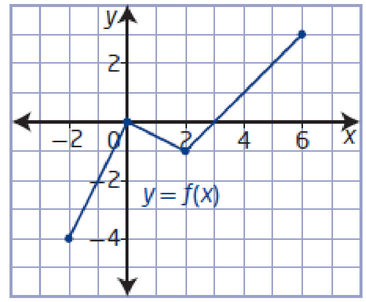
Pre-Calculus 30 – Final Exam Review - Chapters 1-9

Chapter 1 - Outcome 30-7 Review

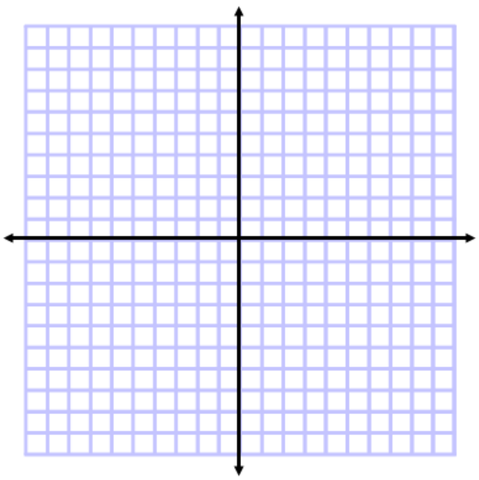
Level 2

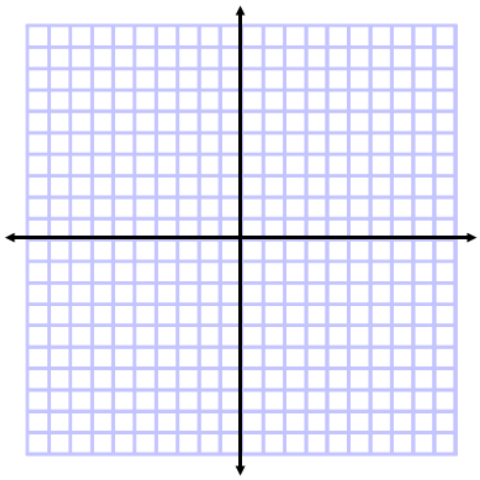


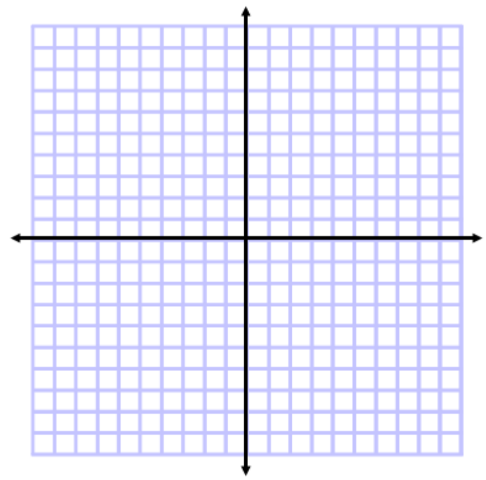
1. Consider the graph of $y = f(x)$.

   a) Sketch each of the transformed function:

      i) $y = f(x+3)$ ii) y = -2 f(x) iii) y = $f(\frac{1}{2}x) - 2$







2. For each equation, describe how the graph was translated, reflected or stretched.

   a) y = -2*f(3(x-4))*

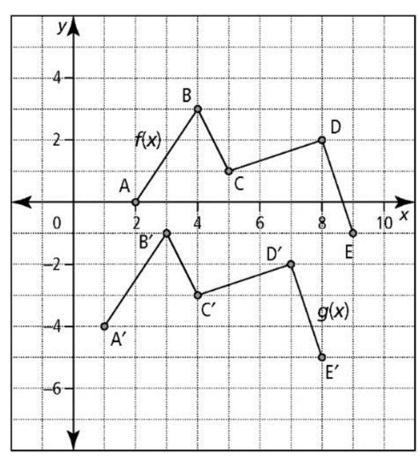
   b) y = *f(x – 5)-3*

   c_ y = *f(-2x)+5*

3. Consider the graph of $y = f(x)$ and $y = g(x)$.

   Determine the equation of the translated function in the form

   $y = af(b(x - h)) + k.$

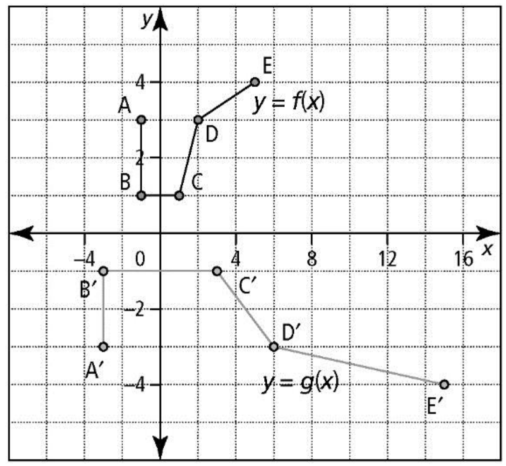


5. Determine algebraically the equation of the inverse of each function.

a) $f(x) = 3x - 6$ **b)** $f(x) = x^2 - 7$ **c)** $y = (x - 5)^2 - 9$

Level 3

6. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine an equation for $g(x)$.



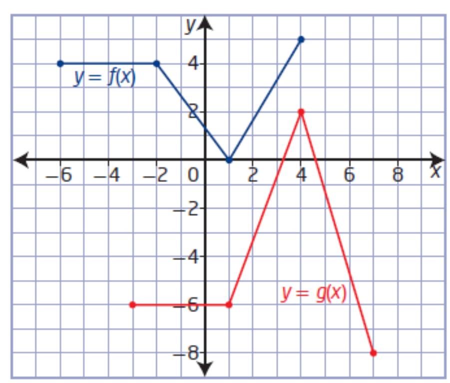
7. Write the equation for each transformation of $y = x^2$ in the form

$y = af(b(x - h)) + k$.

a) a vertical stretch by a factor of 3, reflected in the y-axis, and translated 3 units left and 2 units down

b) a horizontal stretch by a factor of 2, reflected in the x-axis, and translated 7 units up

9. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

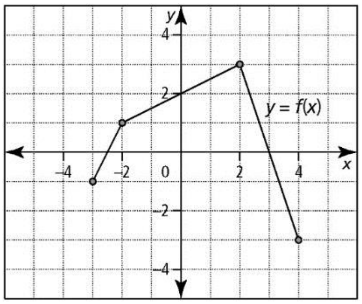


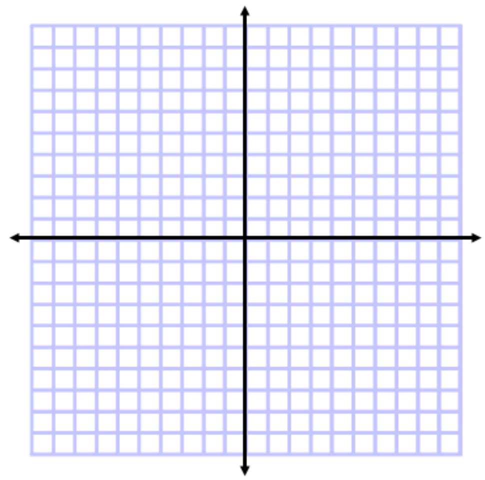
10. The key point (–18, 12) is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

a) $-3f(x + 5) + 4$ **b)** $y = 2f(6x)$

11. Consider the graph of the function $y = f(x)$.

Sketch $y = f(x)$ to $y = 3f(-2(x - 1)) + 4$.

