Written exercises 6.1

2. $m(x) = -3x^2 + 38x + 500$

a) $m(2) = -3(2)^2 + 38(2) + 500$

$= -12 + 76 + 500 = 564$ grams

b) $m(6) = -3(6)^2 + 38(6) + 500$

$= -108 + 228 + 500 = 620$ grams

c) $\frac{m(6) - m(2)}{6-2} = \frac{620 - 564}{4} = 14$ grams/kg

d) $m'(x) = -6x + 38$

$m'(5) = -6(5) + 38 = 8$ grams/kg

4. $V(x) = 150000(1 + .03x)^2$

a) $\frac{V(7) - V(3)}{7-3} = \frac{219615 - 178215}{4} = \frac{41400}{4}$

$= \$10350$/year

b) $V'(x) = 150000(2)(1 + .03x)(.03)$

$V'(8) = 150000(2)(1 + .03(8))(.03) = \$11160$/year

5. $t(x) = \dfrac{20}{1+0.001x} - 1$

a) $t(400) = \dfrac{20}{1+0.001(400)} - 1 = 13.29°C$

b) $\dfrac{t(1800)-t(800)}{1800-800} = \dfrac{6.14-10.11}{1000} = -0.00397°C/m$

c) $t(x) = 20(1+0.001x)^{-1} - 1$

$t'(x) = -20(1+0.001x)^{-2}(.001)$

$t'(1000) = \dfrac{-20(.001)}{[1+0.001(1000)]^2} = -0.005°C/m$

7. $p(t) = \dfrac{2000t^2}{t^2+100}$

a) $\dfrac{p(30)-p(5)}{30-5} = \dfrac{1800-400}{25} = 56$ people/hr

b) $p'(t) = \dfrac{(4000t)(t^2+100) - 2t(2000t^2)}{(t^2+100)^2}$

$\dfrac{4000t[t^2+100-t^2]}{(t^2+100)^2} = \dfrac{400000t}{(t^2+100)^2}$

$p'(10) = \dfrac{4000000}{(200)^2} = 100$ people/hr

written exercises 6.2

1a) $s(t) = t^3 + 4t^2 + 5t + 9$

$v(t) = s'(t) = 3t^2 + 8t + 5$

$a(t) = v'(t) = 6t + 8$

b) $s(t) = t^2\sqrt{t+1} = t^2(t+1)^{1/2}$

$v(t) = s'(t) = 2t(t+1)^{1/2} + \frac{1}{2}(t+1)^{-1/2}(t^2)$

$= \frac{1}{2}t\left[t+1\right]^{-1/2}\left[4(t+1) + t\right]$

$= \frac{1}{2}t(t+1)^{-1/2}(5t+4)$

$a(t) = v'(t) = \frac{1}{2}(t+1)^{-1/2}(5t+4) + (-\frac{1}{2})(t+1)^{-3/2}(\frac{1}{2}t)(5t+4)$

