac{\sin 5x \left(\frac{5x}{5x}\right)}{\sin 2x \left(\frac{2x}{2x}\right)}$

$= \frac{5x\frac{\sin(5x)}{(5x)}}{2x\frac{\sin(2x)}{(2x)}} = \frac{5x(1)}{2x(1)} = \frac{5}{2}$

11. $\lim_{x\to 0} \frac{\sin 3x}{\cos 4x} = \frac{\sin 3(0)}{\cos 4(0)}$

$= \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

13. $\lim_{x\to 0} \frac{(\sin x)^3}{x^3} = \left[\frac{\sin x}{x}\right]^3$
$= (1)^3 = 1$

15. $\lim_{x\to 0} \frac{\sin 2x \left(\frac{2x}{2x}\right)}{\sin x \left(\frac{x}{x}\right)}$

$\frac{2x\frac{\sin(2x)}{(2x)}}{x\frac{\sin(x)}{(x)}} = \frac{2x(1)}{x(1)} = 2$

17. $\lim_{x \to 0} \dfrac{4 - 4\cos x}{x}$

$\dfrac{4(1-\cos x)}{x}$

$4(0) = 0$

19. $\lim_{x \to 0} \dfrac{6x - \sin 2x}{2x}$

$= \dfrac{6x}{2x} - \dfrac{\sin(2x)}{(2x)}$

$= 3 - 1 = 2$

21. $\lim_{x \to 0} \dfrac{(1-\cos x)^4}{x^3} = \dfrac{(1-\cos x)^3 (1-\cos x)}{x^3}$

$= \left(\dfrac{1-\cos x}{x}\right)^3 (1-\cos x)$

$= (0)^3 (1-\cos 0)$

$0(0) = 0$

23. $\lim_{x \to 0} \dfrac{1-\cos x}{\sin x} \dfrac{\left(\frac{x}{x}\right)}{\left(\frac{x}{x}\right)} = \dfrac{x\left(\frac{1-\cos x}{x}\right)}{x\left(\frac{\sin x}{x}\right)} = \dfrac{1(0)}{1(1)} = 0$

25. $\lim_{x \to 0} \dfrac{\cos 2x}{\cos 3x} = \dfrac{\cos(2(0))}{\cos(3(0))} = \dfrac{\cos 0}{\cos 0} = \dfrac{1}{1} = 1$

27. $\lim_{x \to \infty} \dfrac{\sin x}{x} \cdot \dfrac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \dfrac{\frac{1}{x}\sin x}{1} = \dfrac{1}{x} \sin x$

as $x \to \infty$ $\dfrac{1}{x} \to 0$ $\therefore$ $= 0$

written exercises 7.4

1. $f(x) = \sin x$

$f'(x) = \cos x$

$f'(10) = \cos 10 \approx -0.83907$

3. $f(x) = \sin x \quad = \frac{5}{3}$

$f'(x) = \cos x$

$f'(\frac{5\pi}{3}) = \cos \frac{5\pi}{3} = \frac{1}{2}$

5. $f(x) = \sin x \quad x = \frac{2\pi}{3}$

$f'(x) = \cos x$

$f'(\frac{2\pi}{3}) = \cos \frac{2\pi}{3} = -\frac{1}{2}$

$\therefore m = -\frac{1}{2} \quad (\frac{2\pi}{3}, f(\frac{2\pi}{3}))$

$(\frac{2\pi}{3}, \frac{\sqrt{3}}{2})$

$$-\frac{1}{2} = \frac{y - \frac{\sqrt{3}}{2}}{x - \frac{2\pi}{3}}$$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}x + \frac{\pi}{3}$

