written exercises 5.1

1. $f(x) = 5x^{10}$ $f'(x) = 50x^9$ $f''(x) = 450x^8$

3. $y = \frac{12}{x} = 12x^{-1}$

$\frac{dy}{dx} = -12x^{-2}$ $\frac{d^2y}{dx^2} = 24x^{-3}$

5. $f(x) = \pi^3$ $f'(x) = 0$ $f''(x) = 0$

7. $y = -4x^3 - 5x^2 + 11x - 81$

$\frac{dy}{dx} = -12x^2 - 10x + 11$

$\frac{d^2y}{dx^2} = -24x - 10$

9. $f(x) = (3x+5)^6$

$f'(x) = 6(3x+5)^5(3) = 18(3x+5)^5$

$f''(x) = 90(3x+5)^4(3) = 270(3x+5)^4$

11. $y = x^3(x^2+1)^6$

$$\frac{dy}{dx} = 3x^2(x^2+1)^6 + 6(x^2+1)^5(2x)(x^3)$$
$$= 3x^2(x^2+1)^5\left[x^2+1 + 4x^2\right] = 3x^2(x^2+1)^5(5x^2+1)$$
$$= (x^2+1)^5(15x^4+3x^2)$$

$$\frac{d^2y}{dx^2} = 5(x^2+1)^4(2x)(15x^4+3x^2) + (60x^3+6x)(x^2+1)^5$$
$$= 30(x^2+1)^4(x)(5x^4+x^2) + (6)(x)(10x^2+1)(x^2+1)^5$$
$$= 6x(x^2+1)^4\left[5(5x^4+x^2)\right] + \left[10x^2+1\right]\left[x^2+1\right]$$
$$= 6x(x^2+1)^4(35x^4+16x^2+1)$$

13. $f(x) = \dfrac{x}{x+2}$

$$f'(x) = \frac{(1)(x+2) - (1)(x)}{(x+2)^2} = \frac{2}{(x+2)^2} = 2(x+2)^{-2}$$

$$f''(x) = -4(x+2)^{-3}$$

15. $y = \dfrac{x^2}{\sqrt{x+1}} = \dfrac{x^2}{(x+1)^{1/2}}$

$$\frac{dy}{dx} = \frac{2x(x+1)^{1/2} + \frac{1}{2}(x+1)^{-1/2}(x^2)}{\left[(x+1)^{1/2}\right]^2}$$

$$= \frac{2x(x+1)^{-1/2}\left[x+1 + \frac{1}{4}x\right]}{(x+1)^1} = \frac{2x\left(\frac{5}{4}x+1\right)}{(x+1)^{3/2}}$$

$$= \frac{\left(\frac{5}{2}x^2+2x\right)}{(x+1)^{3/2}}$$

$$\frac{d^2y}{dx^2} = \frac{(5x+2)(x+1)^{3/2} + \frac{3}{2}(x+1)^{1/2}\left(\frac{5}{2}x^2+2x\right)}{\left[(x+1)^{3/2}\right]^2}$$

$$= \frac{(x+1)^{1/2}\left[(5x+2)(x+1) + \frac{3}{2}\left(\frac{5}{2}x^2+2x\right)\right]}{(x+1)^3}$$

$$= (x+1)^{-\frac{5}{2}}\left(\frac{3}{4}x^2+2x+2\right)$$

written exercises 5.2

4. a) $f(x) = 2x+5$

$f'(x) = 2 \qquad 2 \neq 0 \qquad$ no critical pts

c) $f(x) = 2x^3 - 24x + 21$

$f'(x) = 6x^2 - 24 = 0$

$6(x^2-4) = 0$

$6(x-2)(x+2) = 0$

$x=2 \quad x=-2$

e) $f(x) = 6x^{2/3}$

$f'(x) = 4x^{-1/3} = \dfrac{4}{\sqrt[3]{x}} \qquad x=0$

g) $y = \dfrac{x^2}{x-2}$

$\dfrac{dy}{dx} = \dfrac{2x(x-2) - (1)(x^2)}{(x-2)^2} = \dfrac{x^2 - 4x}{(x-2)^2}$