$+ 5(\frac{1}{2}t)(t+1)^{-1/2}$

$= \frac{1}{4}(t+1)^{-3/2}\left[2(t+1)(5t+4) - t(5t+4) + 10(t+1)\right]$

$= \frac{1}{4}(t+1)^{-3/2}(10t^2 + 18t + 8 - 5t^2 - 4t + 10t^2 + 10t)$

$= \frac{1}{4}(t+1)^{-3/2}(15t^2 + 24t + 8)$

c) $s(t) = \dfrac{3t^2}{t+2}$

$$v(t) = s'(t) = \frac{6t(t+2) - (1)(3t^2)}{(t+2)^2}$$

$$= \frac{3t^2+12t}{(t+2)^2}$$

$$a(t) = v'(t) = \frac{(6t+12)(t+2)^2 - 2(t+2)(3t^2+12t)}{(t+2)^4}$$

$$= \frac{6(t+2)\left[t^2+4t+4-(t^2+4t)\right]}{(t+2)^4}$$

$$= \frac{6(4)}{(t+2)^3} = \frac{24}{(t+2)^3}$$

2 $s(t) = t^3+6t^2+9t$

a) $s(0) = 0\text{m}$

b) $v(t) = s'(t) = 3t^2+12t+9$

$a(t) = v'(t) = 6t+12$

c) $\dfrac{s(6)-s(2)}{6-2} = \dfrac{486-50}{4} = 109\text{ m/s}$

d) $v(2) = 3(2)^2+12(2)+9 = 45\text{ m/s}$

$v(4) = 3(4)^2+12(4)+9 = 105\text{ m/s}$

$v(6) = 3(6)^2+12(6)+9 = 189\text{ m/s}$

e) $\frac{v(6)+v(2)}{2} = \frac{45+189}{2} = 117 m/s$

av vel between 2 and 6 $= 109 m/s$

no they are not

f) $\frac{v(6)+v(2)}{2} = 117 m/s \qquad v(4) = 105 m/s$

no

g) $v(4) = 105 m/s \neq 109 m/s$

h) $144 = 3t^2+12t+9$

$0 = 3t^2+12t-135$

$0 = 3(t^2+4t-45)$

$= 3(t-5)(t+9)$

$t=5 \quad t=-9$

$s(5) = (5)^3+6(5)^2+9(5)$

$= 320m$

i) $a(5) = 6(5)+12 = 42 m/s^2$

j) $30 = 6t+12$

$6t = 18$

$t = 3sec$

$s(3) = (3)^3+6(3)^2+9(3)$

$= 108m$

k) $\frac{v(6)-v(2)}{6-2} = \frac{189-45}{4} = 36 m/s^2$

4. $h(t) = 72 - 4.9t^2$

a) $v(t) = h'(t) = -9.8t$

$v(3) = -9.8(3) = -29.4 m/s$

b) $V_{av} = \frac{h(3) - h(1)}{3-1} = \frac{27.9 - 67.}{2} = -19.6 m/s$

c) $0 = 72 - 4.9t^2$

$4.9t^2 = 72$

$t^2 = 14.69$ $\quad t = 3.8s$

d) $a(t) = v'(t) = -9.8 m/s^2$

5. $h(t) = -5t^2 + 100t + 8$

a) $v(t) = -10t + 100$

$v(0) = 100 m/s$

b) $v(3) = -10(3) + 100$

$= 70 m/s$

c) $v = 0 = -10t + 100$

$t = 10 sec$

d) $h(10) = -5(10)^2 + 100(10) + 8$

$= 508 m$

e) $0 = -5t^2 + 100t + 8$

$0 = 5t^2 - 100t - 8$

$t = \frac{100 \pm \sqrt{(-100)^2 - 4(5)(-8)}}{10}$

$= \frac{100 \pm \sqrt{10160}}{10} = \frac{100 \pm 100.8}{10} = 20.08s$

f) $v(20.08) =$

$-10(20.08) + 100$

$= -100.8 m/s$

written exercises 6.3

3. let x = one #
$x+10$ = other #

$S = x^2 + (x+10)^2$, $(-\infty, \infty)$

minimize sum S

$\frac{ds}{dx} = 2x + 2(x+10)$

$0 = 4x + 20$

$x = -5$

$S(-5) = (-5)^2 + (-5+10)^2$

$= 25 + 25$

$= 50$

the numbers are -5 and 5
the minimum sum is 50

4. let x = one #
let y = other #

minimize sum S

$S = x^2 + y^2$

$xy = 16$

$y = \frac{16}{x}$

$S = x^2 + \left(\frac{16}{x}\right)^2$

$S = x^2 + 256x^{-2}$

$(-\infty, \infty)$ except 0

$\frac{ds}{dx} = 2x - 512x^{-3}$

$0 = 2x - \frac{512}{x^3}$

$\frac{512}{x^3} = 2x$

$512 = 2x^4$

$x^4 = 256$

$x = \pm 4$

$S(4) = (4)^2 + \left(\frac{16}{4}\right)^2$

$= 16 + 16 = 32$

$S(-4) = (-4)^2 + \left(\frac{16}{-4}\right)^2$

$= 32$

the #'s are 4 and 4 or -4 and -4
their minimum sum is 32

6. Let x = a number — minimize difference S
Let $\sqrt{x}$ = its root

$$S = x - \sqrt{x} \qquad [0, \infty)$$

$$\frac{dS}{dx} = 1 - \frac{1}{2}x^{-1/2}$$

$$0 = 1 - \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = 1$$

$$1 = 2\sqrt{x} \qquad 1 = 4x$$

$$x = \frac{1}{4}$$

$S(0) = 0 - \sqrt{0} = 0$

$S\left(\frac{1}{4}\right) = \frac{1}{4} - \sqrt{\frac{1}{4}} = -\frac{1}{4}$

the # is $\frac{1}{4}$

7. Let x = one #
Let $21 - x$ = other #

$$P = x^2(21 - x)$$
$$= 21x^2 - x^3, \quad [0, 21]$$

maximize product P

$$\frac{dP}{dx} = 42x - 3x^2$$

$$0 = 3x(14 - x)$$

$$x = 0 \qquad x = 14$$

$P(0) = 0(21 - 0) = 0$

$P(14) = 196(7)$
$= 1372$

$P(21) = 0$

the #s are 14 and 7
the max product is 1372

10. let x = a number
$10-x$ = other #
minimize sum S

$$S = \frac{1}{x} + \frac{1}{10-x}, (0,10)$$

$$S = x^{-1} + (10-x)^{-1},$$

$$\frac{ds}{dx} = -x^{-2} - (10-x)^{-2}(-1)$$

$$0 = -\frac{1}{x^2} + \frac{1}{(10-x)^2}$$

$$\frac{1}{x^2} = \frac{1}{(10-x)^2} \qquad x^2 = (10-x)^2$$

$$\pm x = \pm(10-x)$$

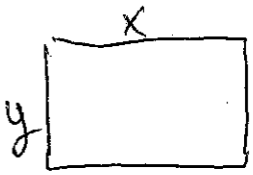
$$2x = 10$$

$$x = 5$$

$$S(5) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

the least possible sum is $\frac{2}{5}$

11.



max area A

$$A = xy$$

$$A = x(200-x)$$

$$A = 200x - x^2 \quad (0, 200)$$

$$2x + 2y = 400$$

$$x + y = 200$$

$$y = 200 - x$$

$$\frac{dA}{dx} = 200 - 2x$$

$$0 = 200 - 2x$$

$$2x = 200$$

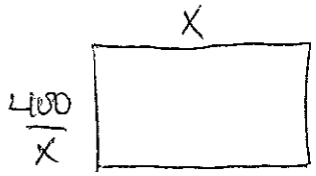
$$x = 100$$

$$A(100) = 100(100) = 10000$$

the max area is $10000\ m^2$ if the rectangle is a $100m \times 100m$ square

12. let x = width minimize perimeter P

$\frac{400}{x}$ = length $P = 2x + 2\left(\frac{400}{x}\right)$ $(0, \infty)$



$$\frac{dP}{dx} = 2 - \frac{800}{x^2} = 0$$

$$2 = \frac{800}{x^2}$$

$$x^2 = 400$$

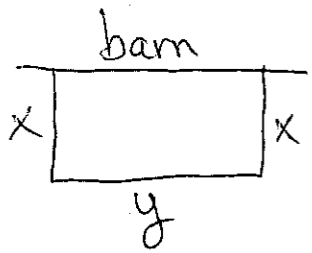
$$x = \pm 20$$

$P(20) = 40 + 2\left(\frac{40}{20}\right)$

$= 80$

the minimum perimeter is 80 m
if the rectangle is 20 m x 20 m

13.



