Written Exercises 3.3

1. $\lim_{x \to -1} (x^3 - x^2 - x) = (-1)^3 - (-1)^2 - (-1)$

$= -1 - 1 + 1$

$= -1$

3. $\lim_{x \to -2} \frac{x^2 - 2x - 8}{2x+1} = \frac{(-2)^2 - 2(-2) - 8}{2(-2)+1} = \frac{4+4-8}{-3}$

$= \frac{0}{-3} = 0$

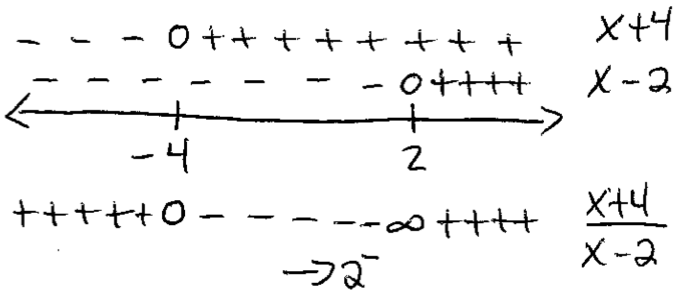
5. $\lim_{x \to 5} x = 5$

7. $\lim_{x \to \frac{\pi}{4}} \sin^2 x = \left(\sin \frac{\pi}{4}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

9. $\lim_{x \to \frac{\pi}{3}} \left[\frac{3x}{\pi} \left(\tan^4 x\right)\right] = \frac{3\left(\frac{\pi}{3}\right)}{\pi} \left[\left(\tan \frac{\pi}{3}\right)^4\right]$

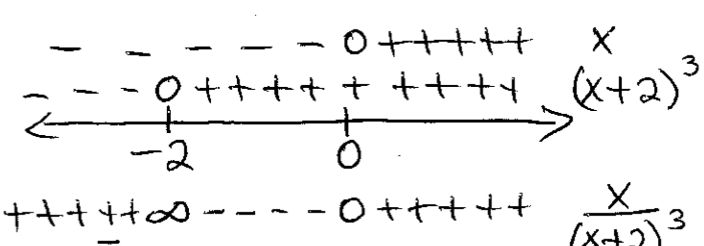
$= 1 \left(\sqrt{3}\right)^4 = \left[\left(\sqrt{3}\right)^2\right]^2 = 9$

11. $\lim_{x \to 2^-} \frac{x+4}{x-2}$



$= -\infty$

13. $\lim_{x \to -2^-} \frac{x}{(x+2)^3}$



$\to -2^- = \infty$

15. $\lim\limits_{x \to 1^-} \dfrac{10^x}{\log x}$

+ + + + + + + + + + 10^x

− − − − 0 + + + + + + + + $\log x$

1

− − − ∞ + + + + + $\dfrac{10^x}{\log x}$

$\to 1^- = -\infty$

17. $\lim\limits_{x \to 1} \dfrac{2x-1}{(x-1)^3}$

− − − 0 + + + + + + + $2x-1$

− − − − − 0 + + + + $(x-1)^3$

$\frac{1}{2}$ 1

+ + + + + 0 − − − ∞ + + + + + $\dfrac{2x-1}{(x-1)^3}$

$\lim\limits_{x \to 1^+} = \infty$ $\lim\limits_{x \to 1^-} = -\infty$ $\therefore \lim\limits_{x \to 1}$ does not exist

11. $\lim\limits_{x \to 4} \dfrac{x-4}{x^2-4x} = \dfrac{(x-4)}{x(x-4)} = \dfrac{1}{x} = \dfrac{1}{4}$

21. $\lim\limits_{x \to -5} \dfrac{x+5}{x^2-25} = \dfrac{(x+5)}{(x-5)(x+5)} = \dfrac{1}{x-5} = \dfrac{-1}{10}$

23. $\lim\limits_{w \to -2} \dfrac{3w^2+4w-4}{2w^2+7w+6} = \dfrac{(3w-2)(w+2)}{(2w+3)(w+2)} = \dfrac{3w-2}{2w+3}$

$= \dfrac{3(-2)-2}{2(-2)+3} = \dfrac{-8}{-1} = 8$

25. $\lim\limits_{v \to \frac{5}{3}} \dfrac{27v^3-125}{3v-5} = \dfrac{(3v-5)(9v^2+15v+25)}{(3v-5)}$