$y = -\frac{1}{2}x + \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

7. $f(x) = \sin 6x$

$f'(x) = \cos 6x\,(6)$

$= 6\cos 6x$

9. $f(\theta) = \cos \frac{2}{3}\theta$

$f'(\theta) = -\sin \frac{2}{3}\theta\,(\frac{2}{3})$

$= -\frac{2}{3}\sin\frac{2}{3}\theta$

11. $y = 4\sin 10x$

$\frac{dy}{dx} = 4(\cos 10x)(10)$

$= 40\cos 10x$

13. $y = 14\cos\frac{\theta}{7} = 14\cos\frac{1}{7}\theta$

$\frac{dy}{dx} = 14(-\sin\frac{1}{7}\theta)(\frac{1}{7})$

$= -2\sin\frac{\theta}{7}$

15. $f(t) = 8\cos(4t-9)$

$f'(t) = 8(-\sin(4t-9))(4)$

$= -32\sin(4t-9)$

17. $f(x) = \cos 3x + \sin 2x$

$f'(x) = -\sin 3x(3) + \cos 2x(2)$

$= -3\sin 3x + 2\cos 2x$

19. $y = \cos^2 x = (\cos x)^2$

$\frac{dy}{dx} = 2(\cos x)^1(-\sin x)$

$= -2\sin x\cos x$

21. $y = \sin(x^5)$

$\frac{dy}{dx} = \cos(x^5)(5x^4)$

$= 5x^4\cos(x^5)$

23. $y = -6\sin(3x^2)$

$\frac{dy}{dx} = -6\cos(3x^2)(6x)$

$= -36x\cos(3x^2)$

25. $y = -3\cos(4x^3-3x)$

$\frac{dy}{dx} = -3(-\sin(4x^3-3x))(12x^2-3)$

$= +3(12x^2-3)\sin(4x^3-3x)$

$= 9(4x^2-1)\sin(4x^3-3x)$

27. $f(x) = \sin\left[(5x+3)^2\right]$

$f'(x) = \cos\left[(5x+3)^2\right]\left[2(5x+3)(5)\right]$

$= 10(5x+3)\cos\left[(5x+3)^2\right]$

29. $f(x) = e^{\cos 3x}$

$f'(x) = e^{\cos 3x}(-\sin 3x)(3)$

$= -3\sin 3x\, e^{\cos 3x}$

31. $f(x) = 4\ln(\cos x)$

$f'(x) = 4\left(\frac{1}{\cos x}\right)(-\sin x)$

$= -4\frac{\sin x}{\cos x} = -4\tan x$

33. $f(x) = \ln(\sin^2 x) = \ln\left[(\sin x)^2\right]$

$f'(x) = \frac{1}{(\sin x)^2}\left[2\sin x(\cos x)\right]$

$= \frac{2\sin x\cos x}{(\sin x)^2} = \frac{2\cos x}{\sin x} = 2\cot x$

35. $y = x \sin x$

$$\frac{dy}{dx} = (1)\sin x + (\cos x)(x)$$
$$= \sin x + x\cos x$$

37. $y = x^3 \sin(x^3)$

$$\frac{dy}{dx} = 3x^2 \sin(x^3) + \cos(x^3)(3x^2 x^3)$$
$$= 3x^2\left[\sin(x^3) + x^3\cos(x^3)\right]$$

39. $y = \dfrac{3x}{\cos x}$

$$\frac{dy}{dx} = \frac{3(\cos x) - (-\sin x)(3x)}{(\cos x)^2}$$
$$= \frac{3\left[\cos x + x\sin x\right]}{\cos^2 x}$$

41. $y = -6\sqrt{\sin 8x}$

$$= -6(\sin 8x)^{1/2}$$

$$\frac{dy}{dx} = -6\left(\tfrac{1}{2}\right)(\sin 8x)^{-1/2}(\cos 8x)(8)$$
$$= -\frac{24\cos 8x}{\sqrt{\sin 8x}}$$

43. $f(t) = \dfrac{\sin t}{\cos 2t}$

$$f'(t) = \frac{(\cos t)(\cos 2t) - (-\sin 2t)(2)(\sin t)}{(\cos 2t)^2}$$
$$= \frac{\cos t\cos 2t + 2\sin t\sin 2t}{\cos^2 2t}$$

45. $f(x) = \cos(e^4)$

$f'(x) = 0$ since e^4 is a constant

47. $f(r) = \sin r\cos r$

$$f'(r) = (\cos r)(\cos r) + (-\sin r)(\sin r)$$
$$\cos^2 r - \sin^2 r$$

49. $y = \sin^2 x + \cos^2 x$

$y = 1$ (identity)

$\frac{dy}{dx} = 0$

51. $y = \sqrt{\cos\sqrt{x}} = \left[\cos(x)^{1/2}\right]^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}\left[\cos(x)^{1/2}\right]^{-1/2}\left[-\sin(x)^{1/2}\right]\left[\frac{1}{2}x^{-1/2}\right]$

