written exercises 4.3

1. $f(x) = x^2$; $P(2, f(2))$

$$m = \frac{f(2+h) - f(2)}{2+h-2} = \frac{(2+h)^2 - (2)^2}{h} = \frac{4+4h+h^2-4}{h}$$

$$= \frac{4h+h^2}{h} = \frac{h(4+h)}{h} = 4+h$$

$$m = \lim_{h\to 0} 4+h = 4$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y - 4 = 4(x-2) \qquad y-4 = 4x-8$$

$$y = 4x-4$$

2. $f(x) = x^3$; $P(-2, f(-2))$

$$m = \frac{f(-2+h) - f(-2)}{-2+h-(-2)} = \frac{(-2+h)^3 - (-2)^3}{h} = \frac{-8+12h-6h^2+h^3+8}{h}$$

$$= \frac{12h-6h^2+h^3}{h} = \frac{h(12-6h+h^2)}{h} = 12-6h+h^2$$

$$m = \lim_{h\to 0} 12-6h+h^2 = 12$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y-(-8) = 12(x-(-2))$$

$$y+8 = 12(x+2)$$

$$y+8 = 12x+24 \qquad y = 12x+16$$

3. $f(x) = x^2 - 4x$; $P(1, f(1))$

$$m = \frac{f(1+h) - f(1)}{h} = \frac{(1+h)^2 - 4(1+h) - \left[(1)^2 - 4(1)\right]}{h}$$

$$= \frac{1 + 2h + h^2 - 4 - 4h + 3}{h} = \frac{h^2 - 2h}{h} = \frac{h(h-2)}{h}$$

$$m = \lim_{h \to 0} h - 2 = -2$$

$$y - (-3) = -2(x-1)$$
$$y + 3 = -2x + 2 \qquad y = -2x - 1$$

4. $f(x) = 6x - 2x^3$; $P(-1, f(-1))$

$$m = \frac{f(-1+h) - f(-1)}{h} = \frac{6(-1+h) - 2(-1+h)^3 - \left[6(-1) - 2(-1)^3\right]}{h}$$

$$= \frac{-6 + 6h + 2 - 6h + 6h^2 - 2h^3 - (-4)}{h} = \frac{-2h^3 + 6h^2}{h} = -2h^2 + 6h$$

$$m = \lim_{h \to 0} -2h^2 + 6h = 0$$

$\therefore$ horizontal line equation is $y = -4$

5. $f(x) = \frac{4}{x}$; $P(-2, f(-2))$

$$m = \frac{f(-2+h) - f(-2)}{h} = \frac{\frac{4}{-2+h} - \left(\frac{4}{-2}\right)}{h} = \frac{\frac{4}{-2+h} + 2}{h}$$

$$= \frac{\frac{4+2(-2+h)}{-2+h}}{h} = \frac{\frac{4-4+2h}{-2+h}}{h} = \frac{2h}{-2+h} \cdot \frac{1}{h}$$

$$= \frac{2}{-2+h} \qquad m = \lim_{h \to 0} \frac{2}{-2+h} = -1$$

$$y-(-2) = -1(x-(-2))$$
$$y+2 = -(x+2)$$
$$y+2 = -x-2 \qquad y = -x-4$$

6. $f(x) = \sqrt{x+3}$; $P(1, f(1))$

$$m = \frac{f(1+h)-f(1)}{h} = \frac{\sqrt{1+h+3} - \sqrt{4}}{h} = \frac{\sqrt{4+h} - 2}{h}\left[\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right]$$

$$= \frac{4+h-4}{h(\sqrt{4+h}+2)} = \frac{h}{h(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4+h}+2}$$

$$m = \lim_{h \to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

$$y-2 = \frac{1}{4}(x-1)$$
$$y-2 = \frac{1}{4}x - \frac{1}{4}$$

$$4y-8 = x-1$$
$$4y = x+7$$
$$y = \frac{1}{4}x + \frac{7}{4}$$

written exercise 4.4

1a) $f(x) = 2x^2 - 3x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) - [2x^2 - 3x]}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h} = 4x + 2h - 3$$

