Chapter 7 Differentiating Transcendental Functions

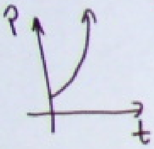
Transcendental functions are non-algebraic functions

2 categories

1) Exponential and Logarithmic

$f(x) = 2^x$

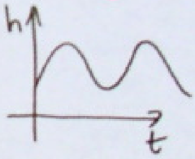
$f(x) = \log x$



2) Trigonometric

$f(x) = \sin x$

$f(x) = \tan x$



7.1 Logarithmic Functions

$\underset{\text{base}}{2}^{\overset{\text{exponent}}{3}} = 8 \iff \log_2 8 = \boxed{3}$ * the answer to a log is an exponent

exponential form — logarithmic form

evaluate the following logarithms

1) $\log_5 25 = x$ $\quad 5^x = 25$
$\quad 2 \qquad 5^x = 5^2$

2) $\log_7 1 = x$ $\quad 7^x = 1$ $\quad * \log_a 1 = 0 \quad a^0 = 1$
$\quad 0 \qquad 7^x = 7^0$

3) $\log_3 3^8 = 8$ $\quad * \log_a a^m = m$

4) $\log_6 6 = 1$ $\quad * \log_a a = 1$

5) $\log_2 0$ $\quad 2^1 = 2 \quad 2^0 = 1 \quad 2^{-1} = \frac{1}{2} \quad 2^{?} = 0$
no answer

6) $\log_{10} 1000 = 3$

7) $\log_3 \sqrt{3}$ $\quad 3^x = 3^{\frac{1}{2}}$
$\quad \frac{1}{2}$

Properties of Exponents and Logarithms

$a^m \cdot a^n = a^{m+n}$

$2^3 \cdot 2^4 = 2^7$

$2 \cdot 2 \cdot 2 \quad 2 \cdot 2 \cdot 2 \cdot 2$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$$\left.\begin{aligned} \log_a x + \log_a y &= \log_a xy \\ \log_a x - \log_a y &= \log_a\left(\frac{x}{y}\right) \\ k \log_a x &= \log_a x^k \end{aligned}\right\}$$

* change of base formula. $\log_a x = \dfrac{\log x}{\log a}$

$\log_4 31 = \dfrac{\log 31}{\log 4}$

ex 1) $3\log a + 2\log b$

$\log a^3 + \log b^2$

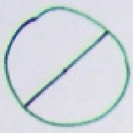
$\log(a^3 b^2)$

2) $\log_2 10 + \log_2 5 - \log_2 25$

$\log_2\left[\dfrac{(10)(5)}{25}\right] = \log_2 2$

$= 1$

$\pi \approx 3.1415$



$\dfrac{C}{d} = \pi$ $\quad 30^\circ = \dfrac{\pi}{6}$ rads.

$A = P\left(1 + \frac{i}{n}\right)^{nt}$ $\quad$ \$5000 6% monthly 30 years

$A = 5000\left(1 + \frac{.06}{12}\right)^{30(12)} \approx \$30\,112.88$

\$1.00 100% 1 year $\quad A = \left(1 + \frac{1}{n}\right)^n$

(\$)A	n
2	1
2.25	2
2.61	12
2.71	365
2.71828...	∞

$A = (1+1)^1$

$A = \left(1 + \frac{1}{2}\right)^2 = 1.5^2 = 2.25$

$A = \left(1 + \frac{1}{12}\right)^{12} = 2.61$

$A = \left(1 + \frac{1}{365}\right)^{365} = 2.71$

$\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e \approx 2.718$ $\quad$ nature's #

$t = \frac{1}{n}$ $\quad$ Euler $\quad \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$

$\log_e m = \ln m$ * natural logarithm

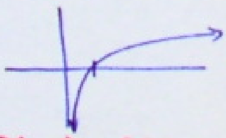
$\log m$ * common logarithm

evaluate logs

1) $\log_2 8 = 3$
2) $\ln e = \log_e e = 1$
3) $\ln e^3 = 3$
4) $\ln \sqrt{e} = \frac{1}{2}$