$2x + y = 32$

$y = 32 - 2x$

$A = xy$ maximize area, A

$A = x(32 - 2x)$

$A = 32x - 2x^2$ $(0, 16)$

$$\frac{dA}{dx} = 32 - 4x = 0$$

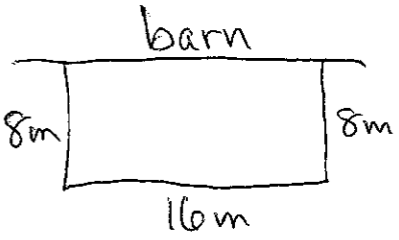
$$x = 8$$

max area is $128 m^2$

$A(8) = 32(8) - 2(8)^2$

$= 256 - 128$

$= 128$



15. max area A

$A = xy$ $\qquad x^2 + y^2 = 20^2$

$x^2 + y^2 = 400$

$A = x\left[400 - x^2\right]^{\frac{1}{2}}$ $\qquad y = (400 - x^2)^{1/2}$

$(0, 20)$

$$\frac{dA}{dx} = (1)(400 - x^2)^{1/2} + \frac{1}{2}(400 - x^2)^{-1/2}(-2x)(x)$$

$$0 = \sqrt{400 - x^2} - \frac{x^2}{\sqrt{400 - x^2}}$$

$x^2 = 400 - x^2$

$2x^2 = 400$

$x^2 = 200$

$x = \pm 10\sqrt{2}$

$A(10\sqrt{2}) = 10\sqrt{2}(400 - 200)^{1/2}$

$= 10\sqrt{2}\,(10\sqrt{2})$

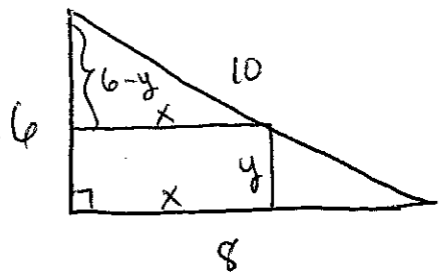
$= 200$

the dimensions are $10\sqrt{2}$ cm x $10\sqrt{2}$ cm

17. max area A

$A = xy$ $\qquad A = x\left(6 - \frac{3}{4}x\right)$

$= 6x - \frac{3x^2}{4}$ $\qquad (0, 8)$



$\frac{dA}{dx} = 6 - \frac{3}{2}x = 0$

$A(4) = 12$

$6 = \frac{3}{2}x$

$x = 4$

by similar $\triangle$s

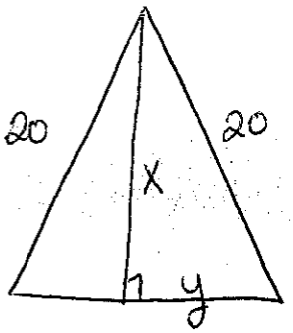
$\frac{6-y}{x} = \frac{6}{8}$

dimensions are 4cm x 3cm

$6x = 48 - 8y$

$4y = 24 - 3x$ $\qquad y = \frac{24 - 3x}{4} = 6 - \frac{3}{4}x$

19. Max Area A $\quad A = \frac{1}{2}bh$



$A = \frac{1}{2}(2\sqrt{400-x^2})(x) \qquad (0, 20)$

$x^2 + y^2 = 400$

$y = \sqrt{400-x^2}$

base $= 2\sqrt{400-x^2}$

$A = x(400-x^2)^{1/2}$

$\frac{dA}{dx} = (400-x^2)^{1/2} + \frac{1}{2}(400-x^2)^{-1/2}(-2x)(x)$

$0 = \sqrt{400-x^2} - \frac{x^2}{\sqrt{400-x^2}}$

$x^2 = 400 - x^2$

$2x^2 = 400$

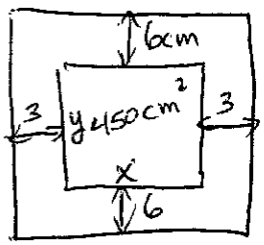
$x^2 = 200$

$x = 10\sqrt{2}$

$A(10\sqrt{2}) = 200$

max area is $200\ cm^2$.

20.



max area A

$A = (x+6)(y+12)$.

$A = xy + 12x + 6y + 72$

$xy = 450$

$y = \frac{450}{x}$

$A = x\left(\frac{450}{x}\right) + 12x + 6\left(\frac{450}{x}\right) + 72$

$A = 450 + 12x + 2700x^{-1} + 72 \qquad (0, \infty)$

$\frac{dA}{dx} = 12 - \frac{2700}{x^2} = 0$

$x^2 = 225$

$x = 15$

$A(15) = 882\ cm$

outside dimensions are 21cm x 42cm

21. max volume V

$$V = x(60-2x)^2 \qquad (0, 30)$$

$$\frac{dV}{dx} = (1)(60-2x)^2 + 2(60-2x)(-2)(x)$$

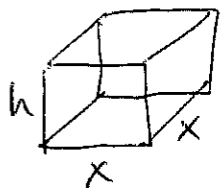
$$0 = (60-2x)[60-2x-4x]$$

$x = 30 \qquad x = 10$

$V(10) = 10(40)^2$
$= 16000$

the dimensions are $40\text{ cm} \times 40\text{cm} \times 10\text{cm}$
the squares are $10\text{cm} \times 10\text{cm}$

23.