$$f'(x) = 4x - 3$$

b) $f(x) = x^3 + 5x^2 - 7x - 2$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 + 5(x+h)^2 - 7(x+h) - 2 - [x^3 + 5x^2 - 7x - 2]}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 5x^2 + 10xh + 5h^2 - 7x - 7h - 2 - x^3 - 5x^2 + 7x + 2}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + 10xh + 5h^2 - 7h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 10x + 5h - 7$$

$$f'(x) = 3x^2 + 10x - 7$$

c) $f(x) = -3x - 8$

$$f'(x) = \lim_{h \to 0} \frac{-3(x+h) - 8 - [-3x-8]}{h}$$

$$= \lim_{h \to 0} \frac{-3x - 3h - 8 + 3x + 8}{h}$$

$$= \lim_{h \to 0} -\frac{3h}{h} = -3$$

$$f'(x) = -3$$

d) $f(x) = 10$

$$f'(x) = \lim_{h \to 0} \frac{10 - 10}{h} = \frac{0}{h}$$

$$f'(x) = 0$$

e) $y = -3x^4 + x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-3(x+h)^4 + (x+h) - [-3x^4 + x]}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-3(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) + x + h + 3x^4 - x}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-3x^4 - 12x^3h - 18x^2h^2 - 12xh^3 - 3h^4 + x + h + 3x^4 - x}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-12x^3h - 18x^2h^2 - 12xh^3 - 3h^4 + h}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} -12x^3 - 18x^2h - 12xh^2 - 3h^3 + 1$$

$$\frac{dy}{dx} = -12x^3 + 1$$

f) $y = 4\sqrt{x}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{4\sqrt{x+h} - 4\sqrt{x}}{h} \left[\frac{4\sqrt{x+h} + 4\sqrt{x}}{4\sqrt{x+h} + 4\sqrt{x}}\right]$$

$$= \lim_{h \to 0} \frac{16(x+h) - 16x}{h\left[4\sqrt{x+h} + 4\sqrt{x}\right]} = \frac{16h}{h\left[4\sqrt{x+h} + 4\sqrt{x}\right]}$$

$$= \lim_{h \to 0} \frac{16}{4\sqrt{x+h} + 4\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{16}{4\sqrt{x} + 4\sqrt{x}} = \frac{16}{8\sqrt{x}} = \frac{2}{\sqrt{x}} = \frac{2\sqrt{x}}{x}$$

g) $y = \frac{2}{x^2}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h} = \frac{\frac{2x^2 - 2(x+h)^2}{x^2(x+h)^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2x^2 - 2x^2 - 4xh - 2h^2}{x^2(x+h)^2}}{h} = \frac{-4xh - 2h^2}{x^2(x+h)^2} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-4x - 2h}{x^2(x+h)^2}$$

$$\frac{dy}{dx} = \frac{-4x}{x^2(x)^2} = \frac{-4x}{x^4} = \frac{-4}{x^3} \text{ or } -4x^{-3}$$

2a) slope of the tangent line is $f'(x)$

$$f(x) = x^2 + 2x - 3$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - 3 - [x^2 + 2x - 3]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - 3 - x^2 - 2x + 3}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$$

$$f'(x) = 2x + 2$$

b) since $f'(x) = 2x+2$, the slope of the tangent at $x = -2$ is $f'(-2) = 2(-2)+2$
$= -2$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y - (-3) = -2(x - (-2)) \qquad y + 3 = -2(x+2)$$

