Final Exam Review

Chapter 3

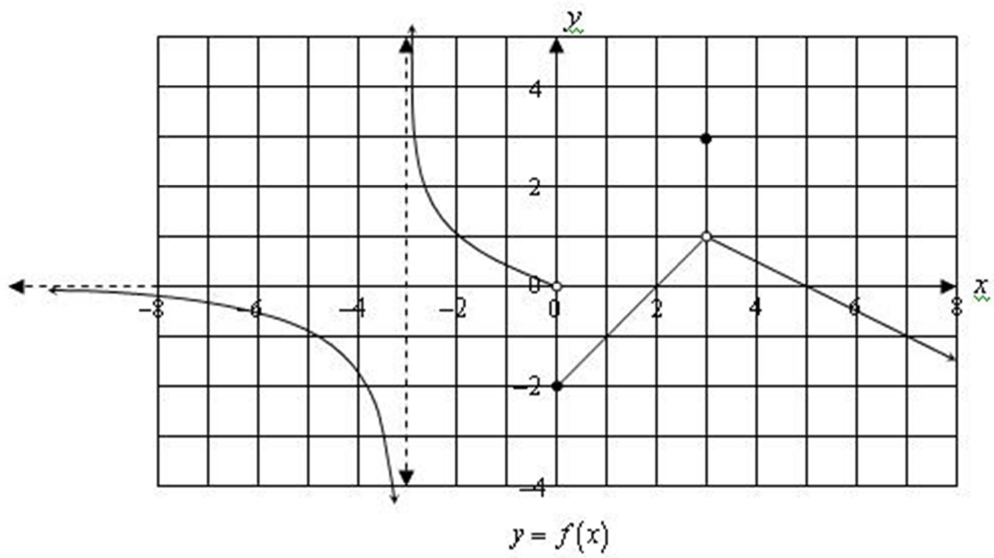
1. By examining the graph of the function below, answer the following questions.

(a) $\lim_{x\to -3^+} f(x)$ (b) $\lim_{x\to -3^-} f(x)$ (c) $\lim_{x\to -3} f(x)$

(d) $\lim_{x\to -2} f(x)$ (e) $\lim_{x\to 0^+} f(x)$ (f) $\lim_{x\to 0^-} f(x)$

(g) $\lim_{x\to 0} f(x)$ (h) $\lim_{x\to 3} f(x)$ (i) $\lim_{x\to 5} f(x)$

(j) $\lim_{x\to -\infty} f(x)$ (k) $\lim_{x\to \infty} f(x)$ (l) $\lim_{x\to 2} f(x)$



Evaluate each of the following limits

2. $\lim_{x\to -3} \dfrac{-x^2-3x+2}{2-x}$

3. $\lim_{w\to 10^+} \dfrac{w}{(w-10)^2}$

4. $\lim_{w\to 10^-} \dfrac{w}{(w-10)^2}$

5. $\lim_{w\to 10} \dfrac{w}{(w-10)^2}$

6. $\lim_{m\to -1} \dfrac{m^2+5m+4}{m^3+1}$

7. $\lim_{x\to 9} \dfrac{x^2-81}{3-\sqrt{x}}$ (don't rationalize, factor)

8. $\lim_{h\to 0} \dfrac{(-2+h)^2-4}{h}$

9. $\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{2x}$

10. $\lim_{h\to 0}\frac{h}{\sqrt{1-h}-1}$

11. $\lim_{x\to\infty}\frac{3-2x}{3+2x}$

12. $\lim_{x\to\infty}\frac{4x^2+4}{(2x-1)(x+5)}$

13. If $h(x)=\begin{cases}-3x-3, & x\in(-\infty,-1]\\ 6x+7, & x\in(-1,1)\\ x^4, & x\in[1,\infty)\end{cases}$ find each of the following limits.

(a) $\lim_{x\to -1^-} h(x)$

(b) $\lim_{x\to -1^+} h(x)$

(c) $\lim_{x\to -1} h(x)$

(d) $\lim_{x\to 1^-} h(x)$

(e) $\lim_{x\to 1^+} h(x)$

(f) $\lim_{x\to 1} h(x)$

Chapter 4

Find the derivative . Do not simplify!

1. f(x) = $\frac{1}{3}x^3-\frac{3}{2}x^2$
2. f(x) = 4 – x^2
3. f(x) = $(x^2 - 2x + 3)^2$
4. f(x) = (x-2)(x-1)(x)
5. f(x) = $\frac{12}{x^2+2}$
6. $x^2 - 3xy - y = 5$
7. f(x) = $x^2\sqrt{-x^2+1}$
8. f(x) = $\sqrt{\frac{x+4}{x-4}}$
9. $x^2 + y = 4$
10. xy= 12

Chapter 5

1. Find the second derivative of $y = -5t^4 + 3x^3 - 9x$

2. Find the second derivative of $f(x) = \dfrac{x}{x+1}$

3. Find $\dfrac{dy}{dx}$ for $2x^2 + y^2 = 7$.

4. Without drawing the graph, determine the absolute extrema for the function on the given interval.

$f(x) = 4 + 2x - x^2$; $[-1, 2]$

5. Find the intervals for which the function $f(x) = x^3 - 3x^2$ is increasing.

6. Determine the relative maximum or minimum point for the function $f(x) = -3x^5 + 5x^3$.

7. Find the relative maximum or minimum points for the function $y = \sqrt[3]{x^2 - 3x - 3}$.

8. Find the inflection point(s) for the function $y = x^4 - 6x^2$.

9. Determine the x and y intercepts for the function $y = x^2 - 5x + 6$

10. Determine any vertical asymptote(s) for $f(x) = \dfrac{x^3 - 4x}{x^2 + 9x}$.

11. Determine any horizontal asymptote(s) for $y = \dfrac{x^2}{x + 5x^2}$.

12. For the function $f(x) = \dfrac{12}{x^2 + 12}$, find the intervals where f(x) is concave up and concave down.

13. With each function you must identify:

- where intervals are increasing and decreasing
- the coordinates of any relative maximum(s) or minimum(s)
- where intervals are concave up and/or concave down
- coordinates of inflection points
- x and y intercepts
- all vertical and horizontal asymptotes.
- sketch the graph