$V = 32$ minimize surface area S

$$32 = x^2 h$$

$$h = \frac{32}{x^2}$$

$$S = x^2 + 4xh$$

$$S = x^2 + 4x\left(\frac{32}{x^2}\right)$$

$$S = x^2 + 128x^{-1} \qquad (0, \infty)$$

$$\frac{ds}{dx} = 2x - \frac{128}{x^2} = 0$$

$$2x = \frac{128}{x^2}$$

$$x^3 = 64$$

$$x = 4$$

$S(4) = 16 + \frac{128}{4}$
$= 48$

the dimensions are $4\text{m} \times 4\text{m} \times 2\text{m}$

24. max volume V

$2x+2y=216$

$x+y=108$

$y=108-x$

$x=2\pi r$

$V=\pi r^2 y$

$V=\pi r^2(108-x)$

$V=\pi r^2(108-2\pi r)$

$V=108\pi r^2-2\pi^2 r^3$ $\quad (0, \frac{54}{\pi})$

$\frac{dV}{dr}=216\pi r-6\pi^2 r^2=0$

$6\pi r[36-\pi r]=0$

$r=0 \quad r=\frac{36}{\pi}$

the dimensions are 72cm x 36cm $\quad V(\frac{36}{\pi})=$

25. max area A

$A=2xy$

$A=2x(9-x^2)$

$A=18x-2x^3$ $\quad (0, 3)$

$\frac{dA}{dx}=18-6x^2=0$

$x^2=3$

$x=\sqrt{3}$

$A(\sqrt{3})=2\sqrt{3}(9-3)$

$=12\sqrt{3}$

max area is $12\sqrt{3}\ u^2$

26. max volume V

$4x+y=300$

$y=300-4x$

$V = x^2y$

$V = x^2(300-4x)$

$V = 300x^2 - 4x^3$

$\frac{dV}{dx} = 600x - 12x^2$

$0 = 12x(50-x)$

$x=0 \quad x=50$

$(0, 75)$

$V(50) = 250000$

max volume is $50\text{cm} \times 50\text{cm} \times 100\text{cm}$

$= 250000\text{cm}^3$

27. let x = # of \$1 drops

max revenue R

$R = (\text{price per ticket}) \times (\#\text{ of fans})$

$= (20-x)(3000+500x)$

$R = 60000 + 7000x - 500x^2 \quad [0, 20]$

$\frac{dR}{dx} = 7000 - 1000x = 0$

$x = 7$

$R(0) = 60000$

$R(20) = 0$

$R(7) = 84500$

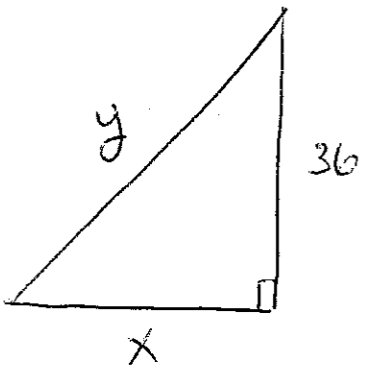
the ticket price is \$13

max revenue is \$84500

6500 fans

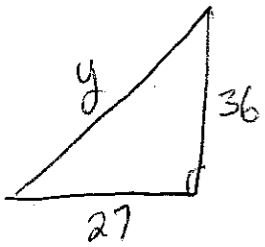
Written exercises 6.4

1.



$\frac{dx}{dt} = -60$

find $\frac{dy}{dt}\Big|_{x=27}$



$x^2 + 36^2 = y^2$

$27^2 + 36^2 = y^2$

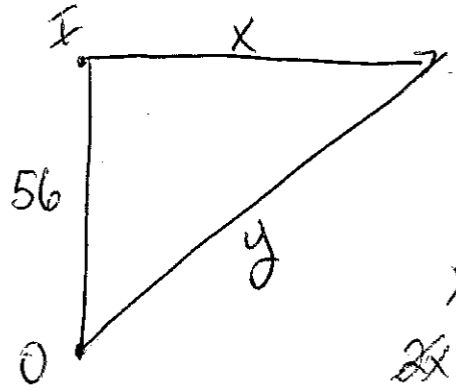
$y = 45$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$(27)(-60) = 45\frac{dy}{dt}$

$\frac{dy}{dt} = -36 m/min$

2.



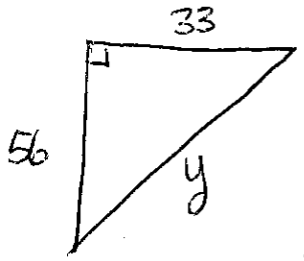
$\frac{dx}{dt} = 30$ find $\frac{dy}{dt}\Big|_{x=33}$

$x^2 + 56^2 = y^2$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$(33)(30) = 65\frac{dy}{dt}$

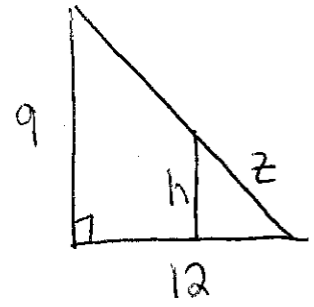
$\frac{dy}{dt} = \frac{990}{65} \approx 15.23 km/h$



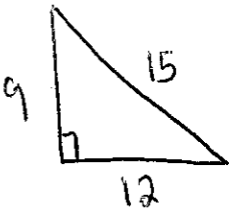
$33^2 + 56^2 = y^2$

$y = 65$

3.