The Derivative of the Logarithmic Function

$f(x) = \log_a x$



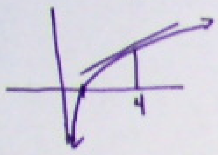
$f'(x)$

* $e = \lim_{t \to 0} (1+t)^{\frac{1}{t}}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h}$ pg297

* $f'(x) = \frac{1}{x} \cdot \log_a e$

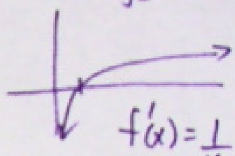
$f(x) = \log_2 x$



$f'(x) = \frac{1}{x} \log_2 e$ ← irrat #

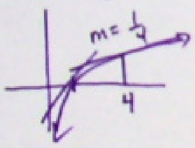
$f'(4) = \frac{1}{4}(\log_2 e)$

$f(x) = \log_3 x$



$f'(x) = \frac{1}{x} \log_3 e$

$f(x) = \log_e x = \ln x$



$f'(x) = \frac{1}{x} \cdot \log_e e$

$\boxed{f'(x) = \frac{1}{x}}$ $f'(4) = \frac{1}{4}$

find the derivative of the following

1) $y = \log_2(3x+1)$ argument

$\frac{dy}{dx} = \frac{1}{3x+1} \cdot (\log_2 e) \cdot 3$

2) $y = \ln(2x-5)$

$\frac{dy}{dx} = \frac{1}{2x-5} \cdot 2 = \frac{2}{2x-5}$

3) $y = x^6 \ln(3x+7)$

$\frac{dy}{dx} = 6x^5 \ln(3x+7) + \frac{1}{3x+7}(3)(x^6)$

4) $f(x) = \ln \sqrt[3]{x+2} = \ln(x+2)^{1/3}$

$f(x) = \frac{1}{3}\ln(x+2)$

$f'(x) = \frac{1}{3}\cdot\frac{1}{x+2}(1)$

5) $f(x) = \ln\left(\frac{3x+1}{2x-3}\right)$

$f(x) = \ln(3x+1) - \ln(2x-3)$

$f'(x) = \frac{1}{3x+1}(3) - \frac{1}{2x-3}(2)$

$\left(\frac{2x-3}{2x-3}\right)\frac{3}{3x+1} - \left(\frac{2}{2x-3}\right)\left(\frac{3x+1}{3x+1}\right) = \frac{6x-9-6x-2}{(2x-3)(3x+1)} = \frac{-11}{(2x-3)(3x+1)}$

* Summary.

1) if $f(x) = \log_a x$, then $f'(x) = \frac{1}{x}\cdot \log_a e$
2) if $f(x) = \log_a u$ where u is a function of x, then $f'(x) = \frac{1}{u}\cdot \log_a e \cdot \frac{du}{dx}$
3) if $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$
4) if $f(x) = \ln u$ where u is a function of x, then $f'(x) = \frac{1}{u}\cdot\frac{du}{dx}$

Solving Logarithmic Equations

Solve: 1) $\log_4 x = 3$

$4^3 = x = 64$

2) $\ln x = 3$

$\log_e x = 3$

$x = e^3$

3) $\ln(x-1) = 0$

$e^0 = x-1$

$1 = x-1$

$x = 2$

ex) find the open intervals on which the function $f(x) = \ln x - x$ is increasing and decreasing. Determine the coordinates of all local extrema.