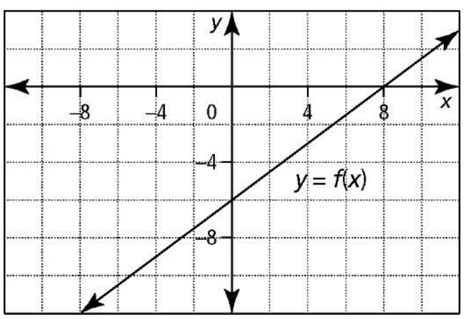


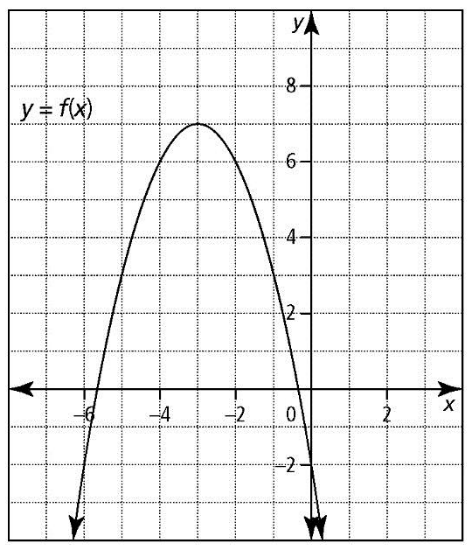


Level 4

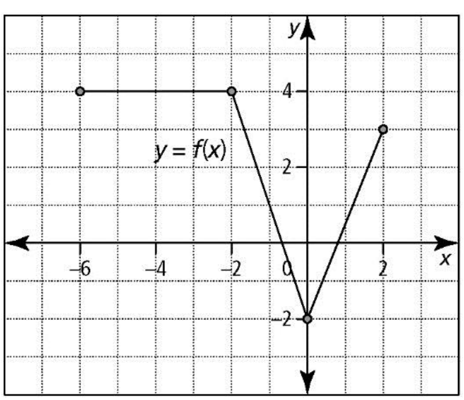
12. Sketch the graph of its inverse, $x = f(y)$. Determine whether the inverse is a function. If the inverse is not a function, restrict the domain of the original graph to make it a function.

a)





c)



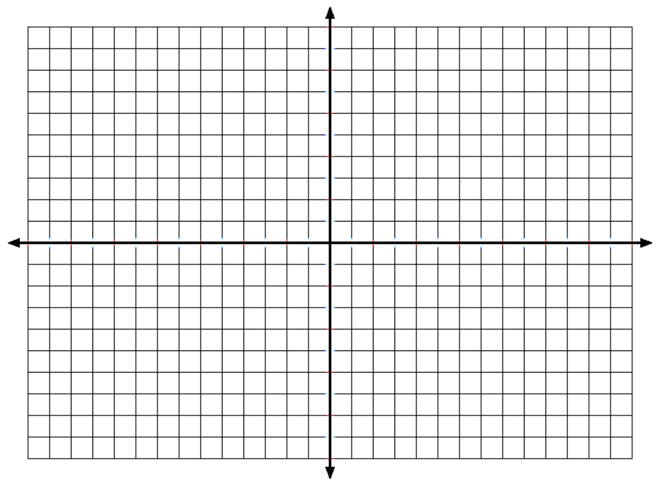
Chapter 2 - Outcome 30-11A

1. Identify a, b, h and k for each of the following

a) $y = 5\sqrt{x+7} - 2$

b) $y = -4\sqrt{-x} + 8$

2. Graph $y = \sqrt{x}$



Level 3

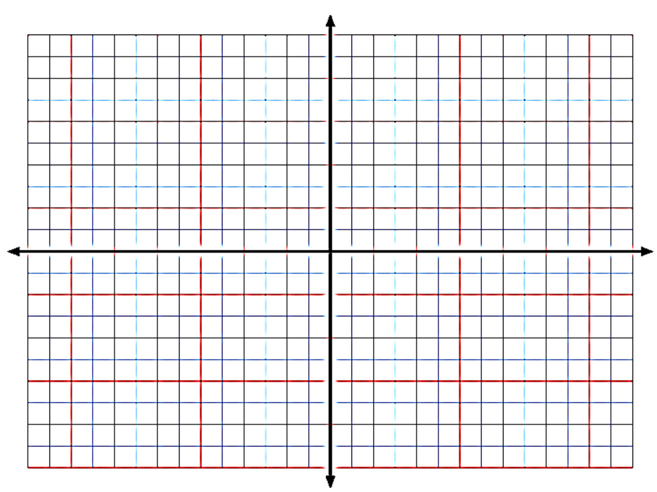
3. Write the equation of a radical function that would result by applying each set of transformations to the graph of .

a) vertical stretch by a factor of 3, and horizontal stretch by a factor of 2

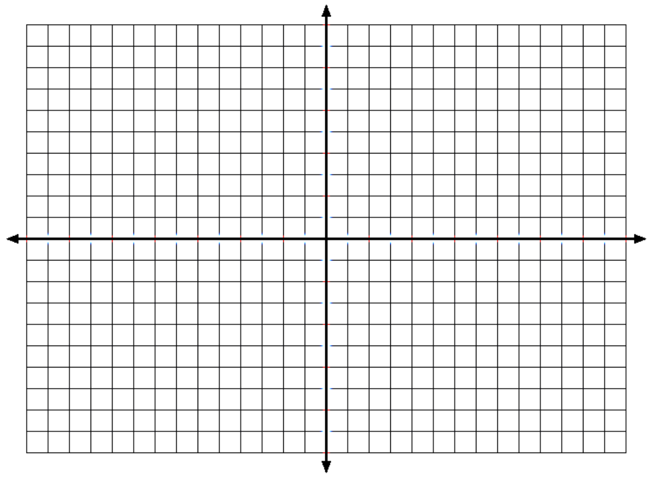
b) horizontal reflection in the *y*-axis, translation up 3 units, and translation left 2 units

