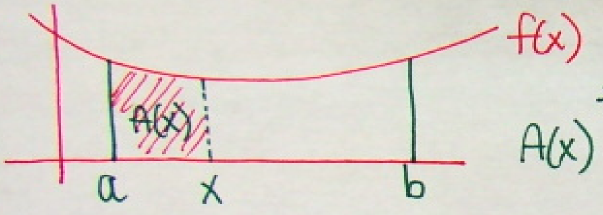
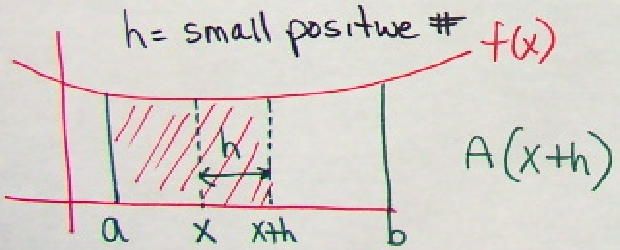
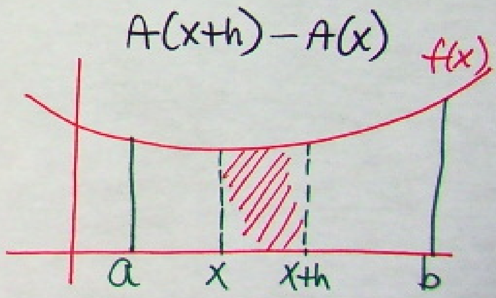
9.1 The Area Under a Curve

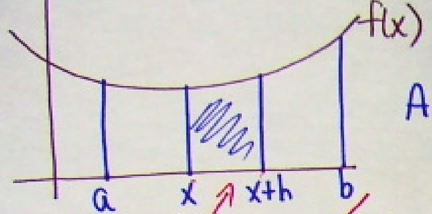
let $A(x)$ be a function that gives the area under the curve $y=f(x)$ from a to x



* note: $A(a)=0$



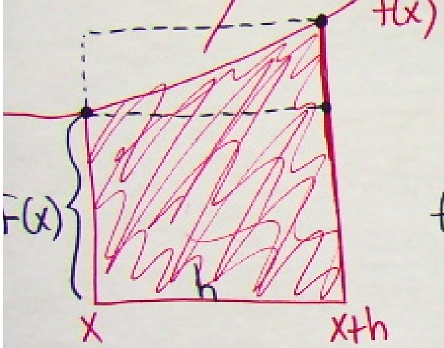


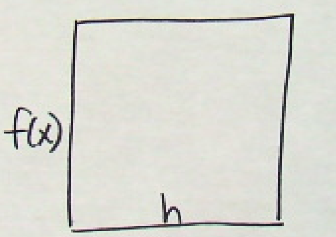


$A(x+h)-A(x)$

$$\text{small} < A(x+h)-A(x) < \text{large}$$

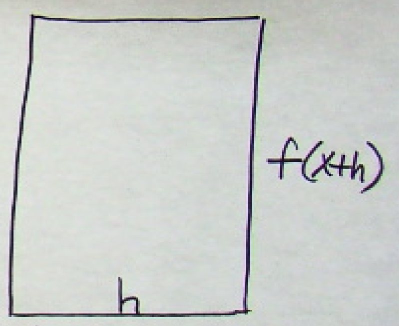
$$h\cdot f(x) < A(x+h)-A(x) < h\cdot f(x+h)$$





small rect

$A = \underline{h\cdot f(x)}$



large rect

$A = h\cdot f(x+h)$

$$h \cdot f(x) < A(x+h) - A(x) < h \cdot f(x+h)$$

divide by h

$$f(x) < \frac{A(x+h) - A(x)}{h} < f(x+h)$$

$\lim\limits_{h \to 0}$

$$\lim_{h\to 0} f(x) < \lim_{h \to 0} \frac{A(x+h) - A(x)}{h} < \lim_{h \to 0} f(x+h)$$

$$f(x) < \boxed{\lim_{h \to 0} \frac{A(x+h) - A(x)}{h}} < f(x)$$

$$f(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h} = A'(x)$$

$$f(x) = A'(x)$$

$$A(x) = \int f(x)\,dx = F(x) + C$$

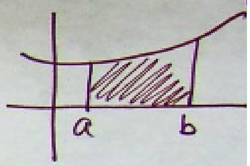
$$A(x) = \int f(x)\,dx = F(x) + C$$

$A(a) = 0$ $\quad$ $A(a) = F(a) + C$

$\qquad\qquad\quad 0 = F(a) + C$ $\quad$ $*\, C = -F(a)$

$$A(x) = F(x) - F(a)$$

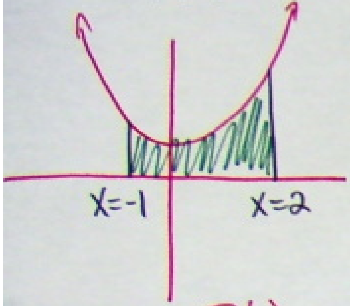
$$A(b) = F(b) - F(a) = \int_a^b f(x)\,dx$$



