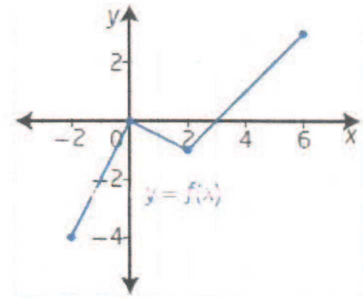
Final Exam Review (AK)

Outcome 1A Review

Level 2

1. Consider the graph of $y = f(x)$.

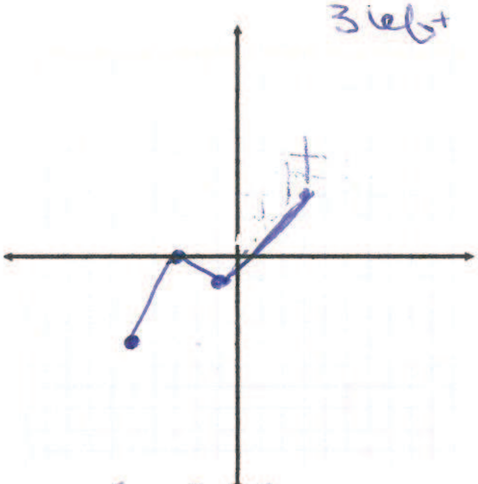


a) Sketch each of the transformed function:

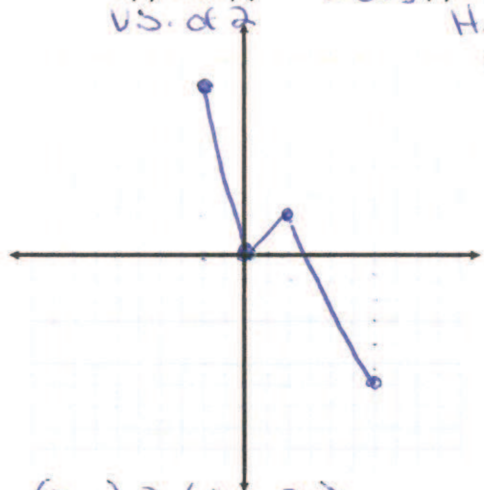
i) $y = f(x+3)$ — 3 left

ii) $y = -2f(x)$ — V. reflection about x-axis, V.S. of 2

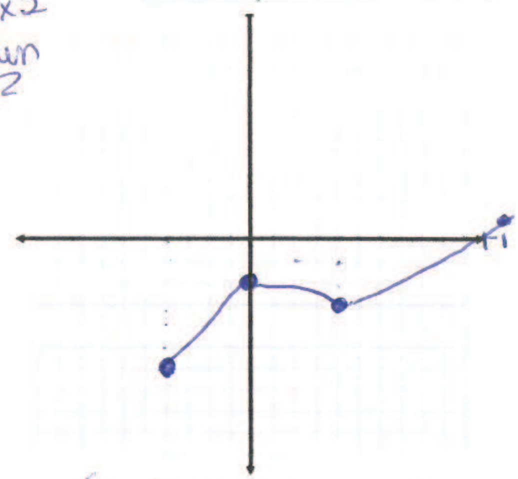
iii) $y = f(\frac{1}{2}x) - 2$ — H.S x2, Down 2



$(x, y) \rightarrow (x-3, y)$



$(x, y) \rightarrow (x, -2y)$



$(x, y) \rightarrow (2x, y-2)$

2. For each equation, describe how the graph was translated, reflected or stretched.

a) $y = -2f(3(x-4))$

V. stretch factor of 2
V. reflection about x-axis
H. stretch factor of $\frac{1}{3}$
H. translation 4 right

b) $y = f(x-5)-3$

H. translation 5 right
V. translation 3 down

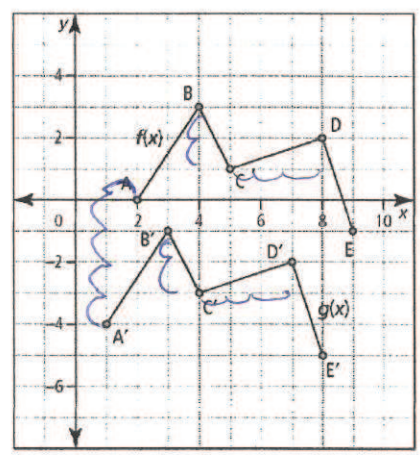
c_ $y = f(-2x)+5$

H. stretch factor $\frac{1}{2}$
H. reflection about y-axis
V. translation 5 ~~down~~ up

3. Consider the graph of $y = f(x)$ and $y = g(x)$.

Determine the equation of the translated function in the form

$y = af(b(x-h)) + k$.



$\boxed{y = f(x+1) - 4}$

4 down $\Rightarrow k = -4$
1 left $\Rightarrow h = -1$
no V. or H stretch.

5. Determine algebraically the equation of the inverse of each function.

a) $f(x)=3x-6$ **b)** $f(x)=x^2-7$ **c)** $y=(x-5)^2-9$

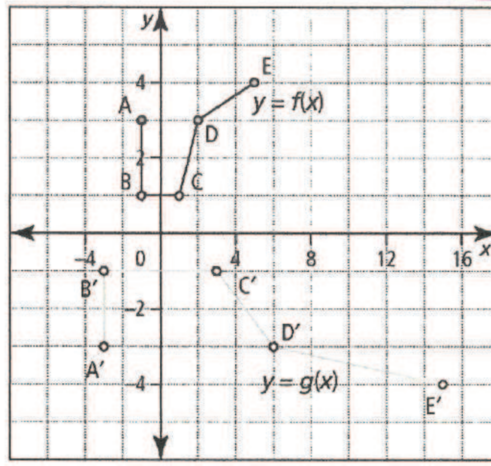
a)
$x = 3y-6$
$x+6 = 3y$
$y = \frac{x+6}{3}$
$y = \frac{x}{3}+2$

b)
$x = y^2-7$
$x+7 = y^2$
$y = \pm\sqrt{x+7}$

c)
$x = (y-5)^2-9$
$\pm\sqrt{x+9} = \sqrt{(y-5)^2}$
$y-5 = \pm\sqrt{x+9}$
$\boxed{y = \pm\sqrt{x+9}+5}$

Level 3

6. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine an equation for $g(x)$.



V. reflection
$a \Rightarrow -ve$
H. stretch $\times 3$
$b = \frac{1}{3}$

$y = -f(\frac{1}{3}x)$

$(x, y) \to (3x, -y)$

7. Write the equation for each transformation of $y = x^2$ in the form

$y = af(b(x - h)) + k$.