$\ln x$

$(0,\infty)$

$f'(x) = \frac{1}{x} - 1 = 0$

$\frac{1}{x} = 1$

$x = 1$

x	$(0,1)$	1	$(1,\infty)$
$f'(x)$	$+$	0	$-$

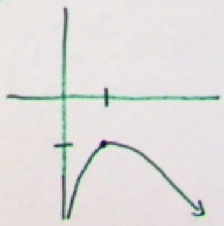
increasing $(0,1)$

decreasing $(1,\infty)$

$f(1) = \ln 1 - 1$

-1

local max $(1,-1)$

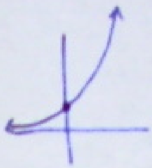


7.2 Exponential Functions

$f(x) = a^x$, $a > 0$, $a \neq 1$

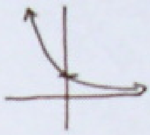
if $a > 1$ exponential growth

$f(x) = 2^x$



if $0 < a < 1$ exponential decay

$f(x) = \left(\frac{1}{2}\right)^x = 2^{-x}$



the derivative of $f(x) = a^x$

$y = a^x$

$\ln y = \ln(a^x)$

$\frac{d(\ln y)}{dx} = \frac{d(x \ln a)}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \ln a$

$\frac{dy}{dx} = \ln a \cdot y$

$\frac{dy}{dx} = a^x \cdot \ln a$

ex $y = 2^x$ $\quad \frac{dy}{dx} = 2^x \cdot \ln 2$

$y = 3^x$ $\quad \frac{dy}{dx} = 3^x \cdot \ln 3$

$y = e^x$

$\frac{dy}{dx} = e^x \cdot \ln e = e^x$

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* summary!

if $y = a^x$

then $\frac{dy}{dx} = a^x \cdot \ln a$

if $y = e^x$

then $\frac{dy}{dx} = e^x$

if $y = a^u$ where u is a function of x

then $\frac{dy}{dx} = a^u \cdot \ln a \cdot \frac{du}{dx}$

if $y = e^u$ where u is a function of x

then $\frac{dy}{dx} = e^u \cdot \frac{du}{dx}$

ex: find the derivative

1) $y = 2^{3x+2}$

$\frac{dy}{dx} = 2^{3x+2} \cdot \ln 2 (3)$

$\cdot 3 \ln 2$

2) $y = 3^{x^2-4x}$

$\frac{dy}{dx} = 3^{x^2-4x} \cdot \ln 3 (2x-4)$

3) $y = e^{6x}$

$\frac{dy}{dx} = e^{6x}(6)$

$= 6e^{6x}$

4) $y = x^2 e^x$

$\frac{dy}{dx} = 2x \cdot e^x + e^x \cdot x^2$

* important properties of logs and exponents

$2^{\log_2 8} = 2^3 = 8$ $\quad$ $2^{\log_2 14} = 14$ $\quad$ $\boxed{* \, a^{\log_a m} = m}$

$e^{\log_e m} = \boxed{e^{\ln m} = m}$

$\log_3 3^5 = 5$ $\quad$ * $\boxed{\log_a a^m = m}$ $\quad$ $\boxed{\ln e^m = m}$

ex|| evaluate

1) $\log_2 2^7 = 7$

2) $\ln e^3 = 3$

3) $e^{2\ln 3} = e^{\ln 3^2}$
$= e^{\ln 9} = 9$

4) $e^{-\ln 2} = e^{\ln 2^{-1}} = 2^{-1} = \frac{1}{2}$

5) $2^{\log_2(x+3)} = x+3$

the number N of bacteria in a culture at time t is

$N = 2000\left[30 + te^{-\frac{t}{20}}\right]$, $[0,50]$ where t is measured in hours.

1) determine the initial number of bacteria

$N(0) = 2000\left[30 + 0e^{0}\right] = 60000$ $\qquad$ $-\frac{t}{20} = -\frac{1}{20}t$

2) find the largest number of bacteria.

