

Pre-Calculus 30

Chapter 2: Radical Functions Review

Name: Answer Key

1. Write the equation of a radical function that would result by applying each set of transformations to the graph of $\sqrt{f(x)}$

a) vertical stretch by a factor of 3, and horizontal stretch by a factor of 2

$$y = 3\sqrt{\frac{1}{2}x}$$

b) horizontal reflection in the y-axis, translation up 3 units, and translation left 2 units

$$y = \sqrt{-(x+2)} + 3$$

c) vertical reflection in the x-axis, horizontal stretch by a factor of $\frac{1}{3}$, and translation down 7 units

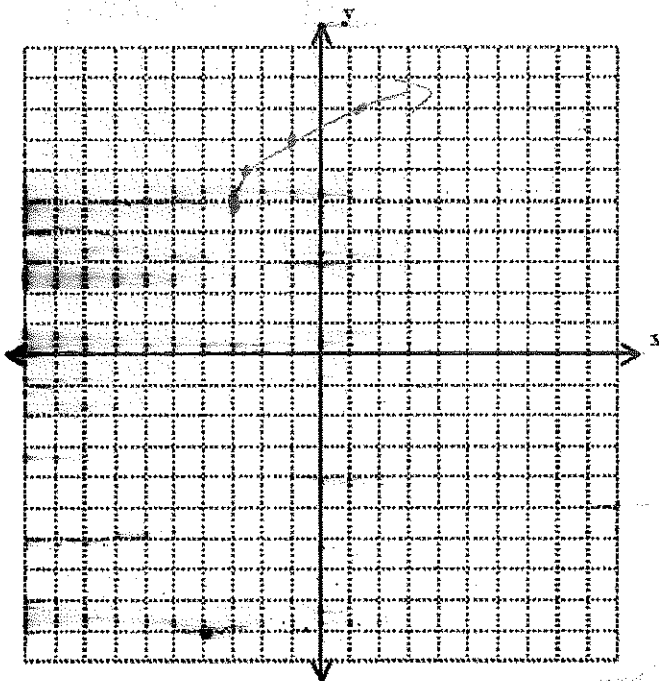
$$y = -\sqrt{3x} - 7$$

d) vertical stretch by a factor of 5, horizontal stretch by a factor of 0.25, and translation right 6 units

$$y = 5\sqrt{4(x-6)}$$

2. Sketch the graph of the following function using transformations.

$$y - 5 = \sqrt{2(x+3)} \quad y = \sqrt{2(x+3)} + 5$$



$$\begin{aligned} (x, y) &\rightarrow (\frac{1}{2}x - 3, y + 5) \\ (0, 0) &\rightarrow (-3, 5) \\ (1, 1) &\rightarrow (-2\frac{1}{2}, 6) \\ (4, 2) &\rightarrow (-1, 7) \\ (9, 3) &\rightarrow (1\frac{1}{2}, 8) \end{aligned}$$