$= \dfrac{x(x-4)}{(x-2)^2} = 0 \qquad x=0$
$x=4$

i) $f(x) = \frac{4}{3}x^3 - 6x^{\frac{2}{3}}$

$f'(x) = 4x^2 - 4x^{-\frac{1}{3}} = 0$

$x^2 - \dfrac{1}{x^{1/3}} = 0$

$x=0$

$x^2 = \dfrac{1}{x^{1/3}}$

$x^{7/3} = 1$

$x = 1$

k) $y = |x^2 - 9|$

recall that $|x| = \sqrt{x^2}$

$\therefore y = \sqrt{(x^2-9)^2} = \left[(x^2-9)^2\right]^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}\left[(x^2-9)^2\right]^{-1/2}\left[2(x^2-9)(2x)\right]$

$= \dfrac{2x(x^2-9)}{\sqrt{(x^2-9)^2}} = 0$ $\quad x=0$ since $f'(0)=0$

and $x = \pm 3$ since $f'(\pm 3)$ is undefined

5a) $f(x) = 3x - 12$, $[-1, 3]$ $\quad f(-1) = -15$

$f'(x) = 3$ no critical points $\quad f(3) = -3$

max is -3

min is -15

c) $y = 4x^3 - 12x - 5$, $[-3, 2]$

$\frac{dy}{dx} = 12x^2 - 12 = 0$

$12(x-1)(x+1) = 0$

$x = 1 \quad x = -1$

$f(-3) = -77$

$f(2) = 3$ $\quad$ max is 3

$f(1) = -13$ $\quad$ min is -77

$f(-1) = 3$

e) $f(x) = \frac{3}{4}x^{\frac{4}{3}} - 2x \; ; [1, 27]$

$f'(x) = x^{\frac{1}{3}} - 2 = 0$

$\sqrt[3]{x} = 2$

$x = 8$

$f(8) = -4$

$f(1) = \frac{-5}{4}$

$f(27) = \frac{27}{4}$

min is -4

max is $\frac{27}{4}$

g) $y = 3x^{4/3} - 12x^{1/3}, \; [-1, 8]$

$\frac{dy}{dx} = 4x^{\frac{1}{3}} - 4x^{-\frac{2}{3}} = 0$

$\sqrt[3]{x} - \frac{1}{\sqrt[3]{x^2}} = 0$

$f(-1) = 15$

$f(8) = 24$

$f(0) = 0$

$f(1) = -9$

Critical point $x = 0$ since $f'(0)$ is undefined

$\frac{\sqrt[3]{x}}{1} = \frac{1}{\sqrt[3]{x^2}}$

$\sqrt[3]{x^3} = 1$

$x = 1$

max is 24

min is -9

i) $f(x) = x\sqrt{50 - x^2} = x(50 - x^2)^{1/2} \; ; [0, 7]$

$f'(x) = (1)(50 - x^2)^{1/2} + \frac{1}{2}(50 - x^2)^{-1/2}(-2x)(x)$

$= \sqrt{50 - x^2} - \frac{x^2}{\sqrt{50 - x^2}} = 0$

$f(0) = 0 \leftarrow$ min

$f(7) = 7$

$f(5) = 25 \leftarrow$ max

$f'(x)$ is undefined at $\sqrt{50}$ which is outside interval

$\frac{\sqrt{50 - x^2}}{1} = \frac{x^2}{\sqrt{50 - x^2}}$

$50 - x^2 = x^2$

$50 = 2x^2 \quad x^2 = 25 \quad x = \pm 5$

$x = -5$ outside interval

$x = 5$

written exercises 5.3

1. $f(x) = 4$
$f'(x) = 0$

this is a horizontal lines, therefore it is never increasing or decreasing and there are no relative extrema

3. $f(x) = x^2 + 6x - 8$
$f'(x) = 2x + 6 = 0$
$x = -3$

x	$(-\infty,-3)$	-3	$(-3,\infty)$
$f'(x)$	$-$	0	$+$

increasing $(-3,\infty)$ decreasing $(-\infty,-3)$
relative minimum at $(-3,-17)$