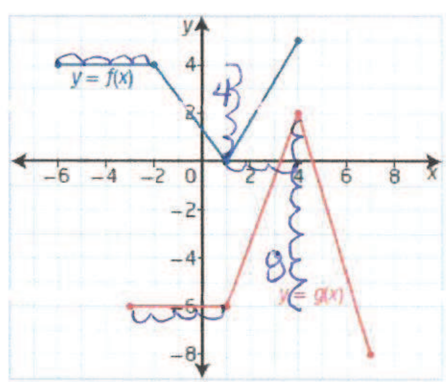
a) a vertical stretch by a factor of 3, reflected in the y-axis, and translated 3 units left and 2 units down

$y = 3f(-(x+3))-2$

b) a horizontal stretch by a factor of 2, reflected in the x-axis, and translated 7 units up

$y = -f(\frac{1}{2}x)+7$

9. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.



$\boxed{y = -2f(x-3)+2}$

V. reflection
$a \Rightarrow -ve$
no H. stretch
V. stretch $\times 2$

3 right, 2 up.

10. The key point $(-18, 12)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

a) $-3f(x + 5) + 4$

$(x, y) \to (x - 5, -3y + 4)$

$(-18, 12) \to (-18 - 5, -3(12) + 4)$

$\to \boxed{(-23, -32)}$

b) $y = 2f(6x)$

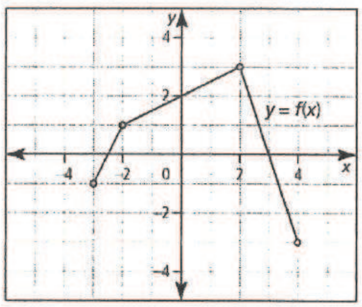
$(x, y) \to (\frac{1}{6}x, 2y)$

$(-18, 12) \to (\frac{1}{6}(-18), 2(12))$

$\to \boxed{(-3, 24)}$

11. Consider the graph of the function $y = f(x)$.

Sketch $y = f(x)$ to $y = 3f(-2(x - 1)) + 4$.



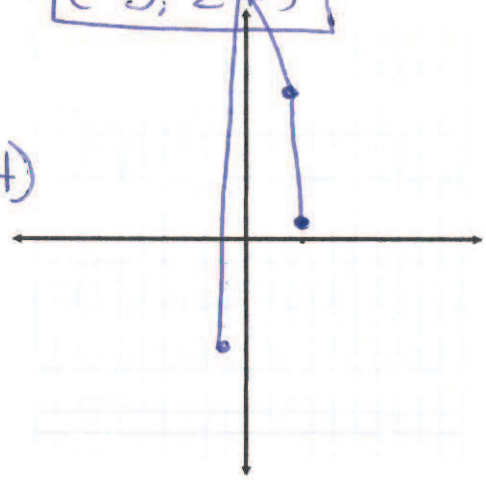
$(x, y) \to (-\frac{1}{2}x + 1, 3y + 4)$

$(-3, -1) \to (\frac{5}{2}, 1)$

$(-2, 1) \to (2, 7)$

$(2, 3) \to (0, 13)$

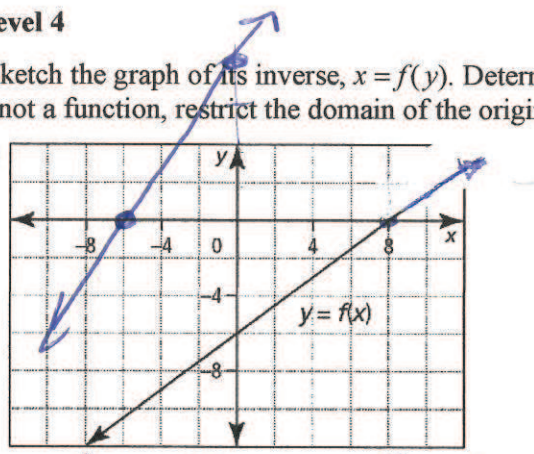
$(4, -3) \to (-1, -5)$



Level 4

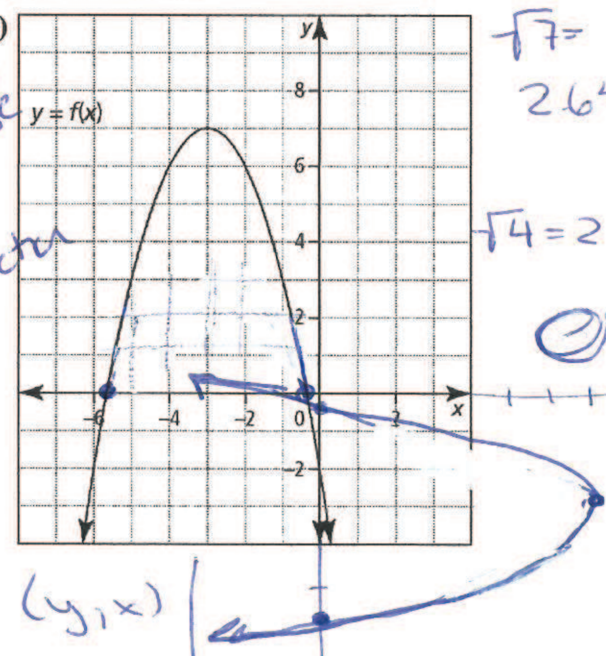
12. Sketch the graph of its inverse, $x = f(y)$. Determine whether the inverse is a function. If the inverse is not a function, restrict the domain of the original graph to make it a function.

a)



Inverse is a function

b)



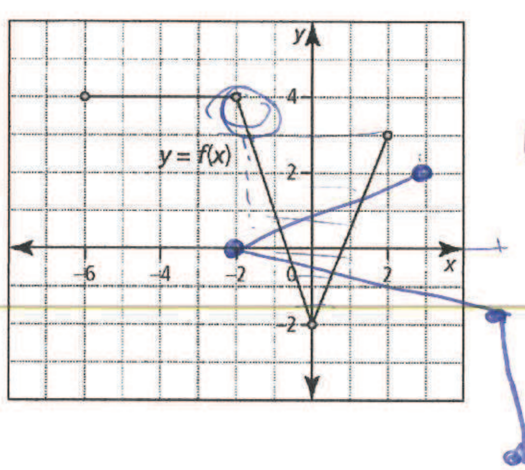
Inverse not a function

$\sqrt{7} = 2.64\ldots$

$\sqrt{4} = 2$

Restrict domain to $(-\infty, -3]$ or $[-3, \infty)$

c)