3. State the domain and range of each function.

a) $y = \sqrt{-x-4}$

D $(-\infty, 0)$

R $[-4, \infty)$

c) $y-4 = -\sqrt{x-4}$

$y = -\sqrt{x-4} + 4$

D $[4, \infty)$

R $(-\infty, 4]$

b) $y = 4\sqrt{x-4}$

D $[4, \infty)$

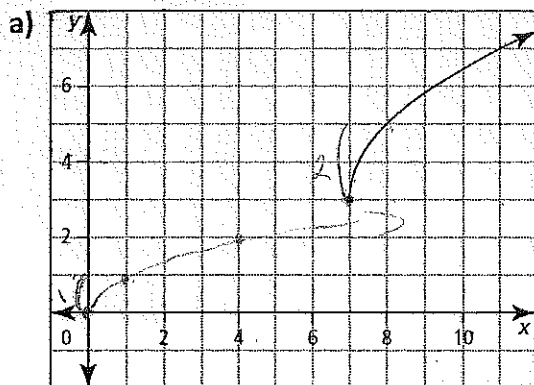
R $[0, \infty)$

d) $y = -\sqrt{4x}$

D $[0, \infty)$

R $(-\infty, 0]$

4. For each function, write an equation of a radical function of the form $y = a\sqrt{b(x-h)} + k$.



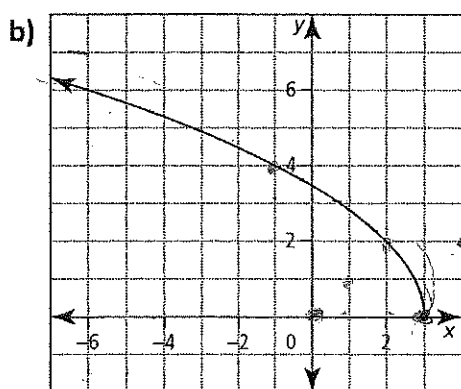
$(x, y) \rightarrow (x+7, 2y+3)$

$(0, 0) \rightarrow (7, 3)$

$(1, 1) \rightarrow (8, 5)$

$(4, 2) \rightarrow (11, 7)$

$y = 2\sqrt{x-7} + 3$



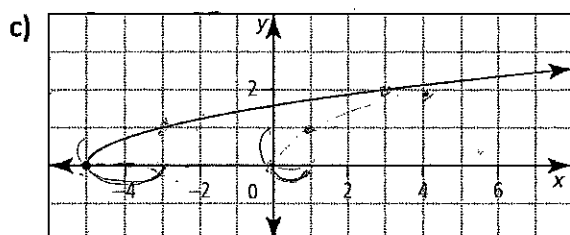
$(x, y) \rightarrow (-x+3, 2y)$

$(0, 0) \rightarrow (3, 0)$

$(1, 1) \rightarrow (2, 2)$

$(4, 2) \rightarrow (-1, 4)$

$y = 2\sqrt{-(x-3)}$



$(x, y) \rightarrow (2x-5, \frac{1}{2}y)$

$(0, 0) \rightarrow (-5, 0)$

$(-1, 1) \rightarrow (-3, 1)$

$(4, 2) \rightarrow (3, 2)$

$y = \sqrt{\frac{1}{2}(x+5)}$

5. Explain how to transform the graph of $y = \sqrt{x}$ to obtain the graph of each function.

a) $y = \sqrt{2x-6} + 5$

Hor. Stretch about y-axis
by factor of 2
Hor. Translation right 3
Vert. Translation up 5

b) $y - 7 = \sqrt{5-x}$

$$y = \sqrt{-x+5} + 7$$

$$y = \sqrt{-(x-5)} + 7$$

Hor. Reflection about y-axis
Hor. Translation right 5
Vert. Translation up 7

6. Determine the domains and ranges of each pair of functions. Explain why the domains and ranges differ.

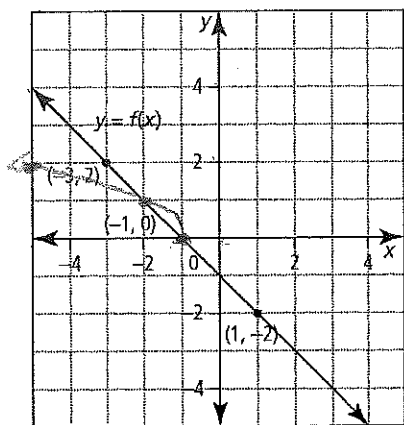
a) $y = x + 5$, $y = \sqrt{x+5}$

D $(-\infty, \infty)$, $[-5, \infty)$
R $(-\infty, \infty)$, $[0, \infty)$

b) $y = x^2 - 9$, $y = \sqrt{x^2 - 9}$

D $(-\infty, \infty)$, $(-\infty, -3] \cup [3, \infty)$
R $[-9, \infty)$, $[0, \infty)$

7. Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



8. Solve.

a) $x+3 = \sqrt{2x^2-7}$

$$x^2 + 6x + 9 = 2x^2 - 7$$

$$0 = x^2 - 6x - 16$$

$$0 = (x-8)(x+2)$$

$$x = 8 \quad x = -2$$

Check

$$\begin{array}{l} 8+3 = \sqrt{2(8)^2-7} \\ 11 = \sqrt{121} \\ 11 = 11 \checkmark \end{array} \quad \begin{array}{l} -2+3 = \sqrt{2(-2)^2-7} \\ 1 = \sqrt{1} \\ 1 = 1 \checkmark \end{array}$$

b) $x = \sqrt{x+10} + 2$

$$x-2 = \sqrt{x+10}$$

$$x^2 - 4x + 4 = x + 10$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6 \quad x = -1$$

Check

$$\begin{array}{l} 6 = \sqrt{6+10} + 2 \\ 6 = 4 + 2 \\ 6 = 6 \checkmark \end{array} \quad \begin{array}{l} -1 = \sqrt{-1+10} + 2 \\ -1 = 3 + 2 \\ -1 = 5 \times \end{array}$$

9. The radius, r , of a sphere is related to the surface area, A , by the equation $r = \frac{1}{2} \sqrt{\frac{A}{\pi}}$.

a) The surface area of a baseball is about 172 cm^2 . Find the radius of a baseball, to the nearest tenth of a centimetre.

$$\begin{aligned} r &= \frac{1}{2} \sqrt{\frac{172}{\pi}} \\ 2r &= \sqrt{\frac{172}{\pi}} \\ 4r^2 &= \frac{172}{\pi} \end{aligned} \quad \begin{aligned} 4r^2 \pi &= 172 \\ \frac{4r^2 \pi}{4\pi} &= \frac{172}{4\pi} \\ r^2 &= \frac{43}{\pi} \\ r &= \sqrt{\frac{43}{\pi}} \end{aligned}$$

$$r = 3.7 \text{ cm}$$

b) The radius of a tennis ball is about 3.3 cm . Find the surface area, to the nearest square centimetre.

$$\begin{aligned} 3.3 &= \frac{1}{2} \sqrt{\frac{A}{\pi}} \\ 6.6 &= \sqrt{\frac{A}{\pi}} \\ 43.56 &= \frac{A}{\pi} \\ A &= 137 \text{ cm}^2 \end{aligned}$$

10. Solve.

$$\sqrt{x+\sqrt{x-2}} = 2$$

$$x + \sqrt{x-2} = 4$$

$$\sqrt{x-2} = 4-x$$

$$x-2 = 16-8x+2x^2$$

$$0 = x^2 - 9x + 18$$

$$0 = (x-6)(x-3)$$

$$x = 6 \quad x = 3$$

Check

$$x=6 \quad \sqrt{6+\sqrt{6-2}} = 2$$

$$\sqrt{6+2} = 2$$

$$\sqrt{8} = 2 \times$$

$x=3$

$$\sqrt{3+\sqrt{3-2}} = 2$$

$$\sqrt{3+1} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2 \checkmark$$