5. $f(x) = x^3 - 27x$
$f'(x) = 3x^2 - 27 = 0$
$3(x-3)(x+3) = 0$
$x = 3$ $x = -3$

x	$(-\infty,-3)$	-3	$(-3,3)$	3	$(3,\infty)$
$f'(x)$	$+$	0	$-$	0	.

increasing $(-\infty,-3) \cup (3,\infty)$
decreasing $(-3,3)$
relative max $(-3,54)$ relative min $(3,-54)$

7. $f(x) = -x^3 - 3x^2 + 24x + 20$
$f'(x) = -3x^2 - 6x + 24 = 0$
$= -3(x^2 + 2x - 8) = 0$
$-3(x+4)(x-2) = 0$
$x = -4$ $x = 2$

x	$(-\infty,-4)$	-4	$(-4,2)$	2	$(2,\infty)$
$f'(x)$	$-$	0	$+$	0	$-$

relative min $(-4,-60)$
relative max $(2,48)$

increasing $(-4,2)$
decreasing $(-\infty,-4) \cup (2,\infty)$

9. $f(x) = 3x^4 + 4x^3 - 36x^2 + 11$

$f'(x) = 12x^3 + 12x^2 - 72x = 0$

$12x(x^2 + x - 6) = 0$

$12x(x+3)(x-2) = 0$

$x=0 \quad x=-3 \quad x=2$

increasing $(-3,0) \cup (2,\infty)$
decreasing $(-\infty,-3) \cup (0,2)$

relative max $(0, 11)$

relative min $(-3, -178)$
$(2, -53)$

x	$(-\infty,-3)$	-3	$(-3,0)$	0	$(0,2)$	2	$(2,\infty)$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

11. $f(x) = 2\sqrt{x} = 2x^{\frac{1}{2}}$

$f'(x) = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}} = 0$

no solution but critical value at $x=0$
since $f'(x)$ is undefined at $x=0$

x	0	$(0,\infty)$
$f'(x)$	und	$+$

increasing $(0,\infty)$
never decreasing
no relative extrema

13. $f(x) = x - 12x^{\frac{1}{3}}$

$f'(x) = 1 - 4x^{-2/3}$

$1 - \frac{4}{\sqrt[3]{x^2}} = 0$

$1 = \frac{4}{\sqrt[3]{x^2}}$ ← does not exist at $x=0$

$\sqrt[3]{x^2} = 4$

$x^2 = 64$

$x = \pm 8$

x	$(-\infty,-8)$	-8	$(-8,0)$	0	$(0,8)$	8	$(8,\infty)$
$f'(x)$	$+$	0	$-$	dne	$-$	0	$+$

increasing $(-\infty,-8) \cup (8,\infty)$
decreasing $(-8,8)$

relative max $(-8, 16)$
relative min $(8, -16)$

15. $f(x) = \dfrac{2x}{x+2}$ f(x) is undefined at $x = -2$ vert asymptote

$f'(x) = \dfrac{2(x+2) - (1)(2x)}{(x+2)^2}$

$= \dfrac{4}{(x+2)^2} = 0$ no critical points

x	$(-\infty,-2)$	-2	$(-2,\infty)$
$f'(x)$	+	und	+

increasing $(-\infty,-2) \cup (-2,\infty)$

never decreasing no relative extrema

19 a) increasing $(0,3)$ since $f'(x) > 0$

b) decreasing $(-2,0)$ since $f'(x) < 0$

c) $x = 0$ is a relative min since $f'(x)$ changes from neg to pos.