$(x, y) \to (y, x)$

Inverse Not a function

Restrict domain to $[-2, 0]$ or $[0, 2]$

Outcome 2A

1. Identify a, b, h and k for each of the following

a) $y = 5\sqrt{x+7} - 2$

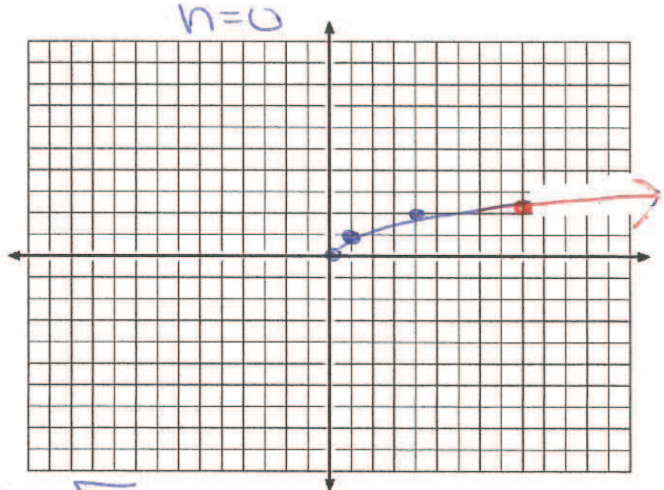
$a = 5$
$b = 1$
$h = -7$
$k = -2$

b) $y = -4\sqrt{-x} + 8$

$a = -4$ $k = 8$
$b = -1$
$h = 0$

2. Graph $y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3



Level 3

3. Write the equation of a radical function that would result by applying each set of transformations to the graph of $f(x) = \sqrt{x}$

a) vertical stretch by a factor of 3, and horizontal stretch by a factor of 2

$y = 3\sqrt{\tfrac{1}{2}x}$ or $y = 3f(\tfrac{1}{2}x)$

b) horizontal reflection in the y-axis, translation up 3 units, and translation left 2 units

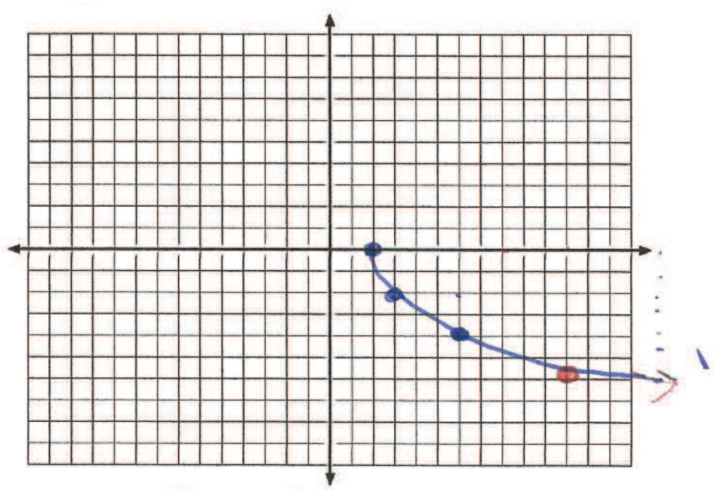
$y = \sqrt{-(x+2)} + 3$ or $y = f(-(x+2)) + 3$

4. Graph the functions below.

Then, identify the domain and range.

a) $y = -2\sqrt{x-2}$

$(x, y) \rightarrow (x+2, -2y)$
$(0,0) \rightarrow (2, 0)$
$(1,1) \rightarrow (3, -2)$
$(4,2) \rightarrow (6, -4)$
$(9,3) \rightarrow (11, -6)$