$= 9v^2+15v+25 = 9\left(\frac{5}{3}\right)^2+15\left(\frac{5}{3}\right)+25 = 75$

27. $\lim_{x \to 1} \frac{x^7-1}{x-1}$

$$\begin{array}{c|cccccccc} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

$$\frac{\cancel{(x-1)}(x^6+x^5+x^4+x^3+x^2+x+1)}{\cancel{(x-1)}}$$

$$= (1)^6+(1)^5+(1)^4+(1)^3+(1)^2+(1)+1$$

$$= 7$$

29. $\lim_{h \to 0} \frac{(3+h)^2-2(3+h)-3}{h} = \frac{9+6h+h^2-6-2h-3}{h}$

$$= \frac{4h+h^2}{h} = 4+h = 4+0 = 4$$

31. $\lim_{x \to 4} \frac{\frac{1}{x-2}-\frac{1}{2}}{x-4} = \frac{\frac{2-(x-2)}{(x-2)(2)}}{x-4} = \frac{-x+4}{2(x-2)} \cdot \frac{1}{x-4}$

$$= \frac{-\cancel{(x-4)}}{2(x-2)} \cdot \frac{1}{\cancel{(x-4)}} = \frac{-1}{2(x-2)} = \frac{-1}{2(4-2)} = \frac{-1}{4}$$

33. $\lim_{a \to 0} \frac{\frac{1}{(a+2)^2}-\frac{1}{4}}{a} = \frac{4-(a+2)^2}{4(a+2)^2} \cdot \frac{1}{a}$

$$\frac{4-a^2-4a-4}{4(a+2)^2} \cdot \frac{1}{a} = \frac{-\cancel{a}(a+4)}{4(a+2)^2} \cdot \frac{1}{\cancel{a}}$$

$$= \frac{-(a+4)}{4(a+2)^2} = \frac{-(0+4)}{4(0+2)^2} = \frac{-4}{16} = \frac{-1}{4}$$

35. $$\lim_{y \to 0} \frac{\sqrt{y+2}-\sqrt{2}}{y}\left[\frac{\sqrt{y+2}+\sqrt{2}}{\sqrt{y+2}+\sqrt{2}}\right] = \frac{y+2-2}{y\left[\sqrt{y+2}+\sqrt{2}\right]}$$

$$= \frac{1}{\sqrt{y+2}+\sqrt{2}} = \frac{1}{\sqrt{0+2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

37. $$\lim_{x \to 5} \frac{x-5}{\sqrt{x+4}-3}\left[\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}\right] = \frac{(x-5)(\sqrt{x+4}+3)}{x+4-9}$$

$$\frac{\cancel{(x-5)}(\sqrt{x+4}+3)}{\cancel{(x-5)}} = \sqrt{x+4}+3 = \sqrt{5+4}+3$$

$$= 6$$

39. $$\lim_{x \to 4} \frac{8\sqrt{x}-x^2}{2-\sqrt{x}}\left[\frac{2+\sqrt{x}}{2+\sqrt{x}}\right] = \frac{(8\sqrt{x}-x^2)(2+\sqrt{x})}{4-x}\left[\frac{8\sqrt{x}+x^2}{8\sqrt{x}+x^2}\right]$$

$$= \frac{(64x-x^4)(2+\sqrt{x})}{(4-x)(8\sqrt{x}+x^2)} = \frac{x\cancel{(4-x)}(16+4x+x^2)(2+\sqrt{x})}{\cancel{(4-x)}(8\sqrt{x}+x^2)}$$

$$= \frac{(4)(16+4(4)+(4)^2)(2+\sqrt{4})}{8\sqrt{4}+(4)^2} = 24$$

41. $$\lim_{x \to \infty} \frac{2x+5}{x+1}\frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \frac{2+\frac{5}{x}}{1+\frac{1}{x}} = \frac{2+0}{1+0} = 2$$

43. $$\lim_{x \to -\infty} \frac{-4x^3}{x^3-2x^2}\frac{\left(\frac{1}{x^3}\right)}{\left(\frac{1}{x^3}\right)} = \frac{-4}{1-\frac{2}{x}} = \frac{-4}{1-0} = -4$$

45. $\lim\limits_{x\to\infty} \frac{2x^2}{x-1} \frac{\left(\cdot\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \frac{2x}{1-\frac{1}{x}} = \infty$