$N(0) = 60000$

$N(50) = 2000\left[30 + 50e^{-\frac{5}{2}}\right] \approx 68208$

$\frac{dN}{dt} = \cancel{2000}\left[(1)e^{-\frac{t}{20}} + e^{-\frac{t}{20}}\left(-\frac{1}{20}\right)(t)\right] = 0$

$e^{-\frac{t}{20}}\left[1 - \frac{t}{20}\right] = 0$ $\qquad$ $N(20) = 2000\left[30 + 20e^{-1}\right]$

$\cancel{e^{-\frac{t}{20}}} = 0$ or $1 - \frac{t}{20} = 0$ $\quad$ $t = 20$ $\qquad$ ≈ 74715

the temp in °C in the chimney of a wood-burning stove, t minutes after a fire has been lit is

$$T(t) = \frac{500e^t}{e^t+200} + 18$$

i) find the rate at which the temp is changing at 2 mins. 6 mins.

$$T'(t) = 500\left[\frac{e^t(e^t+200) - e^t(e^t)}{(e^t+200)^2}\right]$$

$$500\,e^t\left[\frac{200}{(e^t+200)^2}\right]$$

$$T'(2) = 500e^2\left[\frac{200}{(e^2+200)^2}\right] \approx 17.18\ {}^\circ C/min$$

$$T'(6) = 500e^6\left[\frac{200}{(e^6+200)^2}\right] \approx 110.8\ {}^\circ C/min$$

7.3 Limits Involving Trigonometric Functions

Squeeze theorem * not responsible for

Fundamental Trigonometric Limit

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta}$, where θ is measured in rads.

$\frac{\sin 30^\circ}{30^\circ} = \frac{.5}{30^\circ}$

$\frac{\sin\frac{\pi}{6}}{\frac{\pi}{6}} = \frac{.5}{\frac{\pi}{6}}$

θ	$\frac{\sin\theta}{\theta}$
1	.8415
.5	.9589
.1	.9983
.01	.999983

$\frac{\sin 0}{0} = \frac{0}{0}$ indeterminate

$\frac{\sin 1}{1}$

$$\boxed{*\ \lim_{\theta\to 0} \frac{\sin(\theta)}{(\theta)} = 1}$$

1) $\lim_{\theta\to 0} \frac{\theta}{\sin\theta} = 1$

2) $\lim_{x\to 0} \frac{\sin kx}{kx} = 1$

3) $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta} = 0$

4) $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} = 0$

1) $\lim_{x\to 0} \frac{\sin 5x}{5x} = 1$

2) $\lim_{x\to 0} \frac{\sin 5x}{x}\left(\frac{5}{5}\right) = 5\left(\frac{\sin 5x}{5x}\right)$
$= 5(1) = 5$

$\sin^2 x + \cos^2 x = 1$

3) $\lim_{x\to 0} \frac{\sin 2x\left(\frac{2x}{2x}\right)}{\sin 5x\left(\frac{5x}{5x}\right)}$

$\frac{2x\left(\frac{\sin 2x}{2x}\right)}{5x\left(\frac{\sin 5x}{5x}\right)} = \frac{2x(1)}{5x(1)} = \frac{2}{5}$

4) $\lim_{x\to 0} \frac{1-\cos^2 x}{x} = \frac{\sin^2 x}{x}$

$= \left[\left(\frac{\sin x}{x}\right)(\sin x)\right]$

$(1)\sin(0)$

$= 0$

5) $\lim_{x\to 0} \frac{\tan x}{2x} = \frac{\frac{\sin x}{\cos x}}{2x} = \frac{\sin x}{\cos x}\cdot\frac{1}{2x}$

$\frac{\sin x}{2x}\left(\frac{1}{\cos x}\right) = \frac{1}{2}\left(\frac{\sin x}{x}\right)\left(\frac{1}{\cos x}\right)$

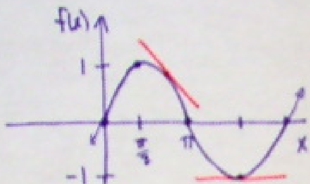
$= \frac{1}{2}(1)\left(\frac{1}{\cos 0}\right) = \frac{1}{2}(1)(1) = \frac{1}{2}$

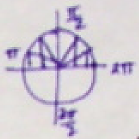
$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{\sin kx}{kx} = 1$

7.4 Sine and Cosine Functions

$f(x) = \sin x$





the derivative $f'(x)$ of $f(x) = \sin x$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$

