8.2 Integration by Sight

Calculus is divided into 2 branches:

1) Differential
2) Integral

$$\begin{tikzcd} f(x) \arrow[r, bend left, "\text{differentiation}"] & f'(x) \arrow[l, bend left, "\text{integration}"] \end{tikzcd}$$

integration is the inverse operation of differentiat
In this section, we are going to find the integral (anti-derivative) by trial and error or sight

find a function $f(x)$ for which the derivative is $f'(x) = 4x^3 + 6x - 10$

$4x^3$ $\quad f(x) = 4x^4 \quad f'(x) = 16x^3$
$\quad f(x) = x^4 \quad f'(x) = 4x^3$

$6x$ $\quad f(x) = 6x^2 \quad f'(x) = 12x$
$\quad f(x) = 3x^2 \quad f'(x) = 6x$

$-10x^0$ $\quad f(x) = -10x \quad f'(x) = -10$

$f(x) = C$
$f'(x) = 0$

$F(x) = x^4 + 3x^2 - 10x + C$ ← any constant power rule of integration

If $f'(x) = x^n$ then $f(x) = \frac{1}{n+1} x^{n+1} + C$

$y = x^2$

$\frac{dy}{dx} = 2x$

$f'(x) = \sin 4x$, $f(x) = ?$

$f(x) = \cos(4x)$, $f'(x) = -\sin 4x \,(4)$

$= \cancel{-4} \sin 4x$

$f(x) = -\frac{1}{4} \cos 4x + C$

$\cancel{-4}$ $f'(x) = +\frac{1}{\cancel{4}}(+\sin 4x)(\cancel{4})$

$\int$ integral sign

$\int x^2 \, dx$ ← variable that you are integrating with respect to

integrand

<u>indefinite</u> integral

$\int x^2 \, dx = \frac{1}{3} x^3 + C$

$\int x^4 \, dx = \frac{1}{5} x^5 + C$ $\int x^3 \, dx = \frac{1}{4} x^4 + C$

$\int \frac{1}{x} dx = \int x^{-1} dx = \frac{1}{0} x^0 + C$

$y = \ln x$ $\frac{dy}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \ln x + C$

if $y = \ln|x|$, then $\frac{dy}{dx} = \frac{1}{x}$

$\int \frac{1}{x} dx = \ln|x| + C$

ex II $\int x^4 (2x-3)\, dx = \int (2x^5 - 3x^4)\, dx$

$= \frac{1}{3} x^6 - \frac{3x^5}{5} + C$

8.3 Integration by u substitution

ex11 $\int (4x+3)^9 dx$

let $u = 4x+3$

$\frac{du}{dx} = 4 \quad du = 4dx$

$\frac{1}{4}du = dx$

$\int u^9 \cdot \frac{1}{4} du = \frac{1}{4}\int u^9 du = \frac{1}{4}\left(\frac{1}{10}u^{10}\right) + C$

$= \frac{1}{40}(u)^{10} + C$

$u = 4x+3 \quad f(x) = \frac{1}{40}(4x+3)^{10} + C$

$f'(x) = \frac{1}{\cancel{40}} \cdot \cancel{10}(4x+3)^9(\cancel{4}) = (4x+3)^9$

$\int x^2(x^3-5)^5 dx$

let $u = x^3 - 5$

$\frac{du}{dx} = 3x^2 \quad du = 3x^2 dx$

$\frac{1}{3}du = x^2 dx$

$\int (x^3-5)^5 x^2 dx$

$\frac{1}{3}\int u^5 du = \frac{1}{3}\cdot\frac{1}{6}u^6 + C = \frac{1}{18}u^6 + C$

$= \frac{1}{18}(x^3-5)^6 + C$

$\int (2x-3)e^{x^2-3x} dx$

let $u = x^2 - 3x$

$du = (2x-3)dx$

$\int e^u du = e^u + C = e^{x^2-3x} + C$

$\int (\sec^4 x \tan x) dx$

let $u = \sec x$

$du = (\sec x \tan x)dx$

$\int (\sec^3 x)\cdot(\sec x \tan x)dx = \int u^3 du = \frac{1}{4}u^4 + C$

$= \frac{1}{4}\sec^4 x + C$

8.4 Definite Integration

Suppose that $f(x)$ is a continuous function on the interval $[a,b]$, and $\int f(x)\,dx = F(x)+C$

The number $F(b)-F(a)$ is called the definite integral of $f(x)$ from a to b

$$* \quad \int_a^b f(x)\,dx = F(b)-F(a)$$

the numbers a and b are called the lower and upper bounds of integration

ex11 evaluate $\int_1^4 (2x+3)\,dx = \underbrace{x^2+3x}_{F}\Big]_1^4 = F(4)-F(1)$

$F(4) = (4)^2+3(4) = 28$

$F(1) = (1)^2+3(1) = 4$

$= 28-4 = 24$

what about the constant of integration C?

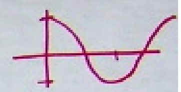
$\int (2x+3)\,dx = x^2+3x+C$

$\int_1^4 (2x+3)\,dx = \left[(4)^2+3(4)+C\right] - \left[(1)^2+3(1)+C\right]$

$(28+C)-(4+C)$

$28+\cancel{C}-4-\cancel{C} = 24$

ex 11 $\int_{-3}^{0} \left(\frac{1}{x+2}\right)dx$ the function $f(x) = \frac{1}{x+2}$ has an infinite discontinuity at $x=-2$. Since $-2 \in [-3,0]$, the definite integral cannot be evaluated



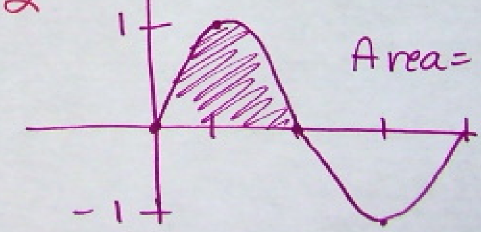
ex 11 $\int_{0}^{\pi} \sin x \, dx = -\cos x \Big]_{0}^{\pi}$

$F(\pi) = -\cos\pi = -(-1) = 1$

$F(0) = -\cos 0 = -(1) = -1$

$F(\pi) - F(0)$

$1 - (-1) = 2$



Area = $2u^2$