a. $f(x) = x^4 - 8x^3$

b. $f(x) = \dfrac{9x}{x^2 - 9}$

Chapter 6

1. What positive number exceeds it's cube by the maximum amount?

2. If $s(t) = 9t^2 - t^3 + 15$.
a. Determine when the velocity will equal zero.

b. Determine when the acceleration will equal zero.

3. What are the dimensions of a rectangle of maximum area if its perimeter is 120m ?

4. A raindrop falls in a puddle and the ripples spread in circles, the radii of which grow at the rate of 2 cm/s. Find the rate of increase of the area of such a circle when its radius is 6 cm.

5. A ladder 10m long leans against a vertical wall. If the bottom slides out at a rate at a rate of 1 m/s, how fast is the top descending when the base of the ladder is 6 m from the wall?

6. The average purchase price of a house in Calgary is $275 000. The predicted value of this house, in *x* years, can be modeled by the function $v(x) = 275000(1+.0325x)^2$.
What will be the instantaneous rate of change in a rate 10 years from now?

7. A ball is thrown vertically upward from the roof of a gymnasium. The height of the ball above the ground (in meters) after *t* seconds is approximated by the function $h(t) = -5t^2 + 100t + 8$

a. Determine the velocity of the ball after 2 seconds.

b. What was the ball's maximum height?

c. When did the ball hit the ground?

8. A rectangular piece of cardboard 20 x 30 cm is to be transformed into a box with a open top by cutting squares of the same size from each corner and folding up the flaps. Find the dimensions of the cutout squares so the volume can be maximized.

9. A cylindrical tank has a radius of 3m and a depth of 10m. It is being filled at the rate of 5 m^3/minute. How fast is the surface rising?
$V = \pi r^2 h$

10. A conical flower vase is 20 cm high and has a radius of 5 cm at the top. $V = \frac{1}{3}\pi r^2 h$
If it is being filled with water at the rate of 8 cm^3/s, find the rate at which the water level is rising when the depth is 12 cm.

Chapter 7

Find the derivative of each of the following

1. $y = 6\sin^2 x$

2. $y = \cos(2t^2 + 1)$

3. $y = (x + 1)\sin(2 - 3x)$

4. $y = \dfrac{\cos 2x}{\sin x}$

5. $y = \sin(7x)\cos(7x)$

6. $f(x) = \ln\sqrt{1-6x}$

7. $f(x) = \left[\ln\left(x^3 - 1\right)\right]^3$

8. $f(x) = x^2 e^x$

9. $f(x) = e^{2x-5}$

10. $f(x) = \dfrac{x+1}{e^{x^2-1}}$

11. Suppose that the temperature of a hot chocolate in degrees Celsius, t minutes after it is poured, is given by the function $T(t) = 70e^{-0.1t} + 15$.

(a) What was the initial temperature of the hot chocolate?

(b) When was the coffee 70° C?

(c) What was the temperature after 8 minutes?

(d) Find $T'(t)$.

(e) Find $T'(5)$ and interpret the result.

Chapter 8&9

1. Determine the following indefinite integrals by sight
   a) $\int(2x^2 - 8x)dx$
   b) $\int(3x^5 + 4x^3 - 7)dx$
   c) $\int \frac{3}{x^5} dx$
   d) $\int(2\cos x - 3)dx$
   e) $\int \frac{10}{x} dx$
   f) $\int(2\sin 3x - \cos 3x)dx$
   g) $\int(3\sqrt{x} - \frac{1}{\sqrt{x}})dx$
   h) $\int 10e^{5x}dx$
   i) $\int(3x^4 - 1 + e^{2x})\, dx$
2. Determine the following indefinite integrals by u substitution
   a) $\int(3x - 5)^{12}dx$
   b) $\int x(x^2 + 3)^6$
   c) $\int x^3 e^{x^4} dx$
3. Evaluate the following definite integrals
   a) $\int_{-1}^{2}(3x^2 + x)dx$
   b) $\int_0^1 (2x + 1)^3 dx$
   c) $\int_2^4 \frac{1}{x} dx$
   d) $\int_0^{\frac{\pi}{3}} \sin 3x\, dx$
   e) $\int_e^{e^2} \frac{4}{x} dx$
   f) $\int_1^8 \left(x^{-\frac{1}{3}} + x^{\frac{1}{3}}\right) dx$
4. Find the area of the region above the x-axis, below the function given, over the given interval.
   a) $f(x) = 2x^2, [0,3]$
   b) $f(x) = 4x - x^3, [0,2]$
   c) $f(x) = 4 - e^{2x}, [0, \ln 2]$
   d) $f(x) = \frac{2}{x+1}, [1,3]$
   e) $f(x) = 3\sin(\frac{x}{2}), [\pi, 2\pi]$
5. Find the area between the two curves over the given interval.
   a) $f(x) = x^2, g(x) = 8\sqrt{x}, [0,4]$
   b) $f(x) = e^x, g(x) = 2, [0, \ln 2]$
   c) $f(x) = \sin x, g(x) = \cos x, [0, \frac{\pi}{4}]$
   d) $f(x) = -x^2 - x, g(x) = x, [-2,0]$