$\frac{dz}{dt} = -20$ find $\frac{dh}{dt}\Big|_t$



$9^2 + 12^2 = 15^2$

$\frac{z}{h} = \frac{15}{9}$

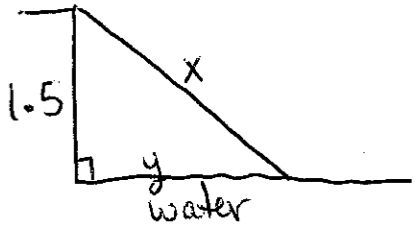
$9z = 15h$

$z = \frac{5}{3}h$

$\frac{dz}{dt} = \frac{5}{3}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3}{5}(-20) = -12 \text{ km/h}$

4.



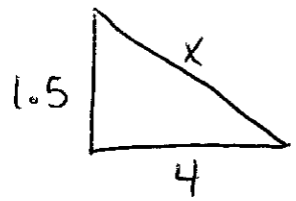
$\frac{dx}{dt} = -1$ find $\frac{dy}{dt}\Big|_{y=4}$

$y^2 + (1.5)^2 = x^2$

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$4\frac{dy}{dt} = 4.27(-1)$

$\frac{dy}{dt} = -1.07 \text{ m/s}$

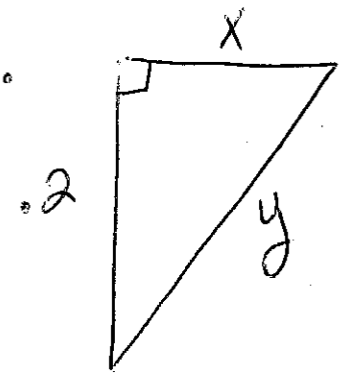


$(1.5)^2 + (4)^2 = x^2$

$x^2 = 18.25$

$x = 4.27$

5.



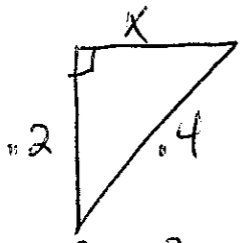
$\frac{dx}{dt} = 15$ find $\frac{dy}{dt}\Big|_{y=.4}$

$x^2 + .2^2 = y^2$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$\frac{(.35)(15)}{.4} = \frac{dy}{dt}$

$\frac{dy}{dt} = 12.99 \text{ km/h}$



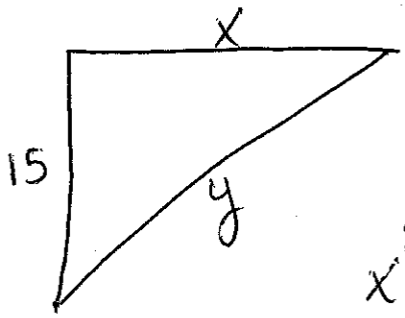
$.2^2 + x^2 = .4^2$

$.04 + x^2 = .16$

$x^2 = .12$

$x = .35$

6.



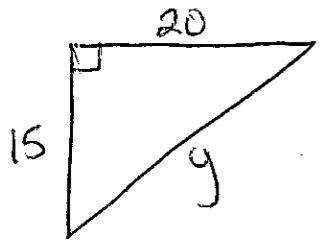
$\frac{dx}{dt} = 8$ find $\frac{dy}{dt}\Big|_{x=20}$

$x^2 + 15^2 = y^2$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

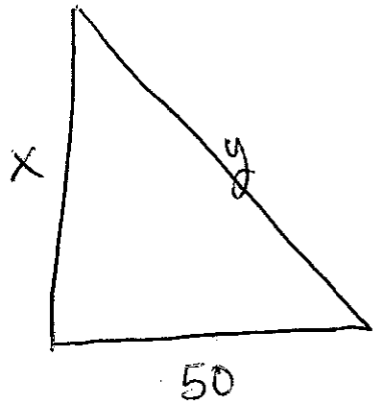
$\frac{(20)(8)}{25} = \frac{dy}{dt}$

$\frac{dy}{dt} = 6.4 \text{ m/s}$



$y = 25$

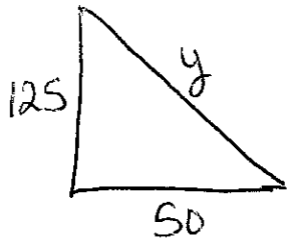
7.



$\frac{dx}{dt} = 2.4$ find $\frac{dy}{dt}\Big|_{x=125}$

$x^2 + 50^2 = y^2$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$



$50^2 + 125^2 = y^2$

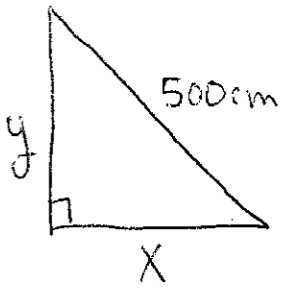
$y^2 = 18125$

$y = 134.63$

$\frac{(125)(2.4)}{134.63} = \frac{dy}{dt}$

$\frac{dy}{dt} = 2.23 \text{ m/s}$

8.



$\frac{dx}{dt} = 12 \text{cm/s}$ find $\frac{dy}{dt}\Big|_{x=400}$

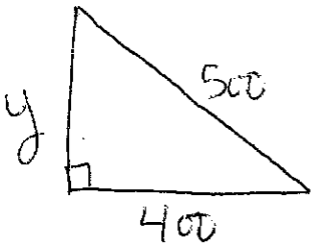
$x^2 + y^2 = 500^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$400(12) + 300\frac{dy}{dt} = 0$

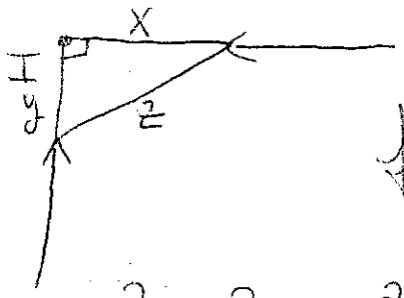
$\frac{dy}{dt} = \frac{-400(12)}{300}$

$= -16 \text{cm/s}$



$y = 300$

9.