4. Graph the functions below. Then, identify the domain and range.

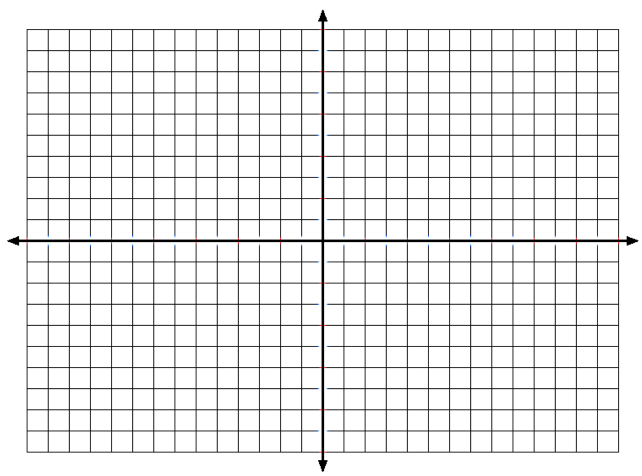
a) $y = -2\sqrt{x-2}$



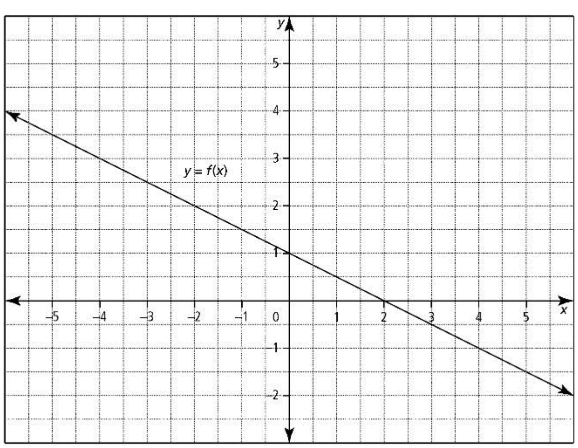
b) $y = \sqrt{2x} - 4$

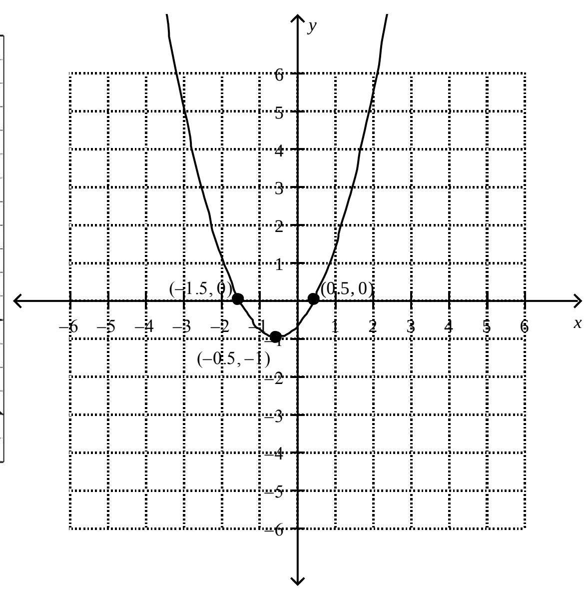


c) $y = 2\sqrt{-(x-3)} + 1$



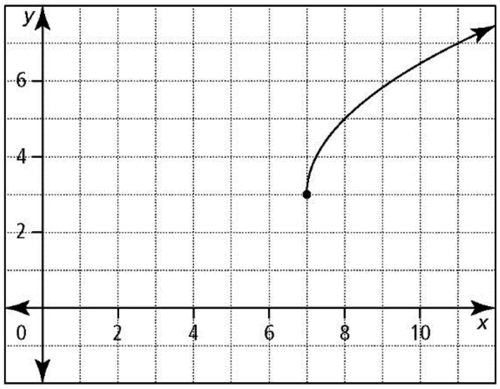
5. Graph $\sqrt{f(x)}$ from the following graphs of *f(x)* and state the **domain and range**



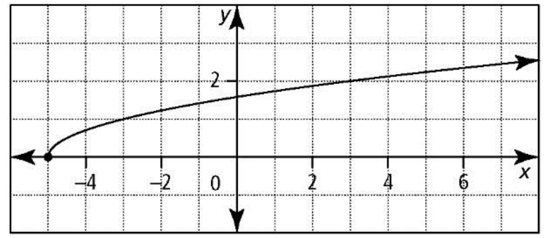


6. For each function, write an equation of a radical function of the form $y = a\sqrt{b(x-h)} + k$.

a)



b)



Chapter 3 - Outcome 30-10A Review

1. Divide the following using long division or synthetic division.

   a) $(2w^4 + 3w^3 - 5w^2 + 2w - 27) \div (w+3)$

   b) $\dfrac{2x^3 - 10x^2 - 15x - 20}{x+5}$

2. Determine the remainder when $x^3 + x^2 - 16x - 16$ is divided by

   a. x + 2

   b) x – 4

   b) Are any of the binomials above a factor of $x^3 + x^2 - 16x - 16$?

3. Factor completely

   a. $x^3 + 2x^2 - 13x + 10$

   b. $x^4 - 26x^2 + 25$

4. Determine the value(s) of k so that the binomial is a factor of the polynomial: $x^2 - 8x - 20$, $x + k$

5. The following polynomial has a factor of $x - 3$. What is the value of k? $kx^3 - 10x^2 + 2x + 3$

Chapter 3 - Outcome 30-10B Review

1. Determine which of the following are polynomials. For each polynomial function, state the degree.

a) $h(x) = 5 - \frac{1}{x}$ **b)** $y = 4x^2 - 3x + 8$ **c)** $g(x) = -9x^6$ **d)** $f(x) = \sqrt[3]{x}$

2. What is the leading coefficient, degree and constant term of each polynomial function?

a) $f(x) = -x^3 + 6x - 8$ **b)** $y = 5 + 2x^2$

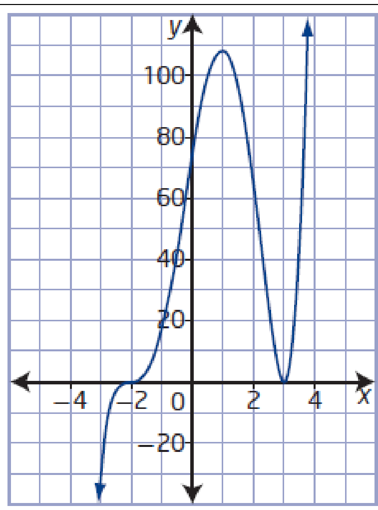
c) $g(x) = 7x^3 + 3x^5 - 8x + 10$ **d)** $k(x) = 9\text{x} - 2x^2$

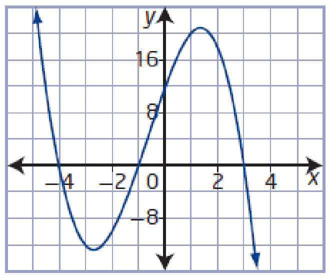
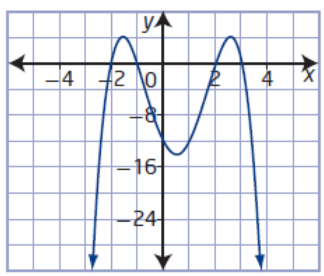
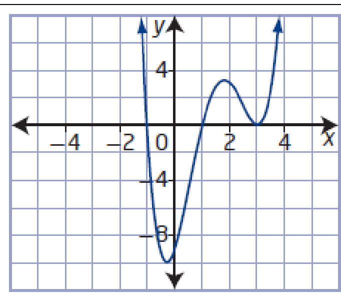
3. Identify the following characteristics for each polynomial function:

- the type of function and whether it is of even or odd degree
- the end behaviour of the graph of the function
- the number of possible x-intercepts
- the *y*-intercept

a) $g(x) = -2x^4 + 6x^2 - 7x - 5$ **b)** $f(x) = 2x^5 + 1x^3 - 12$

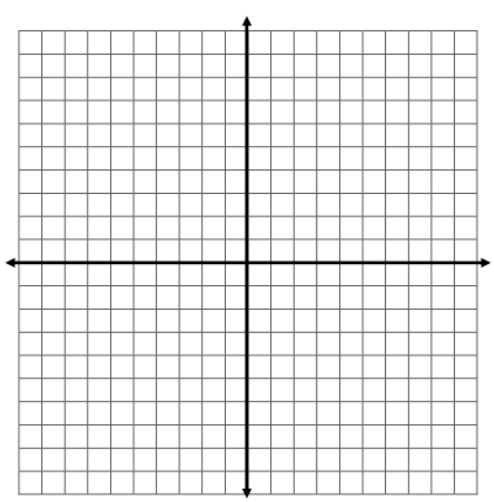
4. Fill in the table below for the following graphs

Graph	Odd or Even	Sign of Leading Coefficient	Number of x-intercepts	Intervals where the function is positive	Intervals where the function is negative
					

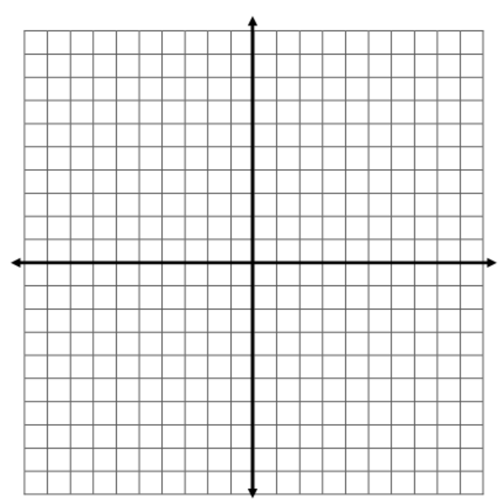
	Odd or Even	Sign of Leading Coefficient	Number of x-intercepts	Intervals where the function is positive	Intervals where the function is negative
					