$$y = -2x - 7$$

the slope of the tangent at $x = 1$ is $f'(1) = 2(1) + 2 = 4$

$$y - 0 = 4(x - 1)$$

$$y = 4x - 4$$

written exercises 4.5

1a) $f(x) = 3x^2 - 7x + 11$

$f'(x) = 6x - 7$

c) $y = \frac{3}{4}x^4 - \frac{5}{3}x^3 + \frac{7}{2}x^2 - x \quad 6$

$\frac{dy}{dx} = 3x^3 - 5x^2 + 7x - 1$

e) $y = 6x^{\frac{2}{3}} - 4x^{\frac{1}{2}} + 3\pi$

$\frac{dy}{dx} = 4x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}}$

g) $f(x) = (x-8)^2$

$f(x) = x^2 - 16x + 64$

$f'(x) = 2x - 16$

i) $y = (3x-4)(2x+5)$

$y = 6x^2 + 15x - 8x - 20$

$y = 6x^2 + 7x - 20$

$\frac{dy}{dx} = 12x + 7$

k) $f(x) = \frac{2x^4 - 3x^3}{x}$

$f(x) = 2x^3 - 3x^2$

$f'(x) = 6x^2 - 6x$

m) $y = 8\sqrt{x} - 6\sqrt[3]{x} - 4\sqrt[4]{x} + 2\sqrt{2}$

$y = 8x^{\frac{1}{2}} - 6x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 2\sqrt{2}$

$\frac{dy}{dx} = 4x^{-\frac{1}{2}} - 2x^{-\frac{2}{3}} - x^{-\frac{3}{4}}$

o) $y = \sqrt{\frac{x}{2}} - \sqrt[3]{\frac{x}{3}} = \frac{x^{\frac{1}{2}}}{\sqrt{2}} - \frac{x^{\frac{1}{3}}}{\sqrt[3]{3}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{2}} x^{-1/2} - \frac{1}{3\sqrt[3]{3}} x^{-2/3}$

3 a) $f(x) = 2x^2 - 5x + 3$ at $x = -2$

$f'(x) = 4x - 5$

$f'(-2) = 4(-2) - 5 = -13$

b) $y = \frac{1}{6}x^3 - \frac{3}{4}x^2 + 2x - 7$ at $x = 4$

$\frac{dy}{dx} = \frac{1}{2}x^2 - \frac{3}{2}x + 2$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2}(4)^2 - \frac{3}{2}(4) + 2 = 8 - 6 + 2$
$= 4$

c) $y = 3\sqrt{x} - 2\sqrt[3]{x} + 6$ at $x = 64$

$y = 3x^{\frac{1}{2}} - 2x^{\frac{1}{3}} + 6$

$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - \frac{2}{3}x^{-2/3}$

$\left.\frac{dy}{dx}\right|_{x=64} = \frac{3}{2}(64)^{-1/2} - \frac{2}{3}(64)^{-2/3}$
$= \frac{3}{2}\left(\frac{1}{8}\right) - \frac{2}{3}\left(\frac{1}{16}\right) = \frac{3}{16} - \frac{1}{24} = \frac{7}{48}$

d) $f(x) = \frac{x-3}{x}$ at $x = -3$

$f(x) = 1 - \frac{3}{x} = 1 - 3x^{-1}$

$f'(x) = 3x^{-2} = \frac{3}{x^2}$ $\quad f'(-3) = \frac{3}{(-3)^2} = \frac{1}{3}$

4a) $f(x) = -x^3 + 8x^2$ at $(5, f(5))$

$f'(x) = -3x^2 + 16x$

$f'(5) = -3(5)^2 + 16(5) = -75 + 80 = 5$

$y_2 - y_1 = m(x_2 - x_1)$

$y - 75 = 5(x - 5)$ $\quad y - 75 = 5x - 25$

$y = 5x + 50$

b) $y = \frac{3}{x} - \frac{2}{x^2} + 7$ at $(-2, f(-2))$

$y = 3x^{-1} - 2x^{-2} + 7$

$\frac{dy}{dx} = -3x^{-2} + 4x^{-3} = \frac{-3}{x^2} + \frac{4}{x^3}$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-3}{(-2)^2} + \frac{4}{(-2)^3} = \frac{-3}{4} - \frac{4}{8} = -\frac{5}{4}$