$D = [2, \infty)$
$R = (-\infty, 0]$

c) $y = \sqrt{2x} - 4$

$(x, y) \to (\frac{1}{2}x, y-4)$

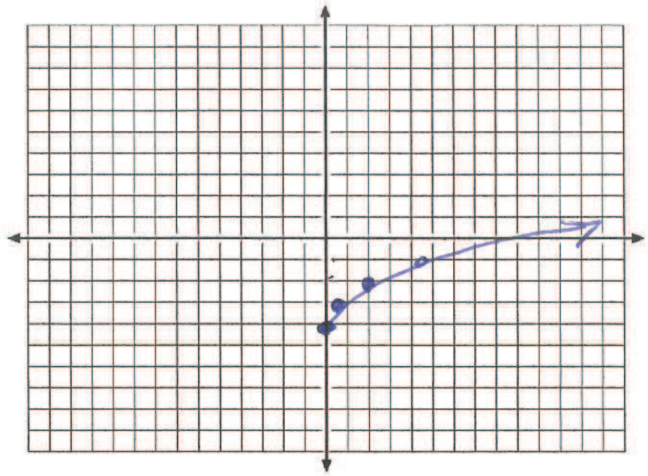
$(0,0) \to (0, -4)$

$(1,1) \to (\frac{1}{2}, -3)$

$(4,2) \to (2, -2)$

$(9,3) \to (4.5, -1)$

$D = [0, \infty)$ $R = [-4, \infty)$



c) $y = 2\sqrt{-(x-3)} + 1$

$(x, y) \to (-x+3, 2y+1)$

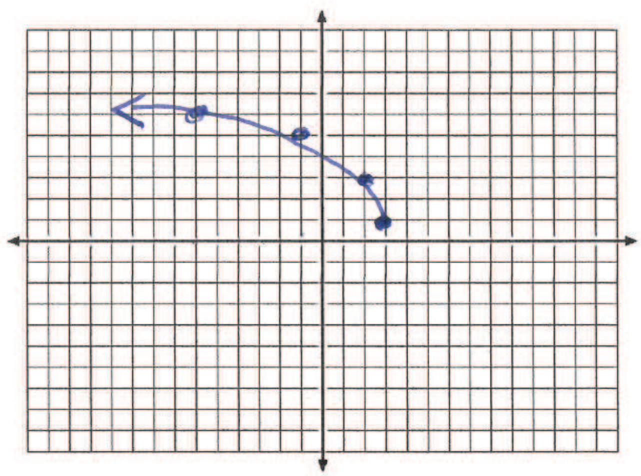
$(0,0) \to (3, 1)$

$(1,1) \to (2, 3)$

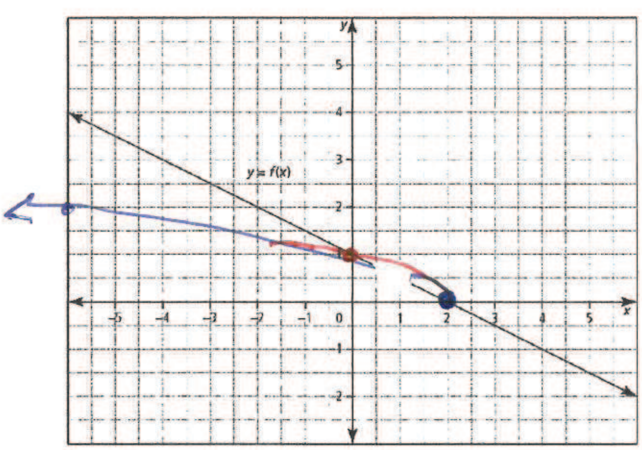
$(4,2) \to (-1, 5)$

$(9,3) \to (-6, 7)$

$D = (-\infty, 3]$ $R = [1, \infty)$

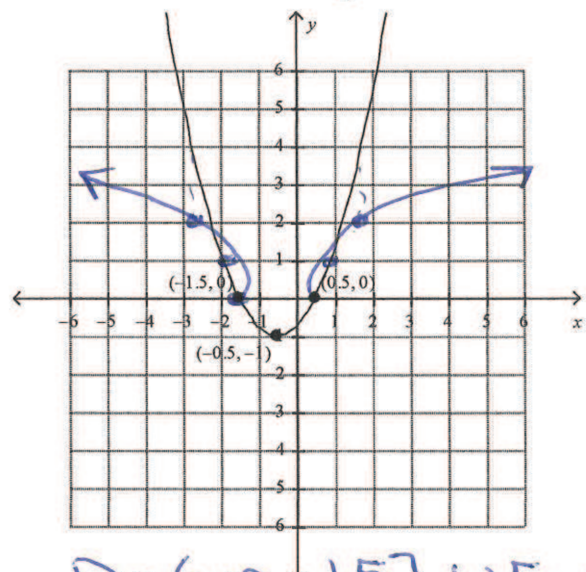


5. Graph $\sqrt{f(x)}$ from the following graphs of *f(x)* and state the **domain and range**



$D = (-\infty, 2]$

$R = [0, \infty)$

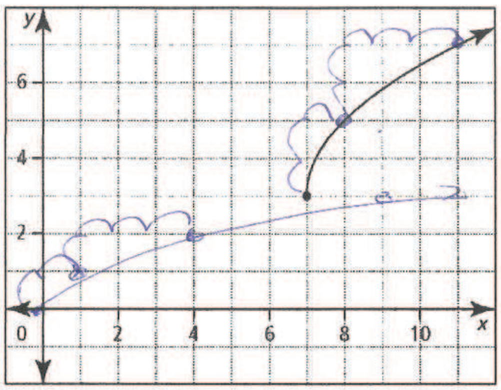


$D = (-\infty, -1.5] \cup [0.5, \infty)$

$R = [0, \infty)$

6. For each function, write an equation of a radical function of the form $y = a\sqrt{b(x-h)} + k$.

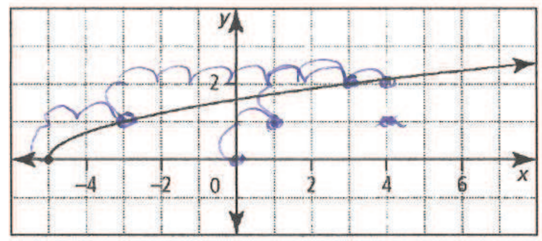
a)



$y = 2f(x-7) + 3$

$y = 2\sqrt{x-7} + 3$

b)



H. stretch × 2

$y = f(\frac{1}{2}(x+5))$

$y = \sqrt{\frac{1}{2}(x+5)}$

Outcome 3A Review

1. Divide the following using long division or synthetic division.

a) $(2w^4 + 3w^3 - 5w^2 + 2w - 27) \div (w+3)$

$$\begin{array}{r|rrrrr} 3 & 2 & 3 & -5 & 2 & -27 \\ & \downarrow & 6 & -9 & 12 & -30 \\ \hline & 2 & -3 & 4 & -10 & 3 \end{array}$$

$\boxed{2w^3 - 3w^2 + 4w - 10 + \left(\frac{3}{w+3}\right)}$

b) $\dfrac{2x^3 - 10x^2 - 15x - 20}{x+5}$

$$\begin{array}{r|rrrr} 5 & 2 & -10 & -15 & -20 \\ & \downarrow & 10 & -100 & 425 \\ \hline & 2 & -20 & 85 & -445 \end{array}$$

$2x^2 - 20x + 85 + \left(\frac{-445}{x+5}\right)$

2. Determine the remainder when $x^3 + x^2 - 16x - 16$ is divided by

a. x + 2

$P(-2) = 12$

b) x – 4

$P(4) = 0$

b) Are any of the binomials above a factor of $x^3 + x^2 - 16x - 16$?

$x - 4$

3. Factor completely

a. $x^3 + 2x^2 - 13x + 10$

$P(1) = 1+2-13+10$
$= 0$
$x-1$ a factor

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -13 & 10 \\ & \downarrow & -1 & -3 & 10 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$(x-1)(x^2+3x-10)$

$\boxed{(x-1)(x+5)(x-2)}$

b. $x^4 - 26x^2 + 25$

$P(1) = 0$ $x-1$ a factor

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -26 & 0 & 25 \\ & \downarrow & -1 & -1 & 25 & 25 \\ \hline & 1 & 1 & -25 & -25 & 0 \end{array}$$