$\frac{dx}{dt} = -80 \quad \frac{dy}{dt} = -100$

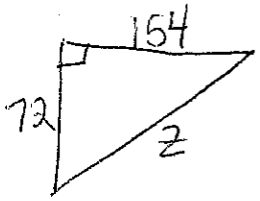
find $\frac{dz}{dt}\Big|_{\substack{x=154\\y=72}}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(154)(-80) + (72)(-100) = 170\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{-19520}{170} = 114.82 \text{ km/h}$

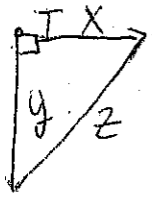


$72^2 + 154^2 = z^2$

$28900 = z^2$

$z = 170$

10.



$\frac{dx}{dt} = 55 \quad \frac{dy}{dx} = 50$

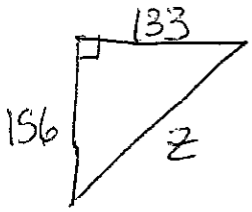
find $\frac{dz}{dt}\Big|_{\substack{y=156\\x=133}}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(133)(55) + (156)(50) = 205\frac{dz}{dt}$

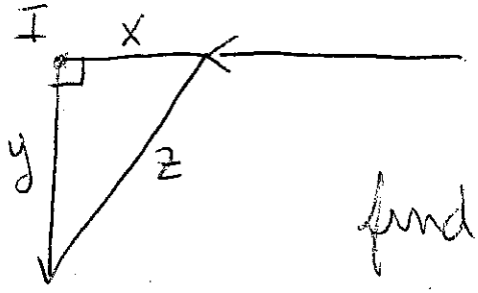
$\frac{15115}{205} = 73.73 \text{km/hr} = \frac{dz}{dt}$



$156^2 + 133^2 = z^2$

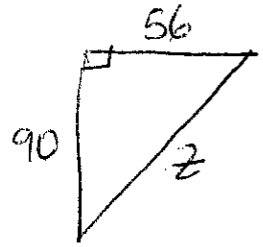
$z = 205$

11.



$\frac{dx}{dt} = -75 \quad \frac{dy}{dt} = 45$

find $\left.\frac{dz}{dt}\right|_{\substack{x=56 \\ y=90}}$



$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

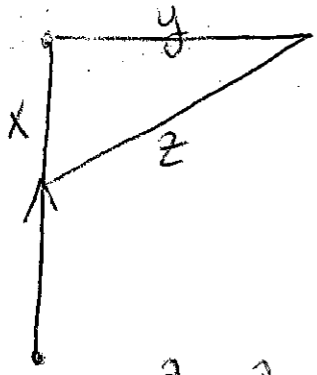
$(56)(-75) + (90)(45) = (106)\frac{dz}{dt}$

$56^2 + 90^2 = z^2$

$z = 106$

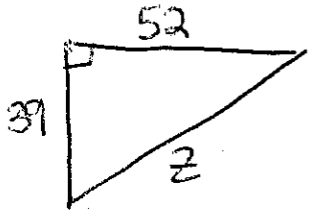
$\frac{dz}{dt} = -1.42 \text{ km/h}$

12.



$\frac{dx}{dt} = -80 \quad \frac{dy}{dt} = 60$

find $\left.\frac{dz}{dt}\right|_{\substack{x=39 \\ y=52}}$



$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(39)(-80) + (52)(60) = 65\frac{dz}{dt}$

$39^2 + 52^2 = z^2$

$z^2 = 4225$

$z = 65$

$0 = 65\frac{dz}{dt}$

$\frac{dz}{dt} = 0$

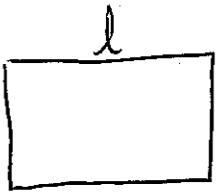
Written exercises 6.5

1. $\frac{dr}{dt} = 25\text{cm/s}$ find $\left.\frac{dA}{dt}\right|_{r=200}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\left.\frac{dA}{dt}\right|_{r=200} = 2\pi(200)(25)$

$= 10000\pi\,\text{cm}^2/\text{s}$

2. 

$\frac{dl}{dt} = 80\text{m/min}$ $w = 13\text{m}$

find $\frac{dA}{dt}$

$A = lw$

$A = 13l$

$\frac{dA}{dt} = 13\frac{dl}{dt} = 13(80)$

$= 1040\,\text{m}^2/\text{min}$

3. $\frac{dr}{dt} = -2\text{mm/day}$ find $\left.\frac{dC}{dt}\right|_{r=25\text{mm}}$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\frac{dr}{dt} = 2\pi(-2)$

$= -4\pi\,\text{mm/day}$

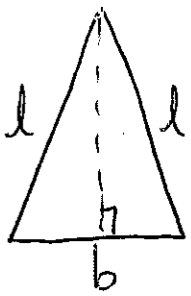
4. $\frac{dl}{dt} = 10\text{cm/s}$ $\frac{dw}{dt} = -15\text{cm/s}$ find $\left.\frac{dA}{dt}\right|_{\substack{l=30\\w=22}}$

$A = lw$

$\frac{dA}{dt} = \frac{dl}{dt}w + \frac{dw}{dt}l = (10)(22) + (-15)(30)$

$= -230\,\text{cm}^2/\text{s}$

5.