$y - 5 = -\frac{5}{4}(x+2)$ $\quad y = -\frac{5}{4}x + \frac{5}{2}$

$y - 5 = -\frac{5}{4}x - \frac{5}{2}$

c) $y = (x^2 - 2x + 3)^2$ at $(-1, f(-1))$

$\frac{dy}{dx} = 2(x^2 - 2x + 3)(2x - 2)$

$\left.\frac{dy}{dx}\right|_{x=-1} = 2(6)(-4) = -48$

$y - 36 = -48(x+1)$ $\quad y = -48x - 12$

$y - 36 = -48x - 48$

5a) $f(x) = 3x^2 - 12x + 5$

$f'(x) = 6x - 12 = 0$

$6x = 12$

$x = 2$

$(2, f(2))$

$(2, -7)$

b) $y = x^3 - 6x^2 + 9x - 1$

$\frac{dy}{dx} = 3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

$(1, f(1))$

$(1, 3)$

$(3, f(3))$

$(3, -1)$

9. $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b = 0$

$2ax = -b$

$x = -\frac{b}{2a}$

Written exercises 4.6

1. $y = (9x+11)(3x-4)$

$\frac{dy}{dx} = 9(3x-4) + 3(9x+11)$

$= 27x - 36 + 27x + 33$

$= 54x - 3$

3. $f(x) = (x^2 - 6x + 5)(x^2 - 3x - 2)$

$f'(x) = (2x-6)(x^2-3x-2) + (2x-3)(x^2-6x+5)$

$= 2x^3 - 6x^2 - 4x - 6x^2 + 18x + 12 + 2x^3 - 12x^2 + 10x - 3x^2 + 18x - 15$

$f'(x) = 4x^3 - 27x^2 + 42x - 3$

5. $y = (-3x^{-2})(2x^5 + 4x^3)$

$\frac{dy}{dx} = +6x^{-3}(2x^5+4x^3) + (10x^4 + 12x^2)(-3x^{-2})$

$= 12x^2 + 24 - 30x^2 - 36 = -18x^2 - 12$

7. $f(x) = (x - x^{-1})(x + x^{-1})$

$f'(x) = (1 + x^{-2})(x + x^{-1}) + (1 - x^{-2})(x - x^{-1})$

$= x + x^{-1} + x^{-1} + x^{-3} - x^{-1} + x - x^{-1} + x^{-3}$

$f'(x) = 2x + 2x^{-3}$

11. $f(x) = (2x-1)(x^2+3)(2x^4)$

$f'(x) = 2(x^2+3)(2x^4) + 2x(2x-1)(2x^4) + 8x^3(2x-1)(x^2+3)$

12. $f(x) = (2x^2-6x-1)(x^2+4x+8)$

$f'(x) = (4x-6)(x^2+4x+8) + (2x+4)(2x^2-6x-1)$

16. $y = (6x^2-5x-4)(x^3+4x^2) \; ; \; x=1$

$\frac{dy}{dx} = (12x-5)(x^3+4x^2) + (3x^2+8x)(6x^2-5x-4)$

$\left.\frac{dy}{dx}\right|_{x=1} = (7)(5) + (11)(-3)$

$= 35 - 33 = 2$

Written exercises 4.7

1. $y = \frac{2x}{x+1}$ $\quad \frac{dy}{dx} = \frac{2(x+1) - 1(2x)}{(x+1)^2} = \frac{2x+2-2x}{(x+1)^2}$

$= \frac{2}{(x+1)^2}$

3. $f(x) = \frac{x+4}{x-4}$ $\quad f'(x) = \frac{(1)(x-4) - 1(x+4)}{(x-4)^2} = \frac{x-4-x-4}{(x-4)^2}$