no relative max values

d) $x = -2$ since f(x) is decreasing from $(-2,0)$

20. see textbook

written exercises 5.4

2 all, 3 all } see solutions in textbook

4. $f(x) = x^2 - 5x - 14$

$f'(x) = 2x - 5$

$f''(x) = 2$ no critical points

f'' is always positive

therefore concave up $(-\infty, \infty)$

never concave down

no inflection points

5. $f(x) = \frac{1}{3}x^3 + 2x^2$

$f'(x) = x^2 + 4x$

$f''(x) = 2x + 4 = 0$

$x = -2$

x	$(-\infty,-2)$	-2	$(-2,\infty)$
$f''(x)$	$-$	0	$+$

concave up $(-2, \infty)$

concave down $(-\infty, -2)$

inflection point $(-2, f(-2))$

$(-2, \frac{16}{3})$

6. $f(x) = \frac{1}{4}x^4 - 6x^2$

$f'(x) = x^3 - 12x$

$f''(x) = 3x^2 - 12 = 0$

$3(x-2)(x+2)$

$x = 2$ $x = -2$

x	$(-\infty,-2)$	-2	$(-2,2)$	2	$(2,\infty)$
$f''(x)$	$+$	0	$-$	0	$+$

concave up $(-\infty, -2) \cup (2, \infty)$

concave down $(-2, 2)$

inflection points $(-2, -20)$ and $(2, -20)$

written exercise 5.5 matching graph

1. $f(x) = \frac{2x}{x+1}$ vertical asymptote (v.a) f

$x+1=0$

$x=-1$

horizontal asymptote (h.a)

$$\frac{2x}{x+1}\frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \frac{2}{1+\frac{1}{x}} \quad \lim_{x\to\infty} = 2$$

h.a $y=2$

y-int

$$y = \frac{2(0)}{0+1} = 0$$

x-int

$$\frac{0}{1} = \frac{2x}{x+1}$$

$2x=0$

$x=0$

2. $f(x) = \frac{2x}{x^2+1}$ v.a none since $x^2+1 \neq 0$

h.a $$\frac{2x}{x^2+1}\frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \frac{\frac{2}{x}}{1+\frac{1}{x^2}} \quad \lim_{x\to\infty} = 0$$

h.a $y=0$ d

y-int

$$y = \frac{2(0)}{(0)^2+1} = 0$$

x-int

$$\frac{0}{1} = \frac{2x}{x^2+1}$$

$x=0$

Matching graph

3. $f(x) = \dfrac{x^2}{x-1}$ V.a $x-1=0$, $x=1$

$$\begin{array}{r} x+1 \\ x-1 \overline{) x^2+0x+0} \\ \underline{x^2-x} \\ x \\ \underline{x-1} \\ 1 \end{array}$$

y-int $=0$

x-int $=0$

l

$$\frac{x^2}{x-1} = x+1+\frac{1}{x-1}$$

as $x \to \pm\infty$ $= x+1$

$\therefore$ slant asymptote of $y=x+1$

4. $f(x) = \dfrac{x^2}{x+1}$ V.a $x+1=0$, $x=-1$

$$\begin{array}{r} x-1 \\ x+1 \overline{) x^2+0x+0} \\ \underline{x^2+1} \\ -x \\ \underline{-x-1} \\ 1 \end{array}$$

$x-1+\dfrac{1}{x+1}$

a

slant asym $y=x-1$

y-int $=0$ x-int $=0$

matching graph

5. $f(x) = \dfrac{x^2-1}{x^2+1}$ v.a none

since $x^2+1 \neq 0$

$\dfrac{x^2-1}{x^2+1}\dfrac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \dfrac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}}$ as $x \to \pm\infty$

$y=1$
h.a

j

x-int

$\dfrac{0}{1} = \dfrac{x^2-1}{x^2+1}$

$x^2-1=0$

$x=\pm 1$

y-int $y = \dfrac{0-1}{0+1}$

$y=-1$

6. $f(x) = \dfrac{x^2+1}{x^2-1}$ v.a $x^2-1=0$

$x=1$ $x=-1$

h.a $y=1$ (same as #5)

h

x-int

$\dfrac{0}{1} = \dfrac{x^2+1}{x^2-1}$

$x^2+1=0$

$x^2=-1$

no solution

$\therefore$ no x-int

y-int

$y=\dfrac{0+1}{0-1}$

$y=-1$

matching graph

7. $f(x) = \dfrac{x^3 + x^2 + 2x + 1}{x+1}$

b

v.a $x+1=0$

$x=-1$

$$\begin{array}{r|l} & x^2+2 \\ \hline x+1 & x^3+x^2+2x+1 \\ & \underline{x^3+x^2} \\ & \quad 2x \\ & \quad \underline{2x+2} \\ & \quad\quad -1 \end{array}$$