$(x-1)\underbrace{(x^3+x^2-25x-25)}_{\text{another division}}$

$(x-1)(x+1)(x+5)(x-5)$

4. Determine the value(s) of k so that the binomial is a factor of the polynomial: $x^2 - 8x - 20$, $x + k$

$P(-k)=0$

$(-k)^2 - 8(-k) - 20 = 0$

$k^2 + 8k - 20 = 0$

$(k+10)(k-2) = 0$

$\boxed{k=-10 \quad k=2}$

5. The following polynomial has a factor of $x - 3$. What is the value of k? $kx^3 - 10x^2 + 2x + 3$

$P(3) = 0$

$k(3)^3 - 10(3)^2 + 2(3) + 3 = 0$

$27k - 90 + 6 + 3 = 0$

$27k - 81 = 0$

$\frac{27k = 81}{27}$

$k = 3$

Outcome 3B Review

1. Determine which of the following are polynomials. For each polynomial function, state the degree.

a) $h(x) = 5 - \frac{1}{x}$

No

b) $y = 4x^2 - 3x + 8$

Yes
$d=2$

c) $g(x) = -9x^6$

yes
$d=6$

d) $f(x) = \sqrt[3]{x}$

No

2. What is the leading coefficient, degree and constant term of each polynomial function?

a) $f(x) = -x^3 + 6x - 8$

LC= -1
deg=3
con= -8

b) $y = 5 + 2x^2$

LC=2
Deg=2
con=5

c) $g(x) = 7x^3 + 3x^5 - 8x + 10$

LC= 3
deg= 5
con=10

d) $k(x) = 9x - 2x^2$

LC= -2
deg= 2
con= 0

3. Identify the following characteristics for each polynomial function:

- the type of function and whether it is of even or odd degree
- the end behaviour of the graph of the function
- the number of possible x-intercepts
- the y-intercept

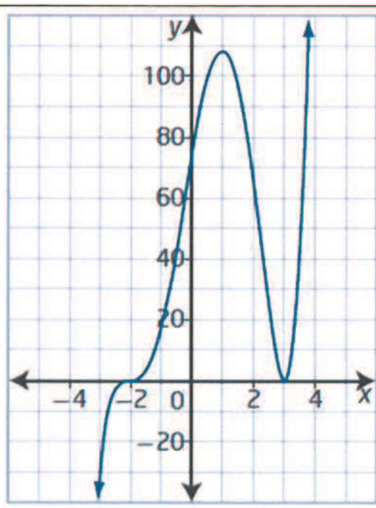
a) $g(x) = -2x^4 + 6x^2 - 7x - 5$

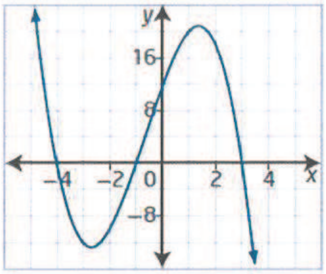
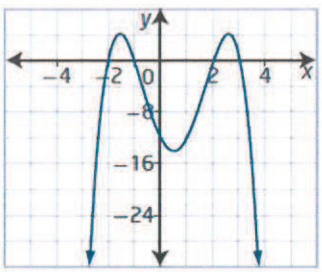
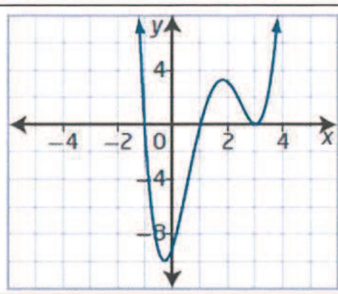
Even 0-4 x int
III → IV y int= -5

b) $f(x) = 2x^5 + 1x^3 - 12$

Odd 1-5 x int
III → I y int= -12

4. Fill in the table below for the following graphs

Graph	Odd or Even	Sign of Leading Coefficient	Number of x-intercepts	Intervals where the function is positive	Intervals where the function is negative
	ODD	+	2	(-2,3) ∪ (3, ∞)	(-∞,-2)

	Odd or Even	Sign of Leading Coefficient	Number of x-intercepts	Intervals where the function is positive	Intervals where the function is negative
	O	−	3	$(-\infty,-4) \cup (-1,3)$	$(-4,-1) \cup (3,\infty)$
	E	−	4	$(-2,-1) \cup (2,3)$	$(-\infty,-2) \cup (-1,2) \cup (3,\infty)$
	E	+	3	$(-\infty,-1) \cup (1,3) \cup (3,\infty)$	$(-1,1)$

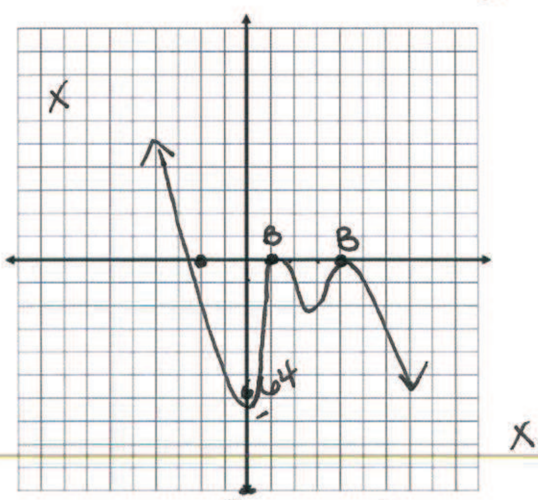
4. Graph the following polynomial functions. The first three have already been factored for you.