$\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

$\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$

$\sin x \underbrace{\left[\frac{\cos h - 1}{h}\right]}_{=0} + \cos x \underbrace{\left[\left(\frac{\sin h}{h}\right)\right]}_{=1}$

* $\lim_{h\to 0} \frac{\sin h}{h} = 1$

$\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

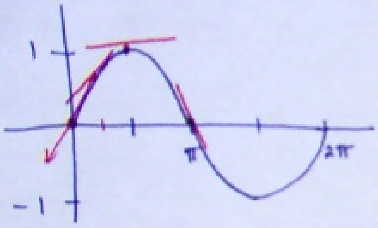
$f'(x) = \sin x(0) + \cos x(1)$

$f'(x) = \cos x$

* if $f(x) = \sin x$ then $f'(x) = \cos x$

find the slope of the tangent line to $f(x) = \sin x$ at $x = 0, \frac{\pi}{4}, \frac{\pi}{2}$ and π

$f'(x) = \cos x$



$f'(0) = \cos 0 = 1$

$f'(\frac{\pi}{4}) = \cos\frac{\pi}{4} \approx .7071 = \frac{\sqrt{2}}{2}$

$f'(\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0$

$f'(\pi) = \cos\pi = -1$

* If $f(x) = \sin(u)$ where u is a function x, then $f'(x) = \cos u \cdot \frac{du}{dx}$

1) $f(x) = \sin 10x$
$f'(x) = \cos 10x\,(10) = 10\cos 10x$

2) $f(x) = \sin(3x^2+5x+1)$
$f'(x) = \cos(3x^2+5x+1)[6x+5]$

3) $y = 2x\sin x$
$\frac{dy}{dx} = 2\sin x + \cos x(2x)$

4) $y = e^{(\sin 3x)}$
$\frac{dy}{dx} = e^{\sin 3x} \cdot \cos 3x \cdot 3$

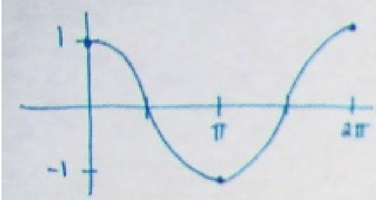
5) $y = \sin^2 x = (\sin x)^2$
$\frac{dy}{dx} = 2(\sin x)^1(\cos x) = 2\sin x\cos x$
$\sin 2x$

$f(x) = \cos x$

cosine
complimentary function of sine

$\cos 20° = \sin 70°$
$\cos 45° = \sin 45$
$\cos x = \sin(\frac{\pi}{2} - x)$

$\cos x^2$
$\cos^2 x$



$f(x) = \cos x$
$f(x) = \sin(\frac{\pi}{2} - x)$
$f'(x) = \cos(\frac{\pi}{2} - x)(-1)$
$f'(x) = -\sin x$

* If $f(x) = \cos x$
then $f'(x) = -\sin x$

If $f(x) = \cos u$ where u is a function of x, then $f'(x) = -\sin u \cdot \frac{du}{dx}$

ex 1) $y = \cos 5x$
$\frac{dy}{dx} = -\sin 5x(5) = -5\sin 5x$

2) $f(x) = \ln(\cos x)$
$f'(x) = \frac{1}{\cos x} \cdot -\sin x = -\tan x$

3) $y = 4\cos(x^2)$
$\frac{dy}{dx} = 4(-\sin x^2)(2x) = -8\sin x^2$

find $\frac{dy}{dx}$ if $x^2 + \sin y = 3$ $\qquad y = f(x)$

$$\frac{d(x^2)}{dx} + \frac{d(\sin y)}{dx} = \frac{d(3)}{dx}$$

$$2x + \cos y \cdot \frac{dy}{dx} = 0$$

$$\cos y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{\cos y} = -2x \sec y.$$

find local extrema of $f(x) = 2\cos x + x$, $[0, 2\pi]$

$$f'(x) = -2\sin x + 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\sin\frac{5\pi}{6} = \frac{1}{2}$