$x^2+2 - \dfrac{1}{x+1}$ parabolic asymptote $y=x^2+2$

as $x \to \pm\infty$

y-int $y = \frac{1}{1} = 1$

13. $f(x) = \dfrac{2x+3}{x-3}$ v.a $x-3=0$

$x=3$

h.a $\dfrac{2x+3\left(\frac{1}{x}\right)}{x-3\left(\frac{1}{x}\right)}$ $\dfrac{2+\frac{3}{x}}{1-\frac{3}{x}}$ as $x \to \pm\infty$ $y=2$

x-int $\dfrac{0}{1} = \dfrac{2x+3}{x-3}$

$0 = 2x+3$

$x = -\frac{3}{2}$

y-int $y = \dfrac{0+3}{0-3}$

$y=-1$

15 $f(x) = \dfrac{x^3+1}{x-1}$ v.a $x-1=0$

$x=1$

$$\begin{array}{r}
x^2+x+1 \\
x-1 \enclose{longdiv}{x^3+0x^2+0x+1} \\
\underline{x^3-x^2} \\
x^2+0x \\
\underline{x^2-x} \\
x+1 \\
\underline{x-1} \\
2
\end{array}$$

$x^2+x+1+\dfrac{2}{x-1}$

as $x \to \pm\infty$

$y = x^2+x+1$

parabolic asymp.

<u>x-int</u> $\dfrac{0}{1} = \dfrac{x^3+1}{x-1}$

$x^3+1=0$

$x^3=-1$

$x=-1$

<u>y-int</u> $y = \dfrac{0+1}{0-1} = -1$

16. $f(x) = \dfrac{6x}{x^2+4}$ no v.a since $x^2+4 \neq 0$

h.a $\dfrac{6x}{x^2+4} \dfrac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \dfrac{\frac{6}{x}}{1+\frac{4}{x^2}}$ as $x \to \pm\infty$

$y=0$

<u>x-int</u> $\dfrac{0}{1} = \dfrac{6x}{x^2+4}$

$6x=0$

$x=0$

<u>y-int</u> $y = \dfrac{6(0)}{0^2+4} = 0$

written exercises 5.6

1. $f(x) = 2x^2 + 5x - 12$

$f'(x) = 4x + 5 = 0$

$x = -\frac{5}{4}$

x	$(-\infty, -\frac{5}{4})$	$-\frac{5}{4}$	$(-\frac{5}{4}, \infty)$
$f'(x)$	$-$	0	$+$

increasing $(-\frac{5}{4}, \infty)$
decreasing $(-\infty, -\frac{5}{4})$
relative minimum $(-\frac{5}{4}, -\frac{121}{8})$