					
					

4. Graph the following polynomial functions. The first three have already been factored for you.

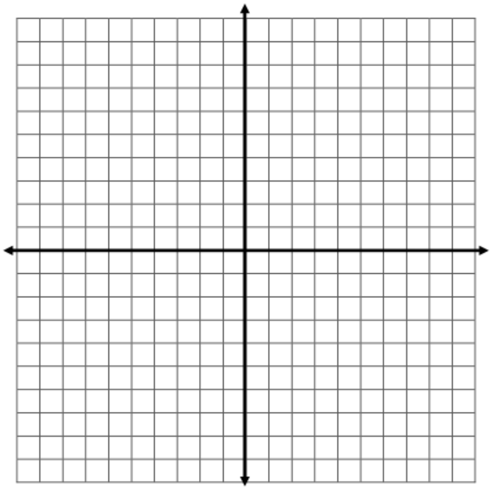
$y = -2(x-1)^2(x+2)(x-4)^2$



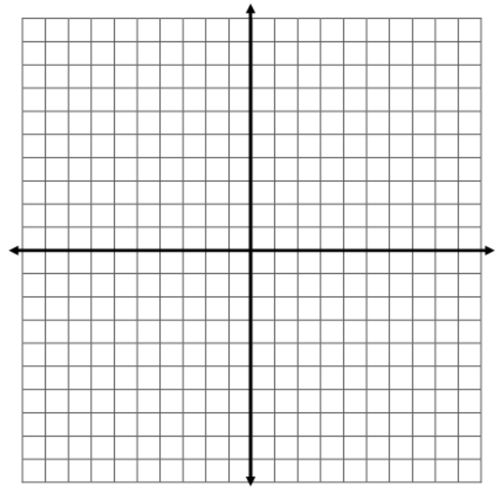
$y = -2x(x+5)^3$



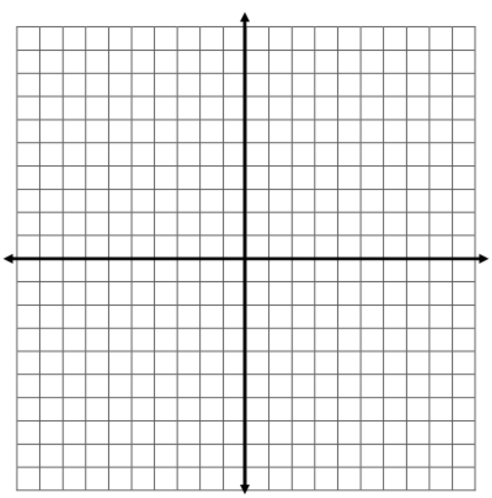
$y = (x + 1)^3(x - 2)$



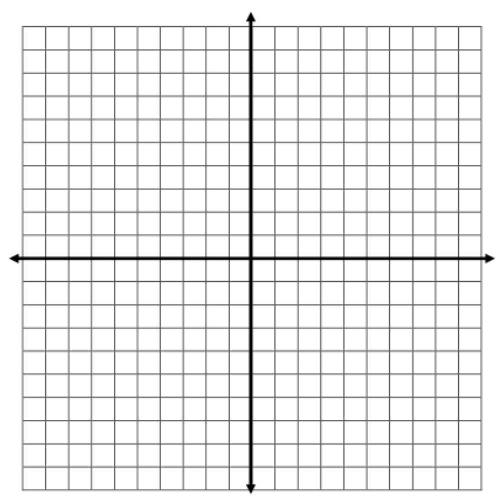
$y = x(x + 4)^3(x - 3)^2$



$f(x) = -x^4 + 19x^2 + 6x - 72$



$y = x^3 + 4x^2 - x - 4$

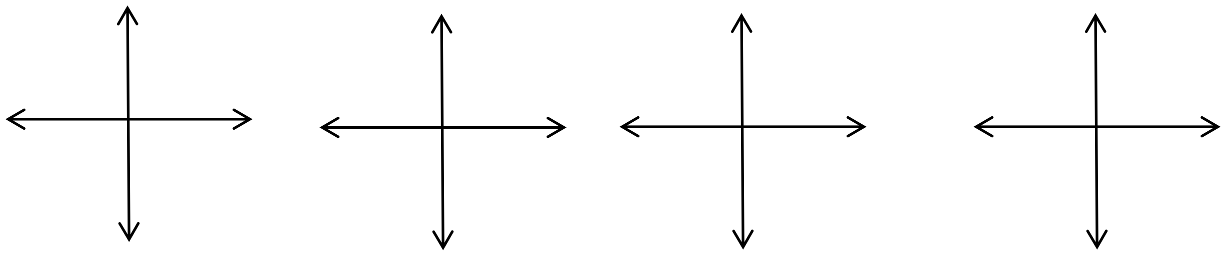


Chapter 4 - Outcome 30.1

Level 2

1. Draw each angle in standard position. In what quadrant does each angle lie?

a) $215°$ **b)** $-70°$ **c)** $\frac{\pi}{6}$ **d)** π



2. Change the degree measures to radians. Give answers as both exact and approximate measures to the nearest hundredth of a unit.

a) $150°$ **b)** $-240°$ **c)** $310°$

3. Change the radian measures to degrees. Round to two decimal places if necessary.

a) $\frac{4\pi}{5}$ **b)** $\frac{5\pi}{6}$ **c)** $-\frac{7\pi}{4}$

4. Determine the one positive and one negative angle that are coterminal with the given angle.

a) $450°$ **b)** $\frac{\pi}{5}$

Level 3

5. Write an expression for all the angles that are coterminal with each given angle.

a) $75°$ **b)** $\frac{\pi}{3}$ **c)** 1

Chapter 4 - Outcome 30.2 Review

1. Which point(s) lies on the unit circle? Explain how you know.

$\left(-\frac{5}{13}, \frac{12}{13}\right)$ $\left(\frac{5}{6}, \frac{1}{2}\right)$ $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

2. Each of the following points lies on the unit circle. Find the missing coordinate satisfying the given conditions.

a) $\left(-\frac{2}{3}, y\right)$ in quadrant III **b)** $\left(x, \frac{4}{5}\right)$ in quadrant II

3. The point (x, y) is located where the unit circle. Determine the coordinates of point for the given angle.

a) $\theta = 45°$ **b)** 2π **c)** $\theta = -60°$ **d)** $\frac{11\pi}{6}$

5. Identify a measure for θ in the interval $0 \le \theta < 2\pi$ for is the given point.

a) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **b)** $(-1, 0)$ **c)** $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **d)** $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

7. Determine the measure of all angles that satisfy the given conditions. **Use exact values when possible**

a) $\tan\theta = -1$, domain $0° \le \theta < 360°$

b) $\sin\theta = \frac{\sqrt{3}}{2}$, domain $0 \le \theta < 2\pi$

c) $\cos\theta = -\frac{1}{2}$, domain $0 \le \theta < 2\pi$

d) $\cot\theta = -4.87$, domain $0 \le \theta < 2\pi$

e) $\sec\theta = 4.87$, domain $0° \le \theta < 360°$

f) $\csc\theta = 2$, domain $0° \le \theta < 360°$

Level 3

8. Determine the value of the following. Use exact values when possible

a) $\csc 60°$

b) $\cos 240°$

c) $\tan 260^{o}$

d) $\cot 137^{o}$

e) $\sin\frac{7\pi}{6}$

f) $\sec 4.5$

9. Determine the **exact** value of each of the following:

a) $\tan\theta+\sqrt{3}=0$, $0 \le \theta \le 360°$.

b) $2\sin\theta + 1 = 2$, $0 \le \theta \le 360°$.

c) $2\cos^2 x - 5\cos x + 2 = 0$, $0 \le x \le 2\pi$.

d) $4\sin^2 x - 3 = 0$, $0 \le x \le 2\pi$.

e) $\sec\theta = -2$, $0 \le \theta \le 360°$.