$\sin\frac{7\pi}{6} = -\frac{1}{2}$

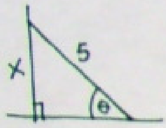
$$f''(x) = -2\cos x$$

$f''\left(\frac{\pi}{6}\right) = -2\cos\left(\frac{\pi}{6}\right)$
$= \text{negative}$
$\therefore \frac{\pi}{6}$ is a local max

$f''\left(\frac{5\pi}{6}\right) = -2\cos\left(\frac{5\pi}{6}\right)$
$= \text{positive}$
$\therefore \frac{5\pi}{6}$ is a local min

<u>related rates</u>

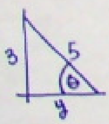
one end of a 5m ladder slides down a vertical wall. when the upper end of the ladder is 3m above the ground, it has a downward velocity of $\frac{1}{2}$ m/s. Find the rate at which the angle of elevation is changing at that time



$\frac{dx}{dt} = -\frac{1}{2}$ $\qquad$ find $\left.\frac{d\theta}{dt}\right|_{x=3}$

$$\sin\theta = \frac{x}{5} = \frac{1}{5}x$$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$$



$\cos\theta = \frac{y}{5} = \frac{4}{5}$

$3^2 + y^2 = 5^2$ $\quad y = 4$

$$\left(\frac{4}{5}\right)\frac{d\theta}{dt} = \frac{1}{5}\left(\frac{-1}{2}\right)$$

$$\frac{d\theta}{dt} = \frac{-1}{10}\left(\frac{5}{4}\right) = -\frac{1}{8} \text{ rad/s}$$

7.5 The Remaining Trigonometric Functions

*recall ① if $y = \sin x$, $\frac{dy}{dx} = \cos x$ ② if $y = \cos x$, $\frac{dy}{dx} = -\sin x$

determine the derivative of:

$y = \tan x$

$y = \frac{\sin x}{\cos x}$

$\frac{dy}{dx} = \frac{\cos x \cdot \cos x - (-\sin x)\sin x}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

③ $\frac{dy}{dx} = \sec^2 x$

pg331 summary.

$y = \sec x = \frac{1}{\cos x}$

$y = (\cos x)^{-1}$

$\frac{dy}{dx} = -(\cos x)^{-2}(-\sin x)$

$= \frac{1}{\cos^2 x} \cdot \sin x$

$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

④ $\frac{dy}{dx} = \sec x \tan x$

⑤ $y = \csc x$, $\frac{dy}{dx} = -\csc x \cot x$

⑥ $y = \cot x$, $\frac{dy}{dx} = -\csc^2 x$

find the derivative of the following

1) $f(x) = \sec(3x)$

$f'(x) = \sec 3x \tan 3x\,[3]$

$= 3\sec 3x \tan 3x$

$y = \sec(x)$, $\frac{dy}{dx} = \sec(x)\tan(x)$

2) $y = 2x\cot^2(5x) = 2x(\cot 5x)^2$

$\frac{dy}{dx} = 2(\cot 5x)^2 + 2(\cot 5x)^1(-\csc^2 5x)(5)(2x)$

$y = \cot x$, $\frac{dy}{dx} = -\csc^2 x$

3) $f(x) = e^{\tan x}$

$f'(x) = e^{\tan x}(\sec^2 x)(1)$

$f(x) = e^x$, $f'(x) = e^x$

3) Solve equations

a) $\ln x = 3$

$\log_e x = 3$

$e^3 = x$

b) $e^{x+3} = 2$

$\log_e 2 = x+3$

$\ln 2 = x+3$

$x = \ln 2 - 3$

4) differentiate * do not simplify

a) $f(x) = \ln(x-4)$

$f'(x) = \frac{1}{x-4} \cdot 1$

b) $f(x) = \ln \sqrt[3]{x}$

$= \ln x^{\frac{1}{3}}$

$= \frac{1}{3} \ln x$

$f'(x) = \frac{1}{3} \cdot \frac{1}{x} = \frac{1}{3x}$

c) $f(x) = 3x \cdot \ln 2x$

$f'(x) = 3 \ln 2x + \frac{1}{\cancel{2}x} \cdot \cancel{2}(3x)$

d) $f(x) = e^{3x+4}$

$f'(x) = e^{3x+4}(3)$

e) $f(x) = e^x \ln x$

$f'(x) = e^x \cdot \ln x + \frac{1}{x} \cdot e^x$

5) applications – related rate

the mass of a radioactive substance, M, in mg after time t, in days, is given by:

$M(t) = 300 \cdot e^{-\frac{\ln 2}{140} t}$

a) find the mass at 50 days

$M(50) = 300 e^{-\frac{\ln 2}{140}(50)} \approx 234.2\text{mg}$

b) find the rate of change at 50 days

$4t$
4

$M'(t) = 300 e^{-\frac{\ln 2}{140} t}\left(-\frac{\ln 2}{140}\right)$

$M'(50) = 300 e^{-\frac{\ln 2}{140}(50)}\left[-\frac{\ln 2}{140}\right] \approx -1.16 \text{ mg/day}$

7.7 Chapter Review

* not responsible for 7.6 Inverse Trig Functions

Part A Logs and Exponents *emphasis on base "e"

1) evaluate logs + exp

a) $\log_{25} 5 = \frac{1}{2}$ $\quad 25^{?} = 5$

b) $\ln e^3 = \log_e e^3 = 3$

c) $\log_2(-8)$ $\quad 2^{?} = -8$ $\quad 2^{-3} = \frac{1}{8}$

not possible

2) properties of logs

a) $\ln 10 + \ln 3 = \ln(10 \times 3)$
$= \ln 30$

b) $3\log 2 - 2\log 5$
$\log 2^3 - \log 5^2$
$\log 8 - \log 25 = \log\left(\frac{8}{25}\right)$

Part B Trig Functions

6) evaluate trig limits

* $\lim_{x \to 0} \frac{\sin(x)}{(x)} = 1$ $\qquad \lim_{x \to 0} \frac{\sin(3x)}{x} = \frac{3}{3}\frac{\sin 3x}{x}$
$= 3\left[\frac{\sin 3x}{3x}\right] = 3(1)$
$= 3$

7) differentiate *do not simplify

6 rules

a) $f(x) = -3\sin(2x)$
$f'(x) = -3\cos 2x\,(2)$

b) $f(x) = \cos^2 x = (\cos x)^2$
$f'(x) = 2(\cos x)^1(-\sin x)$
$= -2\cos x \sin x$

c) $f(x) = \csc\sqrt{x} = \csc x^{\frac{1}{2}}$
$f'(x) = -\csc x^{\frac{1}{2}} \cot x^{\frac{1}{2}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

d) $f(x) = 3x\tan 5x$
$f'(x) = 3\tan 5x + \sec^2 5x(5)(3x)$

8) applications

a) find the local extrema of $f(x) = \frac{1}{2}x + \sin x$, $[0, 2\pi]$ and then classify these values as either max or mins

$f'(x) = \frac{1}{2} + \cos x = 0$ $\quad x = \frac{2\pi}{3}$ $\quad f''(x) = -\sin x$ $\frown$

$\cos x = -\frac{1}{2}$

$x = \frac{4\pi}{3}$ $\quad f''\left(\frac{2\pi}{3}\right) = -\sin\frac{2\pi}{3} = <0$ max

$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{3}$ $\quad f''\left(\frac{4\pi}{3}\right) = -\sin\frac{4\pi}{3} = >0$ min

b) the position of a particle as it moves horizontally is given by $s = 2\sin t - \cos t$ where s is in cm and t is in seconds

i) find the position at 3 sec $\quad s(3) = 2\sin 3 - \cos 3$

$= 1.27\text{cm}$

ii) find the velocity at 3 sec

$v = \frac{ds}{dt} = 2\cos t + \sin t$ $\quad v(3) = 2\cos 3 + \sin 3$

$= -1.84 \text{ cm/s}$