$= \frac{-8}{(x-4)^2}$

5. $y = \frac{x-4}{x^2}$ $\quad \frac{dy}{dx} = \frac{1(x^2) - 2x(x-4)}{(x^2)^2} = \frac{x^2-2x^2+8x}{x^4}$

$= \frac{-x^2+8x}{x^4} = \frac{x(-x+8)}{x^4} = \frac{-x+8}{x^3}$

7. $f(x) = \frac{4-2x}{1-x}$ $\quad f'(x) = \frac{-2(1-x)-(-1)(4-2x)}{(1-x)^2}$

$= \frac{-2+2x+4-2x}{(1-x)^2} = \frac{2}{(1-x)^2}$

9. $f(x) = \frac{2}{\sqrt{x}+1} = \frac{2}{x^{\frac{1}{2}}+1}$ $\quad f'(x) = \frac{(0)(x^{\frac{1}{2}}+1) - \frac{1}{2}x^{-\frac{1}{2}}(2)}{(x^{\frac{1}{2}}+1)^2}$

$= \frac{-x^{-\frac{1}{2}}}{(x^{\frac{1}{2}}+1)^2} = \frac{-1}{\sqrt{x}(\sqrt{x}+1)^2}$

11. $y = \dfrac{x}{\sqrt{x}+1}$ $\qquad \dfrac{dy}{dx} = \dfrac{(1)(x^{\frac{1}{2}}+1) - \frac{1}{2}x^{-\frac{1}{2}}(x)}{(\sqrt{x}+1)^2}$

$= \dfrac{x}{x^{\frac{1}{2}}+1}$ $\qquad = \dfrac{x^{\frac{1}{2}}+1-\frac{1}{2}x^{1/2}}{(\sqrt{x}+1)^2} = \dfrac{\frac{1}{2}\sqrt{x}+1}{(\sqrt{x}+1)^2}$

$= \dfrac{\sqrt{x}+2}{2(\sqrt{x}+1)^2}$

13. $y = \dfrac{x^2}{x+2}$ $\qquad \dfrac{dy}{dx} = \dfrac{2x(x+2) - 1(x^2)}{(x+2)^2} = \dfrac{2x^2+4x-x^2}{(x+2)^2}$

$\dfrac{dy}{dx} = \dfrac{x^2+4x}{(x+2)^2}$ $\qquad \left.\dfrac{dy}{dx}\right|_{x=2} = \dfrac{12}{16} = \dfrac{3}{4}$

$y - 1 = \frac{3}{4}(x-2)$ $\qquad 4y = 3x - 2$

$y = \frac{3}{4}x - \frac{1}{2}$ $\qquad 3x - 4y = 2$

14. $f(x) = \dfrac{\sqrt{x}}{2-x}$ $\qquad f'(x) = \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(2-x) - (-1)(x^{\frac{1}{2}})}{(2-x)^2}$

$f'(4) = \dfrac{(\frac{1}{4})(-2) + (2)}{4} = \dfrac{\frac{3}{2}}{4} = \dfrac{3}{8}$