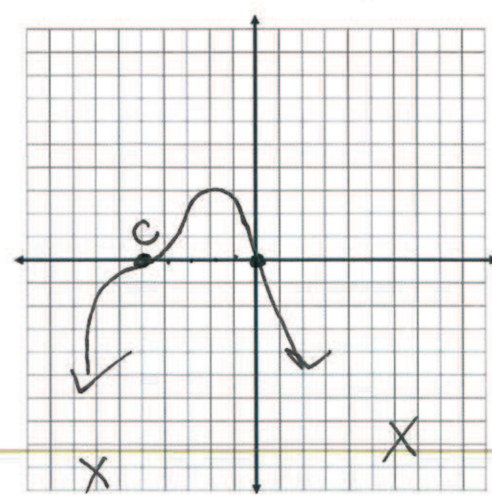
B B

$y = -2(x-1)^2(x+2)(x-4)^2$ $d=5$ $-LC$

C

y= -2x(x+5)^3 $d=4$ $-LC$



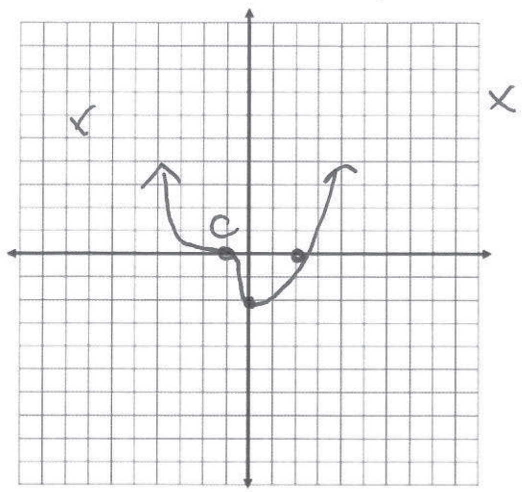


$f(0) = -2(-1)^2(2)(-4)^2$

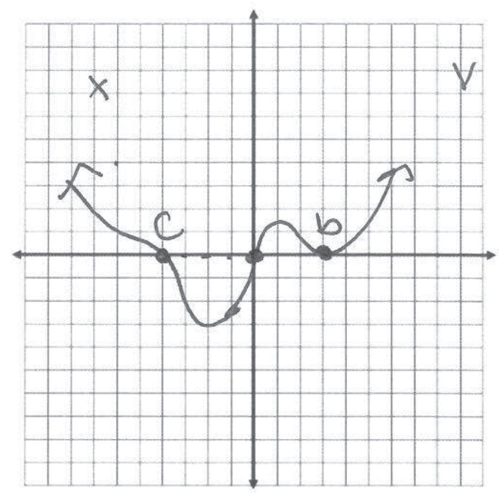
$f(0) = (1)^3(-2) = -2$

c

$y = (x+1)^3(x-2)$ $d = 4$ $+LC$



$y = x(x+4)^3(x-3)^2$ $d = 6$ $+LC$



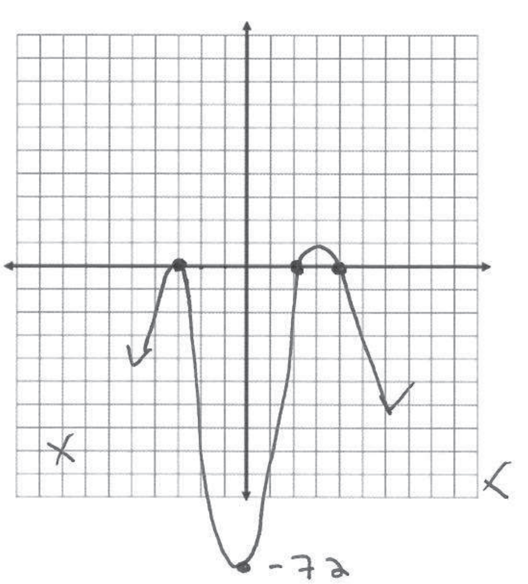
$f(x) = -x^4 + 19x^2 + 6x - 72$

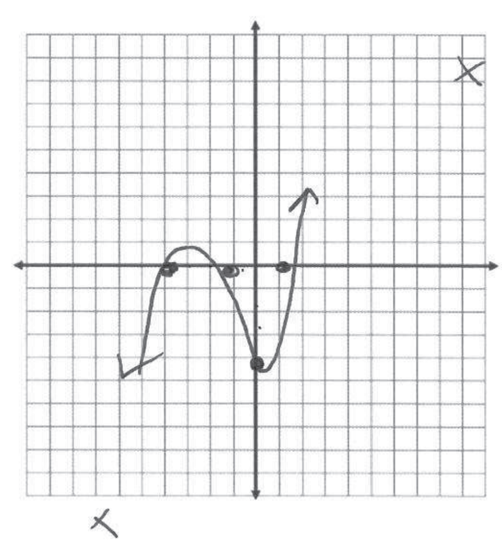
$-(x-2)(x+3)(x-4)(x+3)$

$-(x-2)(x+3)^2(x-4)$

$y = x^3 + 4x^2 - x - 4$

$(x-1)(x+4)(x+1)$





Final Exam Part 2 Review

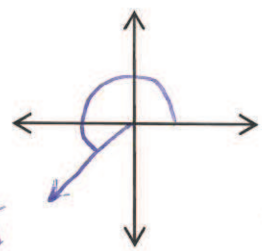
AK

Outcome 4A

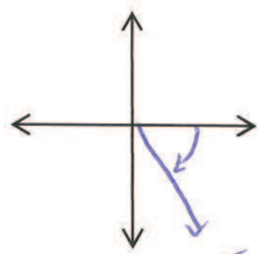
Level 2

1. Draw each angle in standard position. In what quadrant does each angle lie?

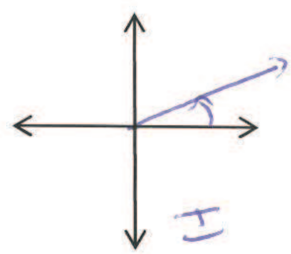
a) 215° **b)** −70° **c)** $\frac{\pi}{6}$ **d)** π



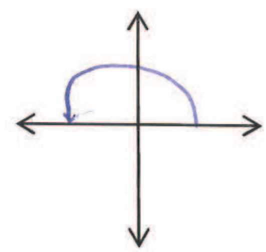
III



IV



I



$-x$ axis

2. Change the degree measures to radians. Give answers as both exact and approximate measures to the nearest hundredth of a unit.

≈ approx equal

a) 150° **b)** -240° **c)** 310°

$\frac{x}{150} = \frac{\pi}{180}$ $x = \frac{5\pi}{6}$

$x = \frac{150\pi}{180}$ $x \approx 2.62$

$x = \frac{-240\pi}{180}$ $x = -\frac{4\pi}{3}$ $x \approx -4.19$

$\frac{310\pi}{180}$ $\frac{31\pi}{18}$ ≈ 5.41

3. Change the radian measures to degrees. Round to two decimal places if necessary.

a) $\frac{4\pi}{5}$ **b)** $\frac{5\pi}{6}$ **c)** 3.8

144° 150°

$\frac{x}{3.8} = \frac{180}{\pi}$ $x = 217.72^\circ$

4. Determine the one positive and one negative angle that are coterminal with the given angle.

a) 450° -360° **b)** $\frac{\pi}{5} \pm 2\pi$

810°, 90°, -270°

$\pm \frac{10\pi}{5}$ $\frac{11\pi}{5}$, $-\frac{9\pi}{5}$

Level 3

5. Write an expression for all the angles that are coterminal with each given angle.

a) 75° **b)** $\frac{\pi}{3}$ **c)** 1

$75^\circ \pm 360^\circ n$, $n \in N$ $\frac{\pi}{3} \pm 2\pi n$, $n \in N$ $1 \pm 2\pi n$, $n \in N$

Outcome 4B Review

Unit Circle $x^2+y^2=1^2$ or $x^2+y^2=1$

1. Which point(s) lies on the unit circle? Explain how you know.

$\left(-\frac{5}{13}, \frac{12}{13}\right)$ $\left(\frac{5}{6}, \frac{1}{2}\right)$ $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