Fundamental theorem of Calculus

Suppose $f(x)$ is a continuous function defined on a closed interval $[a, b]$ and $f(x) \geq 0$ for all points in the interval. The area bounded by the x-axis, the vertical lines $x = a$ and $x = b$ and the function $y = f(x)$ is given by: $\int_a^b f(x)\,dx = F(b) -$

$f(x) = x^2+1 \quad x=-1\ ,\ x=2$



$$A = \int_{-1}^{2}(x^2+1)\,dx = \frac{1}{3}x^3+x\Big]_{-1}^{2} = F(2)-F(-1)$$

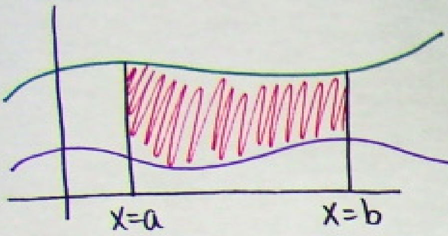
$$\frac{14}{3}-\left(-\frac{4}{3}\right)$$

$$F(2) = \frac{1}{3}(2)^3+2 = \frac{8}{3}+\frac{6}{3} = \frac{14}{3}$$

$$F(-1) = \frac{1}{3}(-1)^3+(-1) = -\frac{1}{3}-\frac{3}{3} = -\frac{4}{3}$$

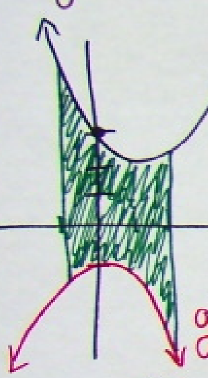
$$\frac{18}{3} = 6u^2$$

9.3 Area Between Two Curves



Theorem: If $f(x)$ and $g(x)$ are two continuous functions on the closed interval $[a,b]$ and $f(x) \geq g(x)$ for every x-value in the interval, then the area of the region formed by the vertical lines $x=a$ and $x=b$, as well as $f(x)$ and $g(x)$ is given by: $\int_a^b [f(x)-g(x)]\,dx$

ex11 find the area bounded by $f(x) = x^2 - 2x + 3$, $g(x) = -1 - x^2$ and the vertical lines $x = -1$ and $x = 2$



$$A = \int_{-1}^{2} \left[(x^2 - 2x + 3) - (-1 - x^2) \right] dx$$

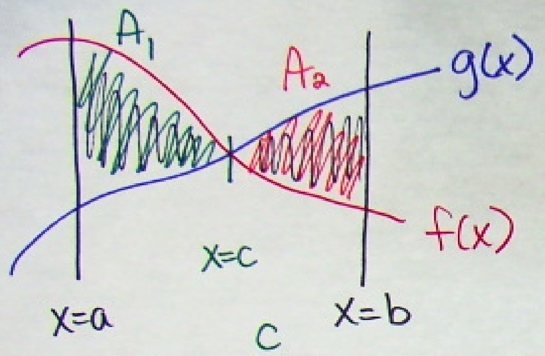
$$\qquad\qquad\qquad +1 \; +x^2$$

$$= \int_{-1}^{2} (2x^2 - 2x + 4)\, dx = \frac{2}{3}x^3 - x^2 + 4x \Big]_{-1}^{2}$$

$$F(2) = \frac{2}{3}(2)^3 - (2)^2 + 4(2) = \frac{16}{3} - 4 + 8 = \frac{28}{3}$$

$$F(-1) = \frac{2}{3}(-1)^3 - (-1)^2 + 4(-1) = -\frac{2}{3} - 1 - 4 = -\frac{17}{3}$$

$$F(2) - F(1) = \frac{28}{3} - \left(-\frac{17}{3}\right) = \frac{45}{3} = 15u^2$$



$$A_1 = \int_a^c [f(x) - g(x)]\, dx \qquad A_2 = \int_c^b [g(x) - f(x)]\, dx$$

$$A_1 + A_2 = \int_a^c [f(x) - g(x)]\, dx + \int_c^b [g(x) - f(x)]\, dx$$