10. The point $\left(\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$ is on the unit circle. Determine the exact value for each of the 6 trigonometric ratios.

Chapter 5 - Outcome 30.3

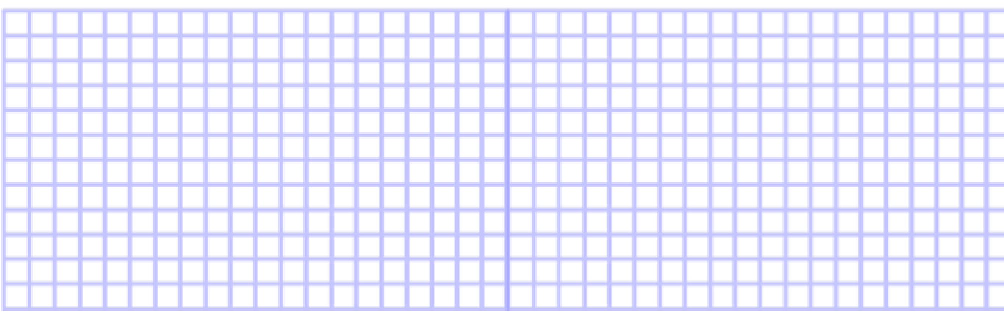
_Level 2

1. Sketch the following:

a) $y = sinx\ ,\quad 0 \leq x \leq 2\pi$

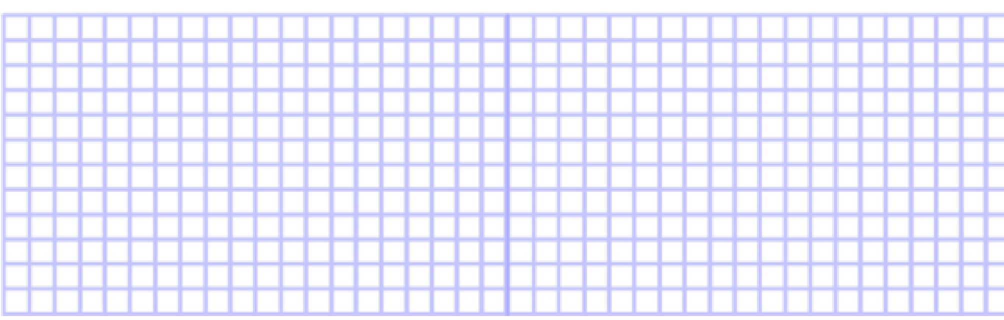
Amplitude: Period: x-intercepts: asymptotes:

b) $y = cosx\ ,\quad 0 \leq x \leq 2\pi$



Amplitude: Period: x-intercepts: asymptotes:

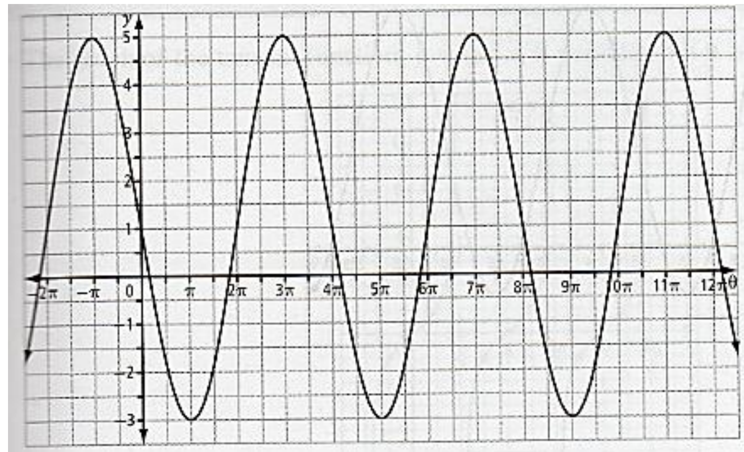
$c)\ y = tanx\ ,\quad 0 \leq x \leq 2\pi$



Amplitude: Period: x-intercepts: asymptotes:

Level 3

2. Determine the following for each graph



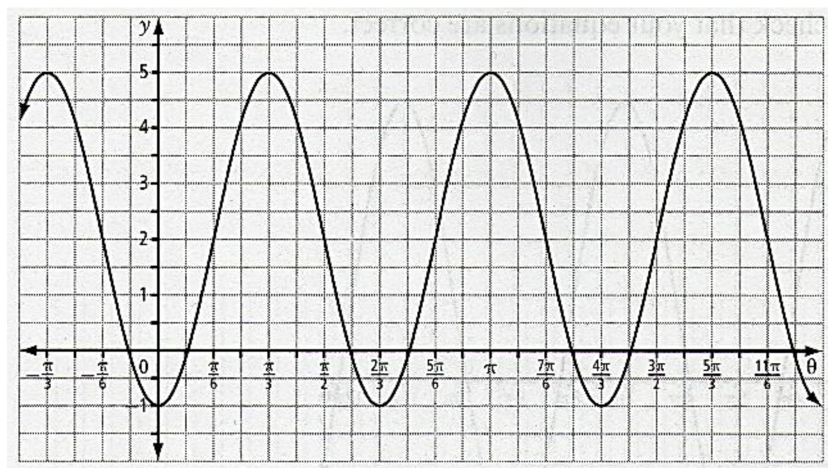
a) Amplitude:

Domain:

Range:

Period:

Write the equation of the graph in form *y = a cos b (x - c)+ d*



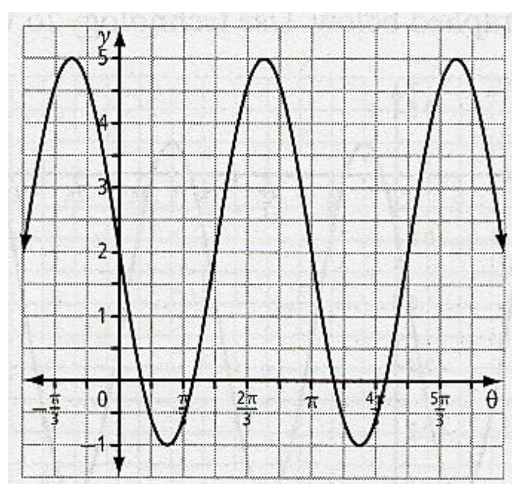
b) Amplitude:

Domain:

Range:

Period:

Write the equation of the graph in form *y = a sin b (x - c)+ d*



c) Amplitude:

Doman:

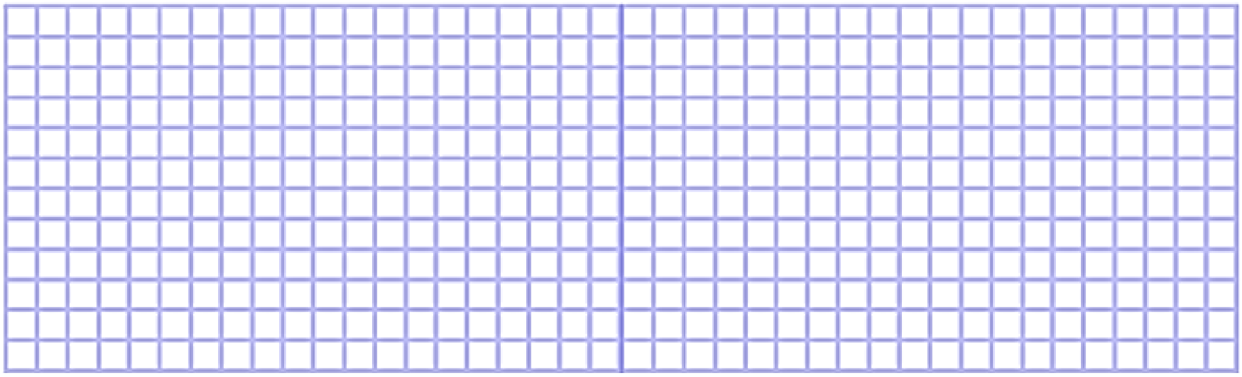
Range:

Period:

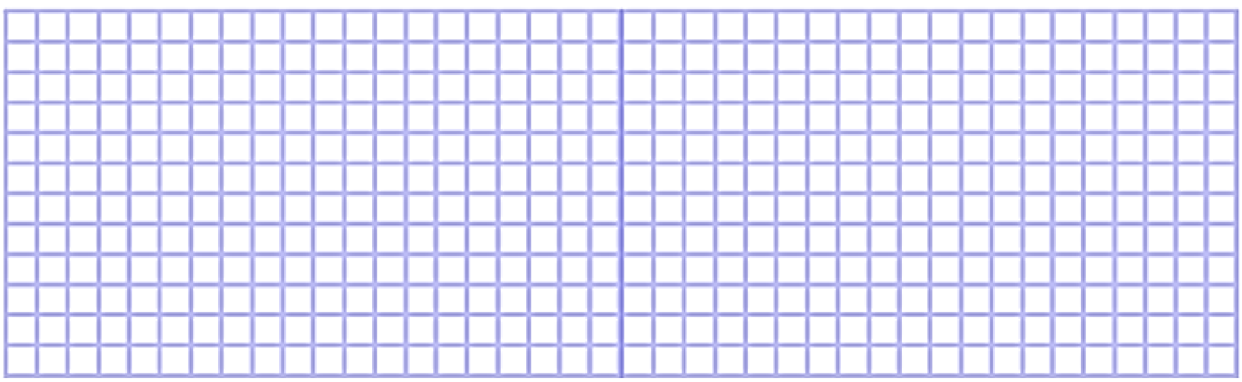
Write the equation of the graph in form *y = a cos b (x - c)+ d*

3. Graph each of the following for at least one cycle. For each state the domain, range, amplitude, and period

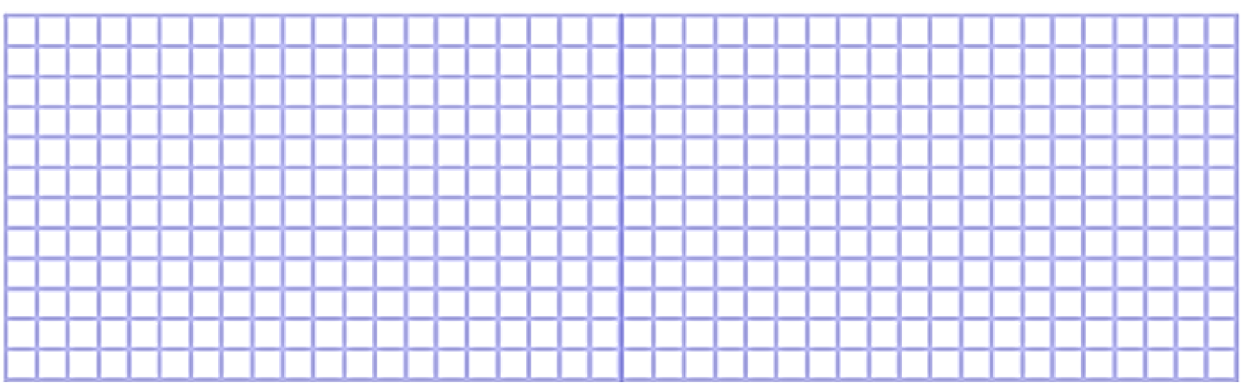
a) $y = 5sin\frac{1}{2}(x + \pi) - 2$



b) $y = -2sin2\left(\theta - \frac{\pi}{2}\right) + 4$



c) $y = 3cos4(x - \pi) - 1$



d) $y = -sin\frac{1}{3}\left(\theta + \frac{\pi}{4}\right) + 3$

e) $y = 3cos\left(2\theta - \frac{\pi}{3}\right) - 4$

Chapter 6 - Outcome 30.5

Level 2:

1) Verify that the equation $(secx + tanx)cosx - 1 = sinx$ is true for $x = 30°$

2) Prove the following identities:

a) $\frac{\cos x \csc x}{\sec x \cot x} = \cos x$

b) $\cot x \sin x = cosx$

c) $\csc x \tan x \sec x \cos x = secx$

3) Determine the <u>exact</u> value of each trigonometric expression

a) $\sin 105°$

b) $cos\frac{\pi}{9}cos\frac{2\pi}{9} - sin\frac{\pi}{9}sin\frac{2\pi}{9}$

Level 3:

4) Prove the following identities.

a) $sin\theta(cot\theta + 1) = sin\theta + cos\theta$

b) $\frac{\sin x+\sin^2 x}{\cos x+\sin x \cos x} = tanx$

$c)\ \sin 2x = \tan x + \tan x \cos 2x$

d) $\frac{tan\theta}{cos\theta + cos\theta tan^2\theta} = sinx$

$$e)\ \frac{1}{1 - cosx} - \frac{1}{1 + cosx} = 2 \cot x\ csc\ x$$

Level 4:

5) Prove:

$$\frac{1 - cos2\theta}{sin2\theta} = tan\theta$$

6) State the non-permissible values for questions 4a, 4d and 5.

Chapter 7 – Outcome30.9c

Level 2

1. Solve

a) $2^x = 64$

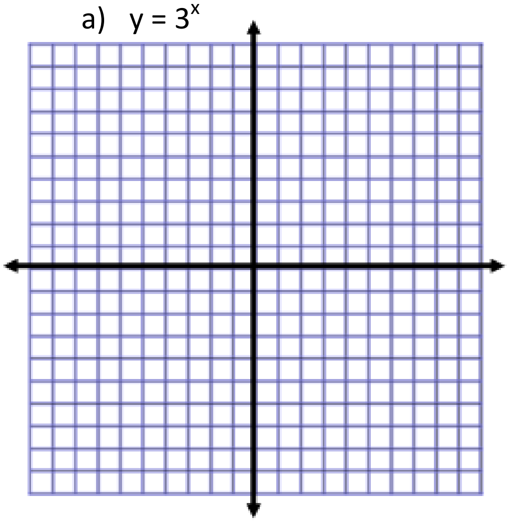
b) $3^x = 27^{x-2}$

c) $8^{2x} = 16^{x+3}$

d) $9^{2x-5} = 27^{x+6}$

2. Graph each of the following, and then determine the:

a) $y = 3^x$



Domain:

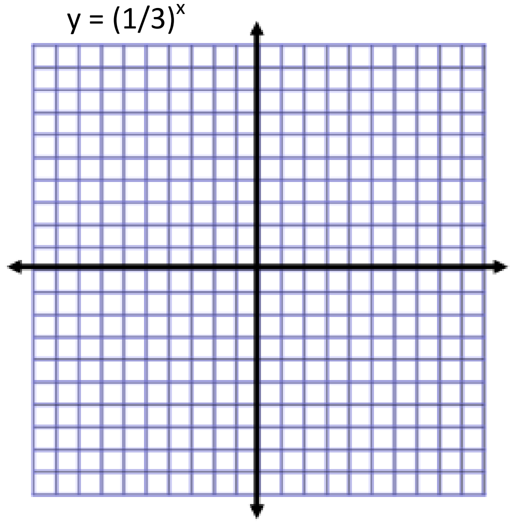
Range:

Horizontal asymptote

Y intercept:

Increasing or Decreasing:

$y = (1/3)^x$



Domain:

Range:

Horizontal asymptote

Y intercept:

Increasing or Decreasing:

2. . Identify all of the transformations of the following: (ie vertical translation up 2)

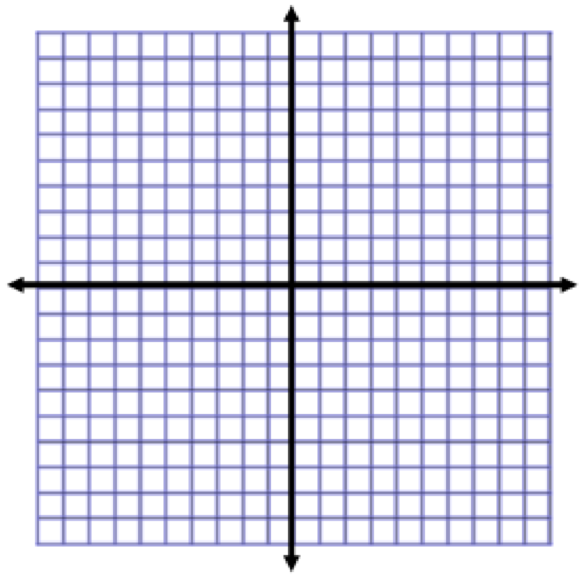
a) $f(x) = 3^{-x} + 5$

b) $h(x) = -2\left(\frac{1}{3}\right)^{x+1}$

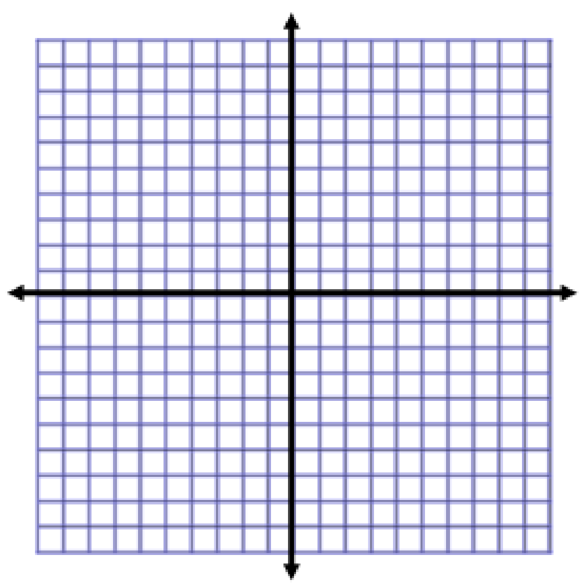
Level 3

4. Sketch the graph of

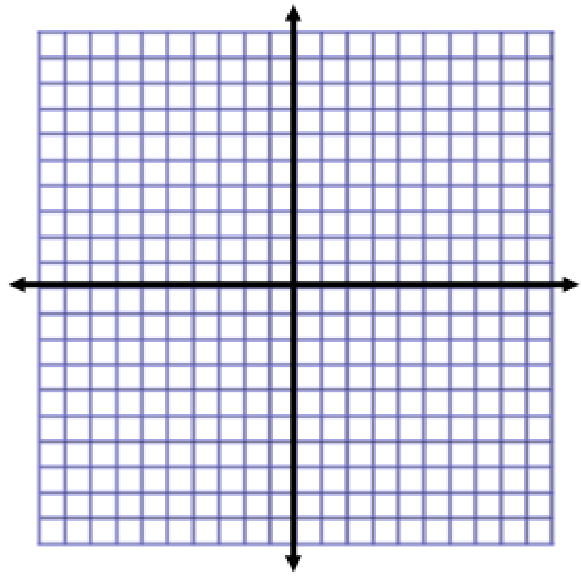
$y = -3^{x-2}$



$y = 3(2^{-x}) - 5$



$y = 2^{2x+4} - 1$



Chapter 8 – Part 1

Level 2

1. Express as a logarithmic statement.

$$2^3 = 8$$

2. Express as an exponential statement.

$$\log_3 81 = 4$$

3. Determine the value of each logarithm.

a) $\log_5 25$

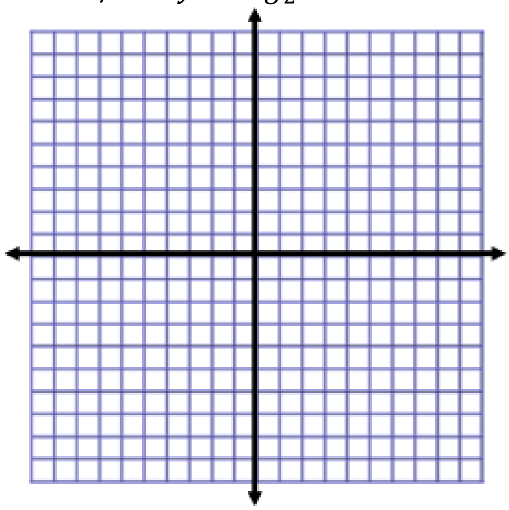
c) $\log_9 1$

b) $\log_2 \frac{1}{8}$

d) $\log_6 6$

4. Graph each of the following and determine

b) $y = log_2 x$



Domain:

Range:

Vertical asymptote

x intercept:

y intercept:

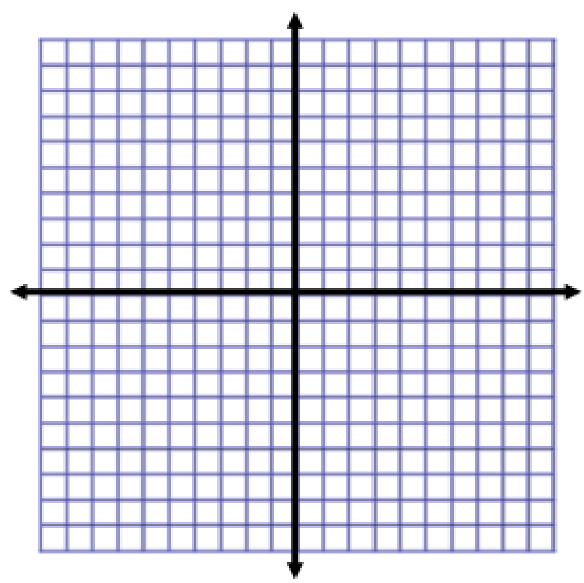
5. Identify all of the transformations of the following: (state all stretches/reflections/translations up, down left or right)

a) $y = -2log_3(x-5)+2$ $y = 2\log_3(-x)+1$

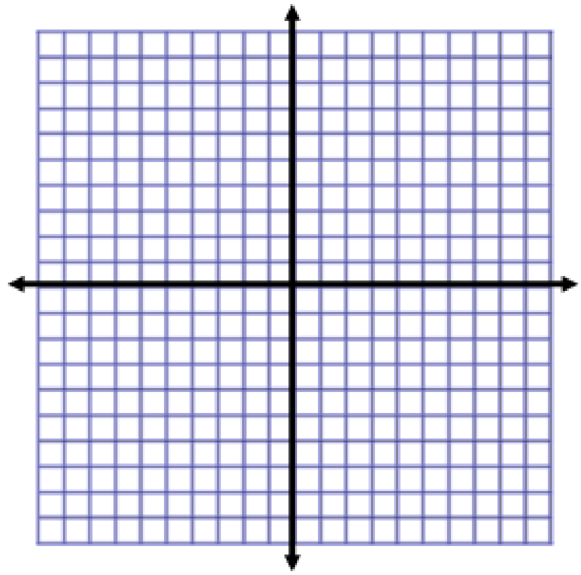
Level 3

6. Sketch

$y = -\log_2(x+1)-2$



$y = 2\log_3(x-2) + 1$



Chapter 8 Part 2

Level 2

1. Use your laws of logarithms to expand each of the following:
   a) $\log_4 \frac{x}{3}$ b) $\log_4 x^5$ c) $\log_2 yx^5$