$= \dfrac{-\sin\sqrt{x}}{4\sqrt{x}\sqrt{\cos\sqrt{x}}}$

53. $f(x) = \sin(\cos 2x)$

$f'(x) = \left[\cos(\cos 2x)\right](-\sin 2x)(2)$

$= -2\sin 2x \cos(\cos 2x)$

55. $y = x^3 \sin x \cos x$

$\frac{dy}{dx} = (3x^2)(\sin x)(\cos x) + (\cos x)(x^3)(\cos x) + (-\sin x)(x^3)(\sin x)$

$= x^2 (3\sin x\cos x + x\cos^2 x - x\sin^2 x$

57. $y = 2x^2 \cos(\frac{1}{x}) = 2x^2 \cos x^{-1}$

$\frac{dy}{dx} = 4x\cos x^{-1} + (-\sin x^{-1})(-x^{-2})(2x^2)$

$= 4x\cos(\frac{1}{x}) + 2\sin(\frac{1}{x})$

58. $2y = \sin(x+y)$

$$\frac{d(2y)}{dx} = \frac{d\left[\sin(x+y)\right]}{dx}$$

$$2\frac{dy}{dx} = \cos(x+y)\left[1 + \frac{dy}{dx}\right]$$

$$2\frac{dy}{dx} = \cos(x+y) + \frac{dy}{dx}\cos(x+y)$$

$$2\frac{dy}{dx} - \frac{dy}{dx}\cos(x+y) = \cos(x+y)$$

$$\frac{dy}{dx}\left(2 - \cos(x+y)\right) = \cos(x+y)$$

$$\frac{dy}{dx} = \frac{\cos(x+y)}{2 - \cos(x+y)}$$

60. $A = \frac{1}{2}ab\sin C$

$$A = \frac{1}{2}(8)(12)\sin\theta$$

$$= 48\sin\theta$$

$$\frac{dA}{d\theta} = 48\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

62. $h(t) = -10\cos\left(\frac{\pi}{10}t\right) + 11$

a) $h(43) = 5.12\text{m}$

b) $v(t) = h'(t) =$

$$-10\left[-\sin\left(\frac{\pi}{10}t\right)\right]\left[\frac{\pi}{10}\right]$$

$$= \pi\sin\left(\frac{\pi}{10}t\right)$$

c) $v(43) = 2.54\text{m/s}$

63. $f(x) = \cos x \quad [0, 2\pi]$

$f'(x) = -\sin x$

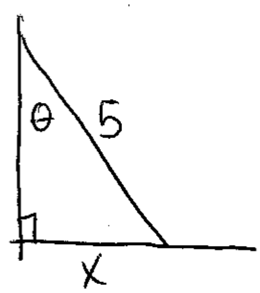
$f''(x) = -\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

x	$[0,\frac{\pi}{2})$	$\frac{\pi}{2}$	$(\frac{\pi}{2},\frac{3\pi}{2})$	$\frac{3\pi}{2}$	$(\frac{3\pi}{2},2\pi]$
$f''(x)$	$-$	0	+	0	$-$

concave up $(\frac{\pi}{2}, \frac{3\pi}{2})$

64.



$\frac{dx}{dt} = .5$ find $\left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{3}}$

$\sin\theta = \frac{x}{5} = \frac{1}{5}x$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

$\cos\frac{\pi}{3}\frac{d\theta}{dt} = \frac{1}{5}\left(\frac{1}{2}\right)$

$\frac{1}{2}\frac{d\theta}{dt} = \frac{1}{10} \quad \frac{d\theta}{dt} = \frac{1}{5}$ rad/s

65. $\frac{d\theta}{dt} = -\frac{1}{4}$

find $\left.\frac{dx}{dt}\right|_{\theta=\frac{5\pi}{6}}$

$x^2 = 6^2 + 12^2 - 144\cos\frac{5\pi}{6}$

$x = \sqrt{180 - 144\left(-\frac{\sqrt{3}}{2}\right)}$

$= 17.5$

$x^2 = 6^2 + 12^2 - 2(6)(12)\cos\theta$

$2x\frac{dx}{dt} = -144(-\sin\theta)\frac{d\theta}{dt}$

$2(17.5)\frac{dx}{dt} = 144\sin\left(\frac{5\pi}{6}\right)\left(\frac{-1}{4}\right)$