$f''(x) = 4$

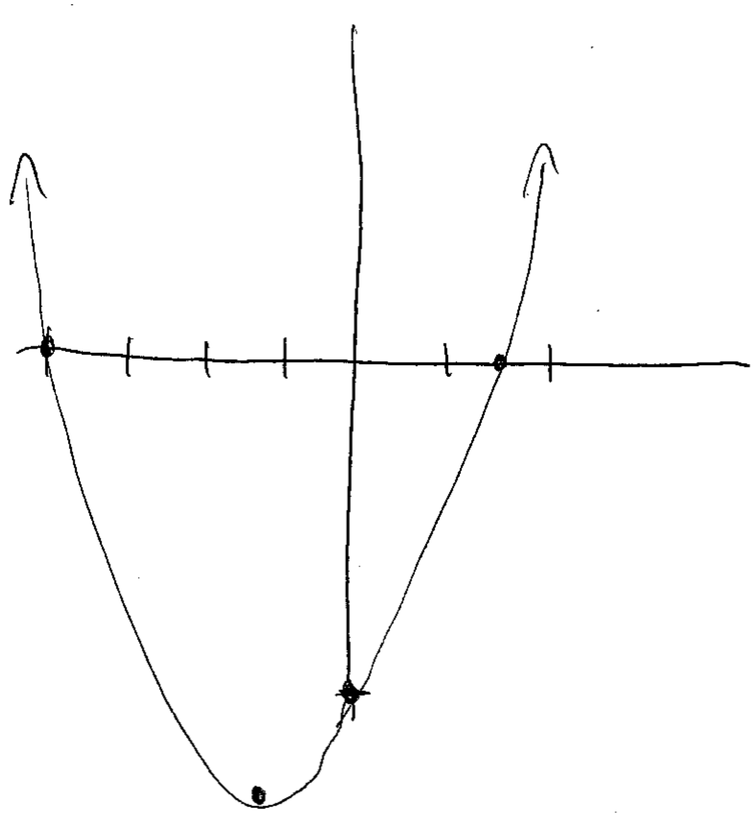
$f''(x) > 0$ concave up $(-\infty, \infty)$
no inflection points

x-int $0 = 2x^2 + 5x - 12$

$0 = (2x-3)(x+4)$

$x = \frac{3}{2}$ $x = -4$

y-int $y = -12$



2. $f(x) = x^3 + 6x^2 - 15x - 90$

$f'(x) = 3x^2 + 12x - 15 = 0$

$3(x^2 + 4x - 5) = 0$

$3(x-1)(x+5) = 0$

$x = 1 \quad x = -5$

x	$(-\infty,-5)$	-5	$(-5,1)$	1	∞
$f'(x)$	$+$	0	$-$	0	$+$

increasing $(-\infty, 5) \cup (1, \infty)$
decreasing $(-5, 1)$

relative max $(-5, 10)$
relative min $(1, -98)$

$f''(x) = 6x + 12 = 0$

$x = -2$

x	$(-\infty,-2)$	-2	$(-2,\infty)$
$f''(x)$	$-$	0	$+$

inflection point
$(-2, -44)$

concave up $(-2, \infty)$
concave down $(-\infty, -2)$

<u>x-int</u>

$0 = (x^3 + 6x^2) - (15x + 90)$

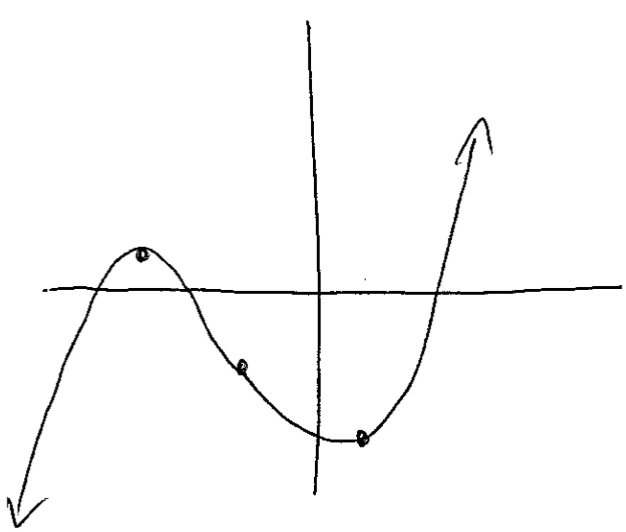
$= x^2(x+6) - 15(x+6)$

$0 = (x^2 - 15)(x+6)$

$x = \pm\sqrt{15} \quad x = -6$

<u>y-int</u>

$y = -90$



3. $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2 = 0$

$2x^2(2x-3)$

$x=0 \quad x=\frac{3}{2}$

x	$(-\infty,0)$	0	$(0,\frac{3}{2})$	$\frac{3}{2}$	$(\frac{3}{2},\infty)$
$f'(x)$	$-$	0	$-$	0	$+$