47. $\lim\limits_{x\to-\infty} \frac{\sqrt{4x^2-x}}{x-2} = \frac{\sqrt{x^2\left(4-\frac{1}{x}\right)}}{x\left(1-\frac{2}{x}\right)} = \frac{|x|\sqrt{4-\frac{1}{x}}}{x\left(1-\frac{2}{x}\right)}$

since $x \to -\infty$, $|x| = -x$

$= -\frac{\sqrt{4}}{1} = -2$

49. $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2-4x+4}}{5-x} = \frac{\sqrt{x^2\left(1-\frac{4}{x}+\frac{4}{x^2}\right)}}{-x\left(1-\frac{5}{x}\right)} = \frac{|x|\sqrt{1-\frac{4}{x}+\frac{4}{x^2}}}{-x\left(1-\frac{5}{x}\right)}$

as $x \to -\infty$, $|x| = -x$ $\quad \frac{-x\sqrt{1-\frac{4}{x}+\frac{4}{x^2}}}{-x\left(1-\frac{5}{x}\right)}$

$= \frac{\sqrt{1}}{1} = 1$

51. $\lim\limits_{x\to\infty} \left(\sqrt{x^2+4x}-x\right)\left[\frac{\sqrt{x^2+4x}+x}{\sqrt{x^2+4x}+x}\right] = \frac{x^2+4x-x}{\sqrt{x^2+4x}+x}$

$\frac{x^2+3x}{\sqrt{x^2\left(1+\frac{4}{x}\right)}+x} = \frac{x(x+3)}{x\left[\sqrt{1+\frac{4}{x}}+1\right]} = \frac{x+3}{\sqrt{1+\frac{4}{x}}+1} = \frac{\infty+3}{1+1}$

$= \infty$

53. $$\lim_{x \to 0^-} \frac{|x|}{x} = \begin{cases} \frac{x}{x}, & \text{if } x > 0 \\ -\frac{x}{x}, & \text{if } x < 0 \end{cases} = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases}$$

since $x \to 0^-$ the $\lim_{x \to 0^-} = -1$

55. $$\lim_{x \to 2^+} \frac{|x-2|}{4-x^2} = \begin{cases} \frac{x-2}{(2-x)(2+x)}, & \text{if } x > 2 \\ \frac{-(x-2)}{(2-x)(2+x)}, & \text{if } x < 2 \end{cases} = \begin{cases} \frac{-1}{x+2}, & x > 2 \\ \frac{1}{x+2}, & x < 2 \end{cases}$$

since $x \to 2^+$ $\lim_{x \to 2^+} = \frac{-1}{2+2} = -\frac{1}{4}$

57. $$\lim_{x \to 1^-} \frac{|x^5-1|}{x-1} = \begin{cases} \frac{(x-1)(x^4+x^3+x^2+x+1)}{(x-1)}, & \text{if } x > 1 \\ \frac{-(x-1)(x^4+x^3+x^2+x+1)}{(x-1)}, & \text{if } x < 1 \end{cases}$$

Since $x \to^{-1}$ $\lim_{x \to 1^-} = -(1+1+1+1+1)$
$= -5$

59. $$\lim_{x \to 3^+} \frac{x^3-27}{|x-3|} = \begin{cases} \frac{(x-3)(x^2+3x+9)}{(x-3)}, & \text{if } x > 3 \\ \frac{-(x-3)(x^2+3x+9)}{(x-3)}, & \text{if } x < 3 \end{cases}$$

since $x \to 3^+$ $\lim_{x \to 3^+} = (3)^2 + 3(3) + 9 = 27$

61. $f(x) = \begin{cases} (x+1)\,2^{x-1}, & \text{if } x \geq 0 \\ 2^{1-x}, & \text{if } x < 0 \end{cases}$

a) $\lim_{x \to 3} f(x) = (3+1)\,2^{3-1}$
$= 4(4) = 16$

b) $\lim_{x \to -3} f(x) = 2^{1-(-3)} = 2^4 = 16$

c) $\lim_{x \to 0^+} f(x) = (0+1)\,2^{0-1}$
$= 1(2^{-1}) = \frac{1}{2}$

d) $\lim_{x \to 0^-} f(x) = 2^{1-0} = 2$

e) $\lim_{x \to 0} f(x)$ does not exist since $\lim_{x \to 0^+} \neq \lim_{x \to 0^-}$