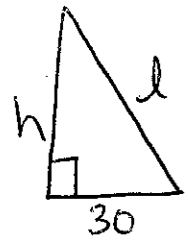
$\frac{dl}{dt} = 16 \text{ cm/s}$ $\quad b = 60$

find $\left.\frac{dA}{dt}\right|_{l=50}$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \cdot 60h$

$A = 30h$



$h^2 + 30^2 = l^2$

$h = (l^2 - 900)^{1/2}$

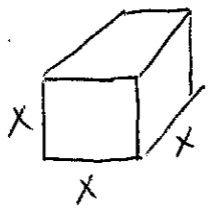
$A = 30(l^2 - 900)^{1/2}$

$\frac{dA}{dt} = 30\left(\frac{1}{2}\right)(l^2 - 900)^{-1/2}(2l)\frac{dl}{dt}$

$\left.\frac{dA}{dt}\right|_{l=50} = 30(2500 - 900)^{-1/2}(50)(16)$

$= \frac{(30)(50)(16)}{\sqrt{1600}} = 600 \text{ cm}^2/\text{s}$

6. $\frac{dV}{dt} = -15 \text{ mm}^3/\text{s}$ $\quad$ find $\left.\frac{dx}{dt}\right|_{x=30}$



$V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$-15 = 3(30)^2\left(\frac{dx}{dt}\right)$

$\frac{dx}{dt} = \frac{-15}{(3)(30)^2} = -\frac{1}{180} \text{ mm/s}$

7.

a) $\frac{dr}{dt} = 3\text{cm/s}$ find $\left.\frac{dA}{dt}\right|_{r=20\text{cm}}$

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(3)(20)$

$= 480\pi \text{ cm}^2/\text{s}$

b) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(20)^2(3)$

$= 4800\pi \text{ cm}^2/\text{s}$

8. find $\frac{dV}{dt}$ when $\frac{dr}{dt} = 2\text{cm/s}$, $r = 8\text{cm}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(8)^2(2)$

$= 512\pi \text{ cm}^3/\text{s}$

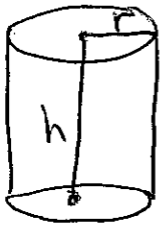
9. find $\frac{dv}{dt}$ $r = 5\text{cm}$ $\frac{dh}{dt} = 0.4\text{cm/s}$

$V = \pi r^2 h$

$= 25\pi h$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$ $= 25\pi(0.4)$

$= 10\pi \text{ cm}^3/\text{s}$



10. $\frac{dA}{dt} = 128\pi \text{ cm}^2/\text{s}$ find $\frac{dV}{dt}$ when $r = 4\text{cm}$

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$128\pi = 8\pi(4)\frac{dr}{dt}$

$\frac{dr}{dt} = 4 \text{ cm/s}$

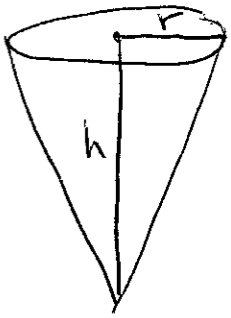
$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt}\Big|_{r=4} = 4\pi(4)^2(4)$

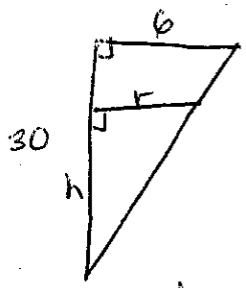
$= 256\pi \text{ cm}^3/\text{s}$

11.



$\frac{dV}{dt} = 50$

find $\frac{dh}{dt}\Big|_{h=10}$



$\frac{30}{6} = \frac{h}{r}$

$\frac{h}{r} = 5$

$h = 5r$

$r = \frac{1}{5}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{1}{5}h\right)^2(h)$

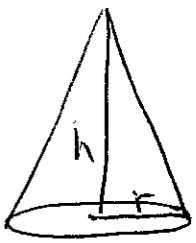
$V = \frac{\pi}{75}h^3$

$\frac{dV}{dt} = \frac{3\pi}{75}h^2\frac{dh}{dt}$

$50 = \frac{\pi(10)^2}{25}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{(50)(25)}{100\pi} = \frac{25}{2\pi} \text{ cm/s}$

12.



find $\frac{dV}{dt}$ $\quad \frac{dr}{dt} = 0.8 \quad r = 3$

$r = 3h$

$h = \frac{1}{3}r$

$V = \frac{1}{3}\pi r^2 h$

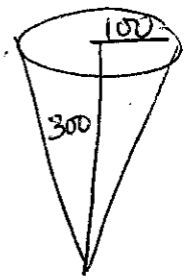
$V = \frac{1}{3}\pi r^2\left(\frac{1}{3}r\right)$

$V = \frac{\pi}{9}r^3$

$\frac{dV}{dt} = \frac{3\pi}{9} r^2 \frac{dr}{dt} = \frac{\pi}{3}(3)^2\left(\frac{4}{5}\right)$

$= \frac{12\pi}{5} m^3/min$

13.



$\frac{dV}{dt} = 20 L/s = 20000 cm^3/s$

find $\frac{dh}{dt}$ when $h = 150$

$\frac{h}{r} = \frac{3}{1}$

$3r = h$

$r = \frac{h}{3}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

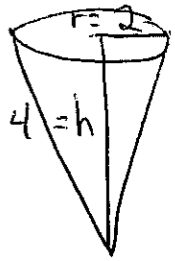
$V = \frac{\pi h^3}{27}$

$\frac{dV}{dt} = \frac{3\pi}{27} h^2 \frac{dh}{dt}$

$20000 = \frac{\pi}{9}(150)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{20000(9)}{(150)^2\pi} = \frac{8}{\pi} cm/s$

14.



find $\frac{dV}{dt}$ when $h = 3m$, $h = 300cm$, $\frac{dh}{dt} = -0.05 cm/s$

$\frac{h}{r} = \frac{4}{2}$

$4r = 2h$

$2r = h$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi r^2 h$

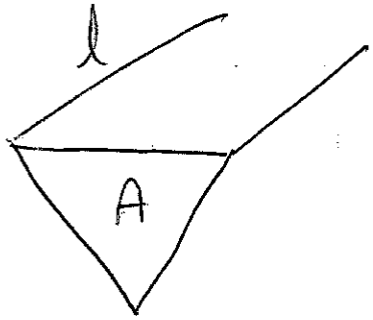
$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$= \frac{\pi}{4}(300)^2(-.05) = -1125\pi\ cm^3/s$

5.