increasing $(\frac{3}{2},\infty)$

decreasing $(-\infty,0)\cup(0,\frac{3}{2})$

relative min $(\frac{3}{2},\frac{-27}{16})$

$f''(x) = 12x^2 - 12x = 0$

$12x(x-1) = 0$

$x=0 \quad x=1$

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
$f''(x)$	$+$	0	$-$	0	$+$

concave up $(-\infty,0)\cup(1,\infty)$

concave down $(0,1)$

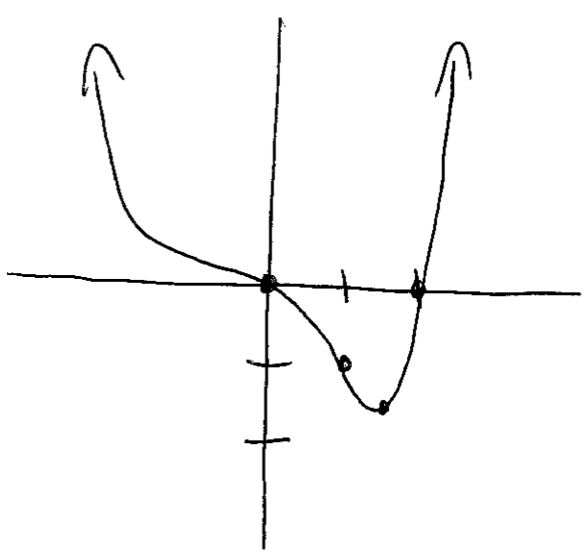
inflection pts $(0,0)$
$(1,-1)$

<u>x-int</u> $0 = x^4 - 2x^3$

$0 = x^3(x-2)$

$x=0 \quad x=2$

<u>y-int</u> $y=0$



6. $f(x) = x - 3x^{\frac{1}{3}}$

$f'(x) = 1 - x^{-2/3} = 0$

$1 - \frac{1}{\sqrt[3]{x^2}}$

$\frac{\sqrt[3]{x^2} - 1}{\sqrt[3]{x^2}} = 0$

x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
$f'(x)$	+	0	−	DNE	−	c.	+

increasing $(-\infty,-1) \cup (1,\infty)$

decreasing $(-1,1)$

critical points at $x=0$
$x=1$
$x=-1$

relative max $(-1,2)$

relative min $(1,-2)$

$\sqrt[3]{x^2} - 1 = 0$

$\sqrt[3]{x^2} = 1$

$x^2 = 1$

$x = \pm 1$

$f''(x) = \frac{2}{3}x^{-\frac{5}{3}} = \frac{2}{3\sqrt[3]{x^5}} = 0$ $\quad f''(x) \neq 0$

critical point at $x=0$

x	$(-\infty,0)$	0	$(0,\infty)$
$f''(x)$	—	und	+

concave down $(-\infty,0)$

concave up $(0,\infty)$

inflection point $(0,0)$

<u>x-int</u> $0 = x - 3\sqrt[3]{x}$

$3\sqrt[3]{x} = x$

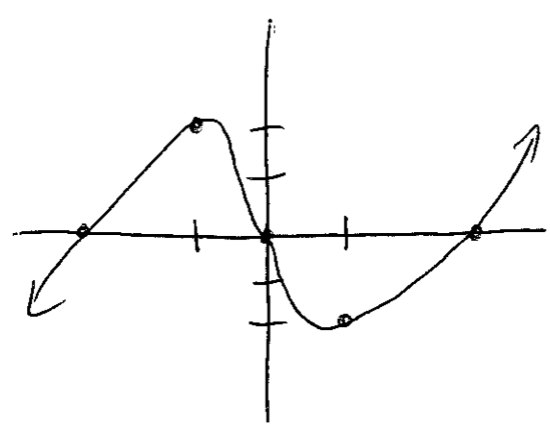
$27x = x^3$

$x^3 - 27x = 0$

$x(x^2 - 27) = 0$

$x = 0 \quad x = \pm 3\sqrt{3}$

<u>y-int</u> $= 0$



8. $f(x) = \dfrac{4x}{x^2-4}$ $\quad f'(x) = \dfrac{4(x^2-4) - 2x(4x)}{(x^2-4)^2}$