63. $h(x) = \begin{cases} 2, & \text{if } x < 0 \\ x+2, & \text{if } 0 \leq x \leq 4 \\ x^2-11, & \text{if } x > 4 \end{cases}$

a) $\lim_{x \to -6} h(x) = 2$

b) $\lim_{x \to 0^-} h(x) = 2$

c) $\lim_{x \to 0^+} h(x) = 0 + 2 = 2$

d) $\lim_{x \to 0} h(x) = 2$

e) $\lim_{x \to 3} h(x) = 3+2 = 5$

f) $\lim_{x \to 4^-} h(x) = 4+2 = 6$

g) $\lim_{x \to 4^+} h(x) = 4^2-11 = 5$

h) $\lim_{x \to 4} h(x)$ does not exist

i) $\lim_{x \to 5} h(x) = 5^2-11 = 14$

written exercise 3.4

8 a) $f(x) = \dfrac{3x - x^2}{x^2 + x} = \dfrac{x(3-x)}{x(x+1)}$

Since $x=0$ results in $\frac{0}{0}$, there is a hole at $x=0, y=3$

Since $x=-1$ results in $\frac{4}{0}$, there is a vertical asymptote at $x=-1$

b) $f(x) = \dfrac{x^3 + 3x^2 - 10x}{x^2 + x - 6} = \dfrac{x(x^2 + 3x - 10)}{x^2 + x - 6} = \dfrac{x(x+5)(x-2)}{(x+3)(x-2)}$

since $x=2$ results in $\frac{0}{0}$ there is a hole at $x=2, y=\frac{14}{5}$

Since $x=-3$ results in $\frac{-6}{0}$ there is a vert asym at $x=-3$

10. a) $f(x) = \sin x + \cos x$

Continuous since all trigonometric functions are continuous

c) $f(x) = \dfrac{x+2}{x^2 - 2x - 3} = \dfrac{x+2}{(x-3)(x+1)}$ since $x=3$ and $x=-1$ result in a <u>non-zero#</u>$_{/0}$, there are infinite discontinuities at $x=3$ and $x=-1$ (vertical asymptotes)

e) $f(x)=\dfrac{|x-1|}{x^2-x} = \begin{cases} \dfrac{x-1}{x(x-1)}, & \text{if } x>1 \\ \dfrac{-(x-1)}{x(x-1)}, & \text{if } x<1 \end{cases} = \begin{cases} \dfrac{1}{x}, & \text{if } x>1 \\ -\dfrac{1}{x}, & \text{if } x<1 \end{cases}$

there is a discontinuity at $x=1$ and $x=0$

since $\lim\limits_{x\to 1^+} f(x)=1 \neq \lim\limits_{x\to 1^-} f(x)=-1$ there is

jump discontinuity at $x=1$

since $x=0$ results in $\frac{1}{0}$ (vertical asym) there is an infinite discontinuity at $x=0$

g) $f(x)=\dfrac{x-4}{\sqrt{x}-2} = \dfrac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}-2)} = \sqrt{x}+2.$

$\lim\limits_{x\to 4^+} f(x) = 4 = \lim\limits_{x\to 4^-} f(x) \therefore \lim\limits_{x\to 4} f(x)$ exists

but $f(x)$ is indeterminate $\frac{0}{0}$ at $x=4$

$\therefore x=4$ is a removable discontinuity.
(a hole)

i) $f(x)=\dfrac{x+5}{\log_2(x^2+1)}$ $\qquad \log_2(x^2+1)=0$ if $x=0$
$\log_2 1=0$

$\therefore x=0$ is an infinite discontinuity

since $x=0$ results in $\frac{5}{0}$

k) $f(x) = \frac{x^2+x}{x^4-x} = \frac{x(x+1)}{x(x^3-1)} = \frac{x(x+1)}{x(x-1)(x^2+x+1)}$

at $x=0$, $f(x) = \frac{0}{0}$ and $\lim_{x \to 0} f(x)$ exists

there is a removable discontinuity at $x=0$ (a hole)

at $x=1$, $f(x) = \frac{2}{0}$ therefore there is an infinite discontinuity at $x=1$ (vert asym)

m) $f(x) = \begin{cases} -4, & x < -3 \\ x-1, & -3 \leq x \leq 2 \\ x^2, & x > 2 \end{cases}$

$\lim_{x \to -3^-} f(x) = -4$ $\quad \lim_{x \to -3^+} f(x) = -3-1 = -4$ $\quad \therefore \lim_{x \to -3} f(x)$ exists

and $f(x)$ is continuous at $x = -3$

$\lim_{x \to 2^-} f(x) = 2-1 = 1$ $\quad \lim_{x \to 2^+} f(x) = 2^2 = 4$ $\quad \therefore \lim_{x \to 2} f(x)$ does not exist

and there is a jump discontinuity at $x=2$

o) $f(x) = \sqrt[3]{9-x^2}$

continuous since all root functions are continuous throughout their domain