$V = A \cdot l$

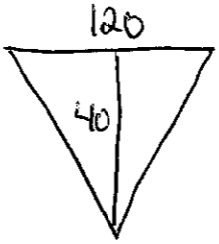
$V = 500A$

$V = 500\left(\frac{1}{2}bh\right)$

$= 250bh$

$\frac{dV}{dt} = 6500$

find $\frac{dh}{dt}$ when $h = 5$



$\frac{b}{h} = \frac{3}{1}$

$b = 3h$

$V = 250(3h)(h)$

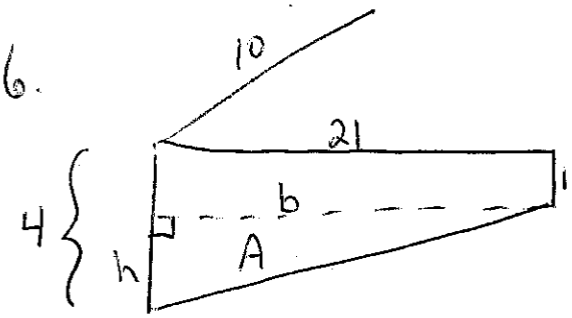
$V = 750h^2$

$\frac{dV}{dt} = 1500h\frac{dh}{dt}$

$\frac{6500}{1500(5)} = \frac{dh}{dt}$

$= \frac{13}{15}\ cm/s$

16.



$V = A \cdot 10$

$V = 10A$

$V = 10\left(\frac{1}{2}bh\right)$

$V = 5bh$

$\frac{dV}{dt} = \frac{3}{2}\ m^3/min$

$\frac{b}{h} = \frac{21}{3} = 7$

find $\left.\frac{dh}{dt}\right|_{h=\frac{5}{2}m}$

$b = 7h$

$\therefore\ V = 5(7h)(h)$

$= 35h^2$

$\frac{dV}{dt} = 70h\frac{dh}{dt}$

$\frac{3}{2} = 70\left(\frac{5}{2}\right)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{(3)(2)}{(2)(350)} = \frac{6}{700}\ m/min$ or $\frac{600}{700}$

$\frac{6}{7}\ cm/min$

17. find $\left.\frac{dV}{dt}\right|_{\substack{r=9\\h=14}}$

$V = \pi r^2 h + \frac{4}{3}\pi r^3$

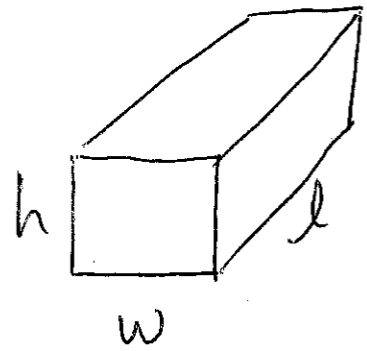
$\frac{dr}{dt} = 3 \quad \frac{dh}{dt} = -2$

$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt}h + \frac{dh}{dt}r^2 + 4r^2\frac{dr}{dt}\right]$

$= \pi\left[(18)(3)(14) + (-2)(81) + (4)(81)(3)\right]$

$= \pi\left[756 - 162 + 972\right]$

$= 1566\pi\ cm^3/s$

18. 

$V = lwh$

$\frac{dl}{dt} = 6 \quad \frac{dw}{dt} = -5 \quad \frac{dh}{dt} = 3$

find $\frac{dV}{dt}\Big|_{\substack{l=14\\w=12\\h=10}}$

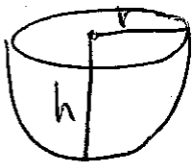
$V = (lw)(h)$

$\frac{dV}{dt} = \left[\frac{dl}{dt}w + \frac{dw}{dt}l\right]h + \frac{dh}{dt}(lw)$

$= \left[(6)(12) + (-5)(14)\right](10) + (3)(14)(12)$

$= (72 - 70)(10) + 504$

$= 524 \text{ cm}^3/\text{s}$

19. 

$\frac{dV}{dt} = -36 \quad r = 20$ $\therefore V = \frac{1}{3}\pi h^2(60 - h)$

find $\frac{dh}{dt}\Big|_{h=4}$ $= 20\pi h^2 - \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = 40\pi h\frac{dh}{dt} - \pi h^2\frac{dh}{dt}$

$-36 = 40\pi(4)\frac{dh}{dt} - \pi(16)\frac{dh}{dt}$

$-36 = 160\pi\frac{dh}{dt} - 16\pi\frac{dh}{dt}$

$= 144\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-36}{144\pi} = \frac{-1}{4\pi} \text{ cm/s}$

20. $\frac{dV}{dt} = -200$ find $\frac{dh}{dt}\Big|_{h=21}$

$\frac{h}{r} = \frac{45}{21}$

$r = \frac{7}{15}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{7}{15}h\right)^2 h$

$V = \frac{49\pi h^3}{675}$

$\frac{dV}{dt} = \frac{(3)(49)\pi h^2}{675}\frac{dh}{dt}$

$\frac{-200(675)}{(3)(49)(\pi(21)^2} = \frac{dh}{dt} = -\frac{5000}{2401\pi}$ cm/s