$y + 1 = \frac{3}{8}(x-4)$

$8y + 8 = 3x - 12$

$3x - 8y = 20$

16. $y = \frac{15x^6}{3x^2}$

1st/

$y = 5x^4$

$\frac{dy}{dx} = 20x^3$

2nd/

$$\frac{dy}{dx} = \frac{90x^5(3x^2) - 6x(15x^6)}{(3x^2)^2}$$

$$= \frac{270x^7 - 90x^7}{9x^4}$$

$$= \frac{180x^7}{9x^4} = 20x^3$$

18. $f(x) = \frac{10x}{x^2+1}$

$$f'(x) = \frac{10(x^2+1) - 2x(10x)}{(x^2+1)^2}$$

$$= \frac{10x^2+10-20x^2}{(x^2+1)^2} = \frac{-10x^2+10}{(x^2+1)^2} = 0$$

$-10x^2+10=0$

$-10x^2 = -10$

$x^2 = 1$

$x = \pm 1$

$(1, 5)$

$(-1, -5)$

written exercises 4.8

1. $y = (3x-4)^6$

$\frac{dy}{dx} = 6(3x-4)^5(3) = 18(3x-4)^5$

3. $y = (x+7)^6$

$\frac{dy}{dx} = 6(x+7)^5(1) = 6(x+7)^5$

5. $f(x) = (x^2+3)^3$

$f'(x) = 3(x^2+3)^2(2x) = 6x(x^2+3)^2$

7. $y = (12-2x^3)^5$

$\frac{dy}{dx} = 5(12-2x^3)^4(-6x^2) = -30x^2(12-2x^3)^4$

9. $f(x) = (x^4-8x)^7$

$f'(x) = 7(x^4-8x)^6(4x^3-8)$

11. $y = (x^2+4)^{-3}$

$\frac{dy}{dx} = -3(x^2+4)^{-4}(2x) = -6x(x^2+4)^{-4}$

13. $f(x) = \frac{3}{x^2+9} = 3(x^2+9)^{-1}$

$f'(x) = -3(x^2+9)^{-2}(2x) = \frac{-6x}{(x^2+9)^2}$

15. $f(x) = \sqrt{x^2+6x} = (x^2+6x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2+6x)^{-\frac{1}{2}}(2x+6) = \frac{x+3}{\sqrt{x^2+6x}}$ or $(x+3)($

17. $y = \frac{6}{5}\sqrt[3]{x-2} = \frac{6}{5}(x-2)^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{6}{5}\left(\frac{1}{3}\right)(x-2)^{-\frac{2}{3}}(1) = \frac{2}{5}(x-2)^{-2/3}$

19. $y = \frac{10}{\sqrt{2x-1}} = 10(2x-1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = 10\left(-\frac{1}{2}\right)(2x-1)^{-3/2}(2) = -10(2x-1)^{-3/2}$

21. $f(x) = \left(x+\frac{1}{x}\right)^{1/2} = (x+x^{-1})^{1/2}$

$f'(x) = \frac{1}{2}(x+x^{-1})^{-1/2}(1-x^{-2})$

23. $f(x) = 6\sqrt{\frac{1}{2}x-4} = 6\left(\frac{1}{2}x-4\right)^{\frac{1}{2}}$

$f'(x) = 6\left(\frac{1}{2}\right)\left(\frac{1}{2}x-4\right)^{-1/2}\left(\frac{1}{2}\right)$

$= \frac{3}{2}\left(\frac{1}{2}x-4\right)^{-1/2}$

25. $f(x) = 2\sqrt{x^2+13}$ at $(6, f(6))$

$= 2(x^2+13)^{\frac{1}{2}}$

$f'(x) = 2\left(\frac{1}{2}\right)(x^2+13)^{-\frac{1}{2}}(2x) = \dfrac{2x}{\sqrt{x^2+13}}$

$f'(6) = \dfrac{12}{\sqrt{49}} = \dfrac{12}{7}$

26. $f(x) = \sqrt[3]{x^2-4x+3} = (x^2-4x+3)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}(x^2-4x+3)^{-2/3}(2x-4) = \dfrac{2x-4}{3\sqrt[3]{(x^2-4x+3)^2}}$