$\frac{dx}{dt} = \frac{144\left(\frac{1}{2}\right)\left(\frac{-1}{4}\right)}{2(17.5)} \approx .052$ cm/s

written exercises 7.5

7. $y = \sec 4x$

$\frac{dy}{dx} = (\sec 4x \tan 4x)(4)$

$= 4 \sec 4x \tan 4x$

9. $y = -3\cot(x^3)$

$\frac{dy}{dx} = -3(-\csc^2(x^3))(3x^2)$

$= 9x^2 \csc^2(x^3)$

11. $y = 6\sqrt{\sec 3x} = 6(\sec 3x)^{1/2}$

$\frac{dy}{dx} = 6\left(\frac{1}{2}\right)(\sec 3x)^{-1/2}(\sec 3x \tan 3x)(3x)$

$= \frac{9x \sec 3x \tan 3x}{\sqrt{\sec 3x}} = 9x\sqrt{\sec 3x}\tan 3x$

13. $y = -4\sec(3x+1)$

$\frac{dy}{dx} = -4[\sec(3x+1)\tan(3x+1)](3)$

$= -12\sec(3x+1)\tan(3x+1)$

15. $y = -5\sec^3 2x = -5(\sec 2x)^3$

$\frac{dy}{dx} = -5(3)(\sec 2x)^2(\sec 2x \tan 2x)(2)$

$= -30 \sec^3 2x \tan 2x$

17. $y = x^4 \cot 8x$

$\frac{dy}{dx} = 4x^3 \cot 8x + (-\csc^2 8x)(8)(x^4)$

$= 4x^3(\cot 8x - 2x\csc^2 8x)$

19. $y = \tan\sqrt{x-1} = \tan(x-1)^{1/2}$

$\frac{dy}{dx} = \sec^2(x-1)^{1/2}\left(\frac{1}{2}\right)(x-1)^{-1/2}$

$= \frac{\sec^2\sqrt{x-1}}{2\sqrt{x-1}}$

21. $y = \csc(e^{2x})$

$\frac{dy}{dx} = \left[-\csc(e^{2x})\cot(e^{2x})\right](e^{2x})(2)$

$= -2e^{2x}\csc(e^{2x})\cot(e^{2x})$

23. $y = 3\ln(\tan 9x)$

$\frac{dy}{dx} = 3\left(\frac{1}{\tan 9x}\right)(\sec^2 9x)(9) = \frac{27\sec^2 9x}{\tan 9x}$

25. $y = (\sec x)(\sec 2x)$

$\frac{dy}{dx} = (\sec x \tan x)(\sec 2x) + (\sec 2x \tan 2x)(2)(\sec x)$

$= \sec x \sec 2x(\tan x + 2\tan 2x)$

27. $y = \ln(\csc^2 6x) = \ln\left[(\csc 6x)^2\right]$

$\frac{dy}{dx} = \frac{1}{\csc^2 6x}(2)(\csc 6x)(-\csc 6x \cot 6x)(6)$

$= \frac{-12\csc^2 6x \cot 6x}{\csc^2 6x} = -12\cot 6x$

29. $y = e^{\ln(\tan x)} = \tan x$

$\frac{dy}{dx} = \sec^2 x$

31. $y = \pi \tan$

$\frac{dy}{dx} = \pi \sec^2 x$

33. $y = \tan 2x \sec 2x$

$\frac{dy}{dx} = (\sec^2 2x)(2)(\sec 2x) + (\sec 2x \tan 2x)(2)(\tan 2x)$

$= 2\sec 2x\left[\sec^2 2x + \tan^2 2x\right]$

35. $y = (e^x)^{\cot 2x} = e^{x \cot 2x}$

$\frac{dy}{dx} = e^{x\cot 2x}\left[(1)(\cot 2x) + (-\csc^2 2x)(2)(x)\right]$

$= e^{x\cot 2x}\left(\cot 2x - 2x\csc^2 2x\right)$

37. $y = \cos^2(\tan x^3) = \left[\cos(\tan x^3)\right]^2$

$\frac{dy}{dx} = 2\left[\cos(\tan x^3)\right]\left[-\sin(\tan x^3)\right]\left[\sec^2 x^3\right](3x^2)$