2. Use the laws of logarithms to simplify each of the following:
   a) $\log 2 + \log 7$ b) $4\log_3 5$ c) $\log_2 42 - \log_2 6$

3. Determine the value of x.
   a) $\log_2 x = 3$ b) $3\log_5 x = \log_5 125$

   c) $6^x = 216$ d) $4^{x+1} = 64$

Level 3

4. Use the laws of logarithms to simplify and then evaluate each of the following:
   a) $\log_3 270 - (\log_3 2 + \log_3 5)$ b) $3\log_2 6 - 3\log_2 3$

5. Write each expression in terms of individual logarithms.
   a) $\log_2 \frac{x^5 \sqrt[3]{y}}{7z}$ b) $\log_5 \sqrt{xy^3}$

6. Write each expression as a single logarithm.
   a) $3 \log w \quad + \log \sqrt{w} - 2 \log w$ b) $\log_2(x+6) + \log_2(x-1)$

7. Solve for x.
   a) $\log_5 x + 6 = 8$ b) $log_4 x + 2log_4 x = 6$

c) $\log_2 x^2 - \log_2 5 = \log_2 20$

d) $log_3(x+7) - log_3(x-3) = 2$

e) $3^x = 100$

f) $7^{x-3} = 517$

Level 4

8. Solve the following. State any restrictions

$$log_6(x+3) - 2 = -log_6(x-2)$$

9. Use what you have learned about logarithms to show how you could use two different transformations to graph the logarithmic function $y = log_2 8x$

10. Simplify the following logarithm. State the restrictions

log $(x^2 - x - 12) - \log(x^2 - 9)$

Chapter 9 Review

Level 2

1. Determine the characteristics of the following functions:

a) $y = \frac{2x-1}{x-4}$

Equation of Vertical Asymptotes:

Points of Discontinuity (holes):

Equation of Horizontal Asymptote:

b) $y = \frac{x+5}{(x+5)(x-3)}$

Equation of Vertical Asymptotes:

Points of Discontinuity (holes):

Equation of Horizontal Asymptote:

c) $y = \frac{x^2-4}{x^2+3x+2}$

Equation of Vertical Asymptotes:

Points of Discontinuity (holes):

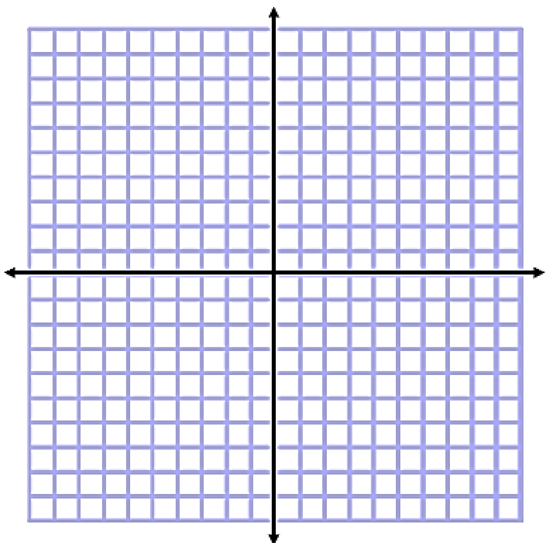
Equation of Horizontal Asymptote:

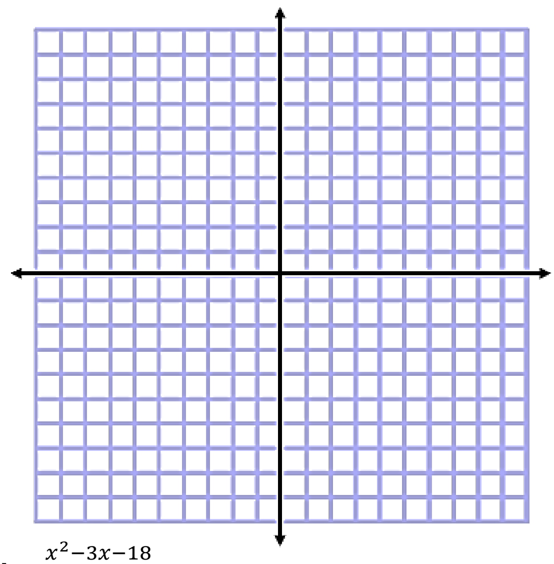
Level 3/Level 4 (Level 4 Questions will have an oblique asymptote. You will need to determine that on your own.)

2. Graph the following functions. Be sure to give the equations of all asymptotes.

a) $y = \frac{-2x+4}{x+5}$

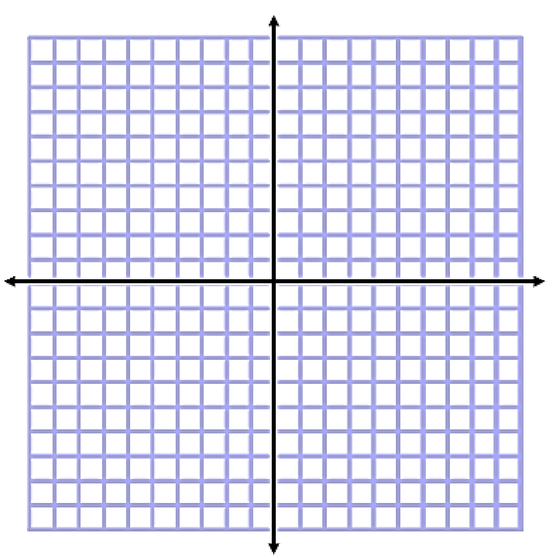
b) $y = \frac{x^2-16}{x+4}$

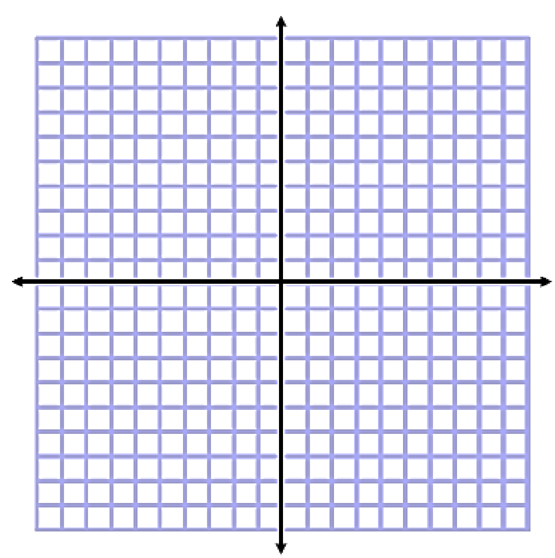




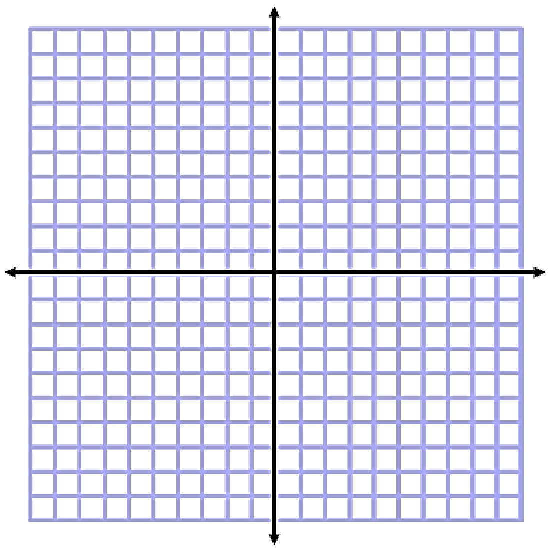
c) $y = \frac{x-5}{x^2-2x-15}$

d) $y = \frac{x^2-3x-18}{x^2+7x+12}$





$y = \frac{x^2-x-6}{x-1}$



$y = \frac{2x+6}{x^2-9}$