$$= \frac{-4x^2-16}{(x^2-4)^2} = \frac{-4(x^2+4)}{(x^2-4)^2} = 0$$

no critical numbers

x	$(-\infty,-2)$	-2	$(-2,2)$	2	$(2,\infty)$
$f'(x)$	$-$	und	$-$	und	$-$

increasing – never

decreasing $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

no relative extrema

$$f''(x) = \frac{-8x(x^2-4)^2 - 2(x^2-4)(2x)(-4x^2-16)}{(x^2-4)^4}$$

$$= \frac{-8x(x^2-4)\left[x^2-4 + (x^2-4)(-2x^2-8)\right]}{(x^2-4)^4} = \frac{8x(x^2+12)}{(x^2-4)^3}$$

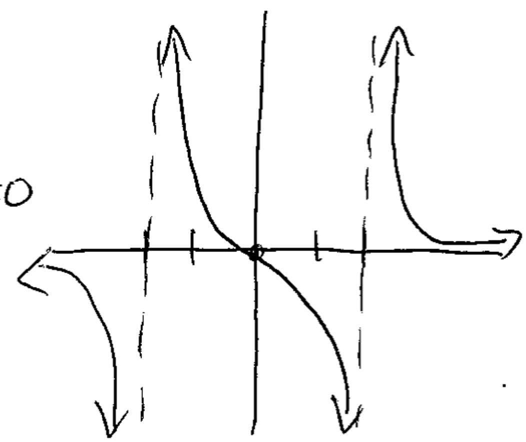
$x = 0$

x	$(-\infty,-2)$	-2	$(-2,0)$	0	$(0,2)$	2	$(2,\infty)$
$f''(x)$	$-$	und	$+$	0	$-$	und	$+$

concave up $(-2,0) \cup (2,\infty)$

concave down $(-\infty,-2) \cup (0,2)$

inflection point $(0,0)$



x-int $= 0$

y-int $= 0$

v.a $x = \pm 2$ since $x^2-4=0$

h.a $\dfrac{4x}{x^2-4}\dfrac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \dfrac{\frac{4}{x}}{1-\frac{4}{x^2}}$

as $x \to \pm\infty$ $y = 0$

10. $f(x) = \dfrac{x^2+3}{x-1}$ $\quad f'(x) = \dfrac{2x(x-1) - (1)(x^2+3)}{(x-1)^2}$

$f'(x) = \dfrac{x^2-2x-3}{(x-1)^2} = \dfrac{(x-3)(x+1)}{(x-1)^2} = 0 \quad x=3, \; x=-1$

x	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,3)$	3	$(3,\infty)$
$f'(x)$	$+$	0	$-$	und	$-$	0	$+$

increasing $(-\infty,-1) \cup (3,\infty)$ $\quad$ decreasing $(-1,1) \cup (1,3)$

relative max $(-1,-2)$ $\quad$ relative min $(3,6)$

$f''(x) = \dfrac{(2x-2)(x-1)^2 - 2(x-1)(x^2-2x-3)}{(x-1)^4}$

$= \dfrac{2(x-1)[x^2-2x+1 - x^2+2x+3]}{(x-1)^4} = \dfrac{8}{(x-1)^3}$ $\quad$ no critical point

x	$(-\infty,1)$	1	$(1,\infty)$
$f''(x)$	$-$	und	$+$

concave up $(1,\infty)$

concave down $(-\infty,1)$

no inflection pts

<u>x-int</u> $\dfrac{0}{1} = \dfrac{x^2+3}{x-1}$

$x^2+3=0$

$x^2=-3$

no x-int

vert asym $x-1=0$

$x=1$

oblique asym $y=x+1$

$$\begin{array}{r} x+1 \\ x-1 \overline{\big) x^2+0x+3} \\ \underline{x^2-x} \\ x+3 \\ \underline{x-1} \\ 4 \end{array}$$

$x+1+\dfrac{4}{x-1}$

<u>y-int</u> $y = \dfrac{3}{-1} = -3$