a) horizontal when $f'(x) = 0$

$\dfrac{2x-4}{3\sqrt[3]{(x^2-4x+3)^2}} = \dfrac{0}{1}$ $\quad 2x-4=0$

$x = 2.$ $\quad (2, f(2))$

$(2, -1)$

b) vertical when $f'(x)$ is undefined

ie: $3\sqrt[3]{(x^2-4x+3)^2} = 0$

$3\sqrt[3]{[(x-3)(x-1)]^2} = 0$

$x = 3 \quad x = 1$

$(3, 0) \quad (1, 0)$

written exercises 4.9

1. $f(x) = 2x^3 + 15x^2 - 36x + 12$

$f'(x) = 6x^2 + 30x - 36 = 6(x^2 + 5x - 6) = 6(x+6)(x-1)$

3. $y = \frac{1}{x} + 4x = x^{-1} + 4x$

$\frac{dy}{dx} = -x^{-2} + 4 = -\frac{1}{x^2} + 4 = \frac{-1 + 4x^2}{x^2}$

$= x^{-2}(4x^2 - 1) = x^{-2}(2x-1)(2x+1)$

5. $f(x) = (2x-3)^3(x+1)^2$

$f'(x) = 3(2x-3)^2(2)(x+1)^2 + 2(x+1)(2x-3)^3$

$= 2(2x-3)^2(x+1)\left[3(x+1) + (2x-3)\right]$

$= 2(2x-3)^2(x+1)(5x) = 10x(x+1)(2x-3)^2$

7. $y = (x-2)\sqrt{x^2 - 3x - 1} = (x-2)(x^2 - 3x - 1)^{\frac{1}{2}}$

$\frac{dy}{dx} = (1)(x^2 - 3x - 1)^{\frac{1}{2}} + \frac{1}{2}(x^2 - 3x - 1)^{-\frac{1}{2}}(2x-3)(x-2)$

$= \frac{(x^2 - 3x - 1)^{\frac{1}{2}}}{1} + \frac{2x^2 - 7x + 6}{2(x^2 - 3x - 1)^{\frac{1}{2}}}$

$= \frac{2(x^2 - 3x - 1) + 2x^2 - 7x + 6}{(x^2 - 3x - 1)^{\frac{1}{2}}} = \frac{4x^2 - 13x + 4}{2(x^2 - 3x - 1)^{\frac{1}{2}}}$

$= \frac{1}{2}(4x^2 - 13x + 4)(x^2 - 3x - 1)^{-1/2}$

9. $f(x) = \dfrac{x^2-3x}{x^2+3}$

$$f'(x) = \frac{(2x-3)(x^2+3) - 2x(x^2-3x)}{(x^2+3)^2}$$

$$= \frac{2x^3+6x-3x^2-9-2x^3+6x^2}{(x^2+3)^2}$$

$$= \frac{3x^2+6x-9}{(x^2+3)^3} = \frac{3(x^2+2x-3)}{(x^2+3)^2} = \frac{3(x+3)(x-1)}{(x^2+3)^2}$$

11. $y = x^3(2x-1)(3x+2) = x^3(6x^2+x-2)$

$$y = 6x^5+x^4-2x^3$$

$$\frac{dy}{dx} = 30x^4+4x^3-6x^2 = 2x^2(15x^2+2x-3)$$

13. $f(x) = \left(\dfrac{2x}{x+2}\right)^{-2} = \left(\dfrac{x+2}{2x}\right)^2$

$$f'(x) = 2\left[\frac{x+2}{2x}\right]^1\left[\frac{(1)(2x)-2(x+2)}{(2x)^2}\right]$$

$$= \frac{2(x+2)(2x-2x-4)}{(2x)(4x^2)} = \frac{2(x+2)(-4)}{8x^3}$$

$$= -\frac{(x+2)}{x^3}$$

15. $y = \dfrac{\sqrt{x}}{x^2+1} = \dfrac{x^{\frac{1}{2}}}{x^2+1}$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}(x^2+1) - 2x(x^{\frac{1}{2}})}{(x^2+1)^2} = \frac{\frac{x^2+1}{2x^{\frac{1}{2}}} - \frac{2x^{\frac{3}{2}}}{1}}{(x^2+1)^2}$$

$$= \frac{\frac{x^2+1-4x^2}{2x^{\frac{1}{2}}}}{(x^2+1)^2} = \frac{1-3x^2}{2x^{\frac{1}{2}}(x^2+1)^2}$$