$\left(-\frac{5}{13}\right)^2+\left(\frac{12}{13}\right)^2=1$ $\left(\frac{5}{6}\right)^2+\left(\frac{1}{2}\right)^2\neq 1$ $\left(-\frac{2}{3}\right)^2+\left(-\frac{\sqrt{5}}{3}\right)^2=1$

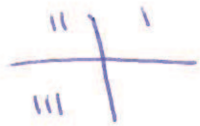
$\frac{25}{169}+\frac{144}{169}=1$ ✓ Yes

No

$\frac{4}{9}+\frac{5}{9}=1$ ✓ Yes

2. Each of the following points lies on the unit circle. Find the missing coordinate satisfying the given conditions.

a) $\left(-\frac{2}{3}, y\right)$ in quadrant III



$\left(-\frac{2}{3}\right)^2+y^2=1$

$\frac{4}{9}+y^2=\frac{9}{9}$

$y^2=\frac{5}{9}$

$y=\pm\frac{\sqrt{5}}{3}$

$y=-\frac{\sqrt{5}}{3}$

b) $\left(x, \frac{4}{5}\right)$ in quadrant II

$x^2+\left(\frac{4}{5}\right)^2=1$

$x^2+\frac{16}{25}=\frac{25}{25}$

$\sqrt{x^2}=\pm\sqrt{\frac{9}{25}}$

$x=\pm\frac{3}{5}$

$x=\frac{-3}{5}$

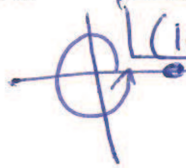
3. The point (x, y) is located where the unit circle. Determine the coordinates of point for the given angle.

$\cos\theta, \sin\theta$

a) $\theta = 45°$

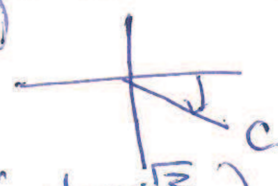
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

b) 2π



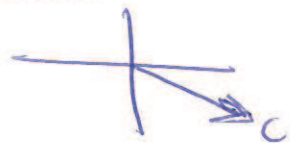
$\boxed{(1, 0)}$

c) $\theta = -60°$



$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

d) $\frac{11\pi}{6}$



$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

5. Identify a measure for θ in the interval $0 \leq \theta < 2\pi$ for is the given point.

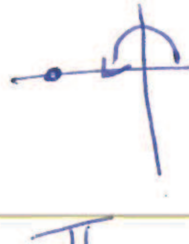
a) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\theta_R = 45°$

$\frac{\pi}{4}$

$\frac{5\pi}{4}$

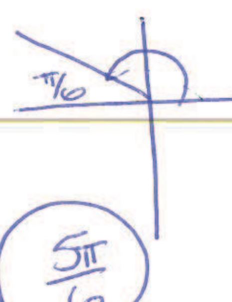
b) $(-1, 0)$



π

c) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

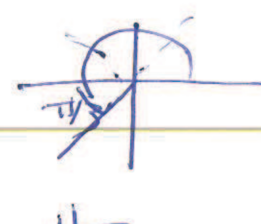
$\theta_R = 30°, \frac{\pi}{6}$



$\frac{5\pi}{6}$

d) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\theta_R = 60°, \pi/3$



$\frac{4\pi}{3}$

7. Determine the measure of all angles that satisfy the given conditions. **Use exact values when possible**

a) $\tan\theta = -1$, domain $0° \le \theta < 360°$

$\theta_R = \tan^{-1}(1) = 45°$

$\{135°, 315°\}$

b) $\sin\theta = \frac{\sqrt{3}}{2}$, domain $0 \le \theta < 2\pi$ Radian.

$\theta_R = 60° = \pi/3$

$\{\frac{\pi}{3}, \frac{2\pi}{3}\}$

c) $\cos\theta = -\frac{1}{2}$, domain $0 \le \theta < 2\pi$ Radian

$\theta_R = 60° = \pi/3$

$\{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

d) $\cot\theta = -4.87$, domain $0 \le \theta < 2\pi$ Radian

$\tan^{-1}(1 \div 4.87)$

$\theta_R = 0.20$

3.14 − 0.2

6.28 − 0.2

$\{2.94, 6.08\}$

e) $\sec\theta = 4.87$, domain $0° \le \theta < 360°$

$\cos^{-1}(1 \div 4.87)$

$\theta_R = 78.2°$

$\{78.2°, 281.8°\}$

f) $\csc\theta = 2$, domain $0° \le \theta < 360°$

$\sin^{-1}(\frac{1}{2})$

$\theta_R = 30°$

$\{30°, 150°\}$

Level 3

8. Determine the value of the following. Use exact values when possible

a) csc 60°

$\sin 60° = \frac{\sqrt{3}}{2}$

$\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

b) cos 240°

$\theta_R = 60$

$-\frac{1}{2}$

c) tan 260°

$\theta_R = 80$

5.67

d) cot 137°

$(\tan 137)^{-1}$

-1.07

e) $\sin\frac{7\pi}{6}$

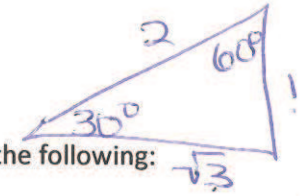
$30°$ $\pi/6$

$-\frac{1}{2}$

f) sec 4.5

$(\cos 4.5)^{-1}$

-4.74

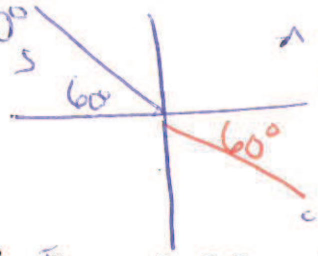


9. Determine the **exact** value of each of the following:

a) $\tan\theta + \sqrt{3} = 0$, $0 \le \theta \le 360°$.

$\tan\theta = -\sqrt{3}$

$\theta_R = 60°$



$\{120°, 300°\}$

b) $2\sin\theta + 1 = 2$, $0 \le \theta \le 360°$.

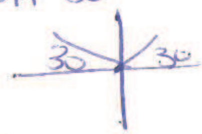
$-1 \quad -1$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\{30°, 150°\}$

$\theta_R = 30°$

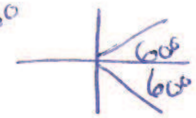


c) $2\cos^2 x - 5\cos x + 2 = 0$, $0 \le x \le 2\pi$.

$(2\cos x - 1)(\cos x - 2) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cancel{\cos x = 2}$

$x_R = 60°$



$\{60°, 300°\}$

$\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

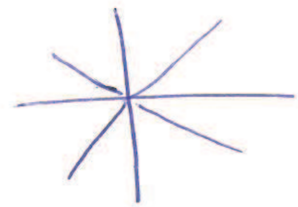
d) $4\sin^2 x - 3 = 0$, $0 \le x \le 2\pi$.

$4\sin^2 x = 3$

$\sqrt{\sin^2 x} = \pm\sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x_R = 60°, \pi/3$

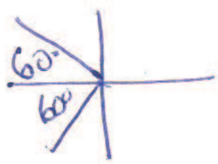


$\left\{\pi/3, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

e) $\sec\theta = -2$, $0 \le \theta \le 360°$.

$\cos\theta = -\frac{1}{2}$

$\theta_R = 60°$



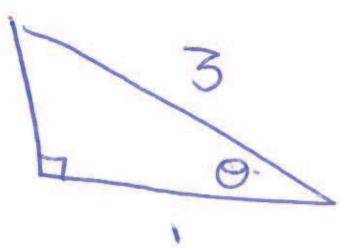
$\{120°, 240°\}$

10. The point $\left(\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$ is on the unit circle. Determine the exact value for each of the 6 trigonometric ratios.

$\cos\theta = \frac{1}{3}$

$\sin\theta = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$



$\sec\theta = \frac{3}{1} = 3$

$\csc\theta = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{3\sqrt{2}}{4}$

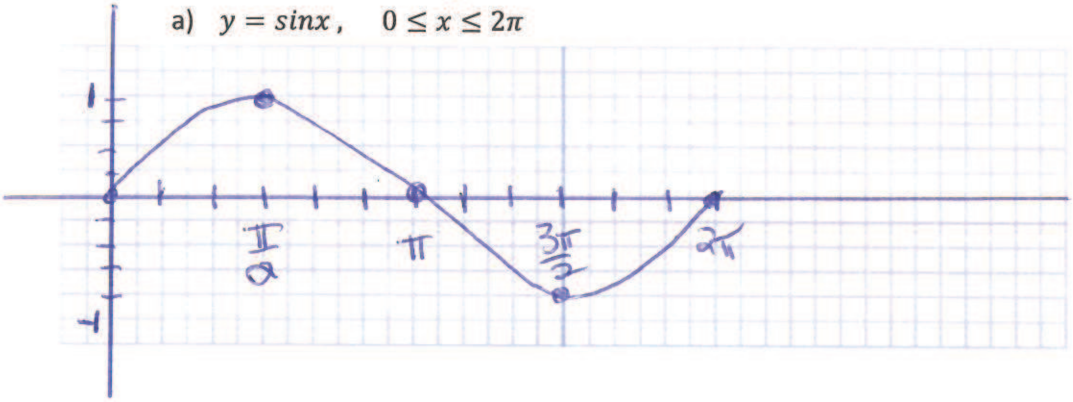
$\cot\theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

Outcome 5A:

Level 2

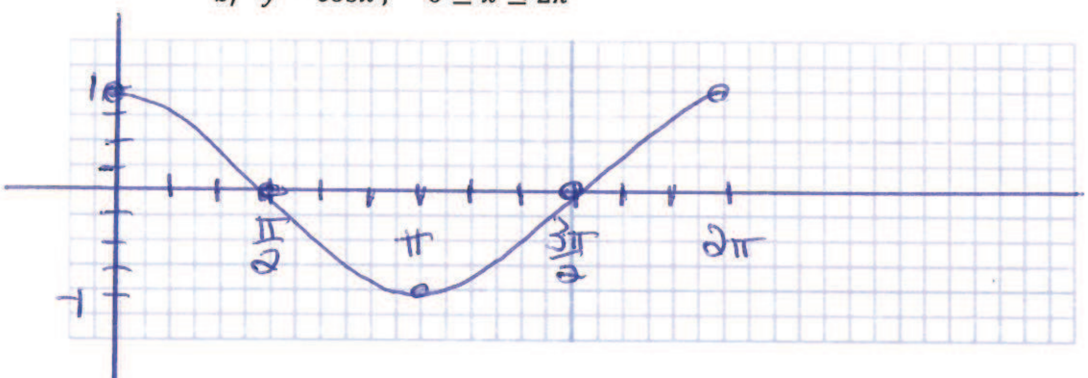
1. Sketch the following:

a) $y = sinx\,, \quad 0 \le x \le 2\pi$



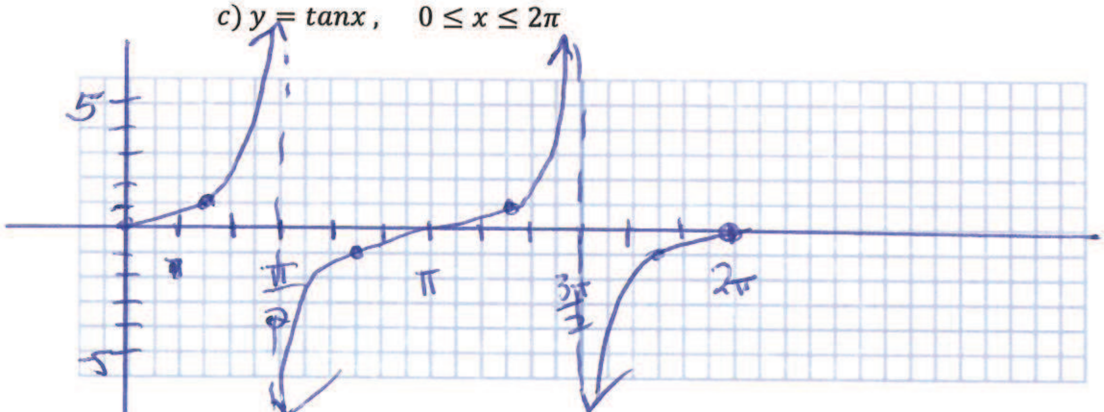
Amplitude: 1 Period: 2π x-intercepts: $0, \pi, 2\pi$ asymptotes: None

b) $y = cosx\,, \quad 0 \le x \le 2\pi$



Amplitude: 1 Period: 2π x-intercepts: $\frac{\pi}{2}, \frac{3\pi}{2}$ asymptotes: None

c) $y = tanx\,, \quad 0 \le x \le 2\pi$



Amplitude: None Period: π x-intercepts: $0, \pi, 2\pi$ asymptotes: $\frac{\pi}{2}, \frac{3\pi}{2}$

Level 3

2. Determine the following for each graph

a) Amplitude: 4