$= -6x^2 \sec^2 x^3 \cos(\tan x^3) \sin(\tan x^3)$

39. $y = \tan\left[\cot(\sec x)\right]$

$\frac{dy}{dx} = \sec^2\left[\cot(\sec x)\right]\left[-\csc^2(\sec x)\right]\left[\sec x \tan x\right]$

40. $y = x + \tan x \qquad x = \frac{\pi}{3}$

$\left(\frac{\pi}{3}, \frac{\pi}{3} + \sqrt{3}\right)$

$\frac{dy}{dx} = 1 + \sec^2 x$

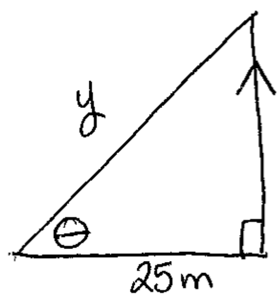
$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} = 1 + \left(\sec\frac{\pi}{3}\right)^2 = 1 + (2)^2 = 5 \quad \therefore m = 5$

$\frac{5}{1} = \frac{y - \left(\frac{\pi}{3} + \sqrt{3}\right)}{x - \frac{\pi}{3}}$

$5x - \frac{5\pi}{3} = y - \frac{\pi}{3} - \sqrt{3}$

$y = 5x - \frac{4\pi}{3} + \sqrt{3}$

44.

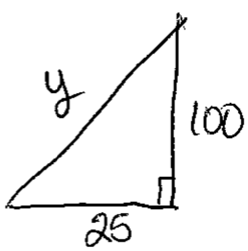


$\frac{dx}{dt} = 3 \qquad \text{find } \left.\frac{d\theta}{dt}\right|_{x=100}$

$\tan\theta = \frac{x}{25}$

$x = 25\tan\theta$

$\frac{dx}{dt} = 25\sec^2\theta \frac{d\theta}{dt}$



$y^2 = 25^2 + 100^2$

$y^2 = 10625$

$y = \sqrt{10625}$

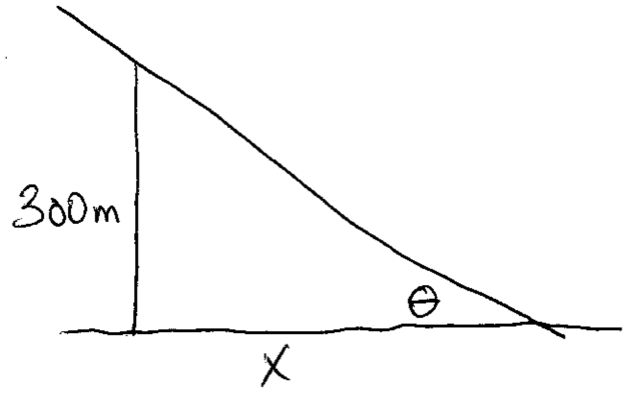
$\sec^2\theta = \frac{10625}{625}$

$= 17$

$3 = 25(17)\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{3}{425}$ rads/s

45.



$$\frac{d\theta}{dt} = -0.262 \text{ rad/hr}$$

find $\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{6}}$

$$\tan\theta = \frac{300}{x} = 300x^{-1}$$

$$\sec^2\theta \frac{d\theta}{dt} = -\frac{300}{x^2}\frac{dx}{dt}$$

$$\left(\sec\frac{\pi}{6}\right)^2(-0.262) = \frac{-300}{(300\sqrt{3})^2}\frac{dx}{dt}$$

$$\left(\frac{4}{3}\right)(-0.262) = \frac{dx}{dt}\left(\frac{-300}{270000}\right)$$

$$\left(\frac{4}{3}\right)(-0.262)(900) = \frac{dx}{dt}$$

$$314.4 \text{ m/hr} = \frac{dx}{dt}$$

$$\tan\frac{\pi}{6} = \frac{300}{x}$$

$$\frac{\sqrt{3}}{3} = \frac{300}{x}$$

$$x = \frac{900}{\sqrt{3}}$$

$$= 300\sqrt{3}$$