17. $f(x) = \dfrac{1}{(x^2-2)\sqrt{2x+3}} = (x^2-2)^{-1}(2x+3)^{-1/2}$

$$f'(x) = -(x^2-2)^{-2}(2x)(2x+3)^{-1/2} - \frac{1}{2}(2x+3)^{-3/2}(2)(x^2)^{-1}$$

$$= -(x^2-2)^{-2}(2x+3)^{-3/2}\left[2x(2x+3) + (x^2-2)\right]$$

$$= -(x^2-2)^{-2}(2x+3)^{-3/2}(4x^2+6x+x^2-2)$$

$$= -(5x^2+6x-2)(x^2-2)^{-2}(2x+3)^{-3/2}$$

written exercises 4.10

1. $3x - 4y - 12 = 0$

$$\frac{d(3x)}{dx} - \frac{d(4y)}{dx} - \frac{d(12)}{dx} = \frac{d(0)}{dx}$$

$$3 - 4\frac{dy}{dx} = 0 \qquad 3 = 4\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3}{4}$$

3. $x^2 - y^2 = 4$

$$\frac{d(x^2)}{dx} - \frac{d(y^2)}{dx} = \frac{d(4)}{dx}$$

$$2x - 2y\frac{dy}{dx} = 0$$

$$2x = 2y\frac{dy}{dx} \qquad \frac{dy}{dx} = \frac{x}{y}$$

5. $xy = 12$

$$\frac{d(xy)}{dx} = \frac{d(12)}{dx}$$

$$(1)(y) + (1)\frac{dy}{dx}(x) = 0$$

$$y = -x\frac{dy}{dx} \qquad \frac{dy}{dx} = -\frac{y}{x}$$

7. $\frac{1}{x} + \frac{1}{y} = 10$

$$x^{-1} + y^{-1} = 10$$

$$\frac{d(x^{-1})}{dx} + \frac{d(y^{-1})}{dx} = \frac{d(10)}{dx}$$

$$-x^{-2} + (-y^{-2})\frac{dy}{dx} = 0$$

$$-\frac{1}{x^2} = \frac{1}{y^2}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2}$$

9. $x^2 + 3xy = 30$

$$\frac{d(x^2)}{dx} + \frac{d(3xy)}{dx} = \frac{d(30)}{dx}$$

$$2x + (3)(y) + (1)\frac{dy}{dx}(3x) = 0$$

$$3x\frac{dy}{dx} = -2x - 3y \qquad \frac{dy}{dx} = \frac{-2x-3y}{3x}$$

11. $x^2 - y^2 - 2xy - 8 = 0$

$$\frac{d(x^2)}{dx} - \frac{d(y^2)}{dx} - \frac{d(2xy)}{dx} - \frac{d(8)}{dx} = \frac{d(0)}{dx}$$

$$2x - 2y\frac{dy}{dx} - \left[(2)(y) + (1)\frac{dy}{dx}(2x)\right] = 0$$

$$2x - 2y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 0$$

$$x - y\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$$

$$x - y = x\frac{dy}{dx} + y\frac{dy}{dx}$$

$$x - y = \frac{dy}{dx}(x+y) \qquad \frac{dy}{dx} = \frac{x-y}{x+y}$$

29. $\frac{x^2}{25} + \frac{y^2}{9} = 1 \qquad 9x^2 + 25y^2 = 225$

$$18x + 50y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{9x}{25y}$$

$$y - \frac{9}{5} = \frac{-4}{5}(x-4)$$

$$5y - 9 = -4(x-4)$$

$$4x + 5y = 25$$

$$\left.\frac{dy}{dx}\right|_{(4,\frac{9}{5})} = \frac{-9(4)}{25(\frac{9}{5})} = -\frac{36}{45} = \frac{-4}{5} \qquad 4x + 5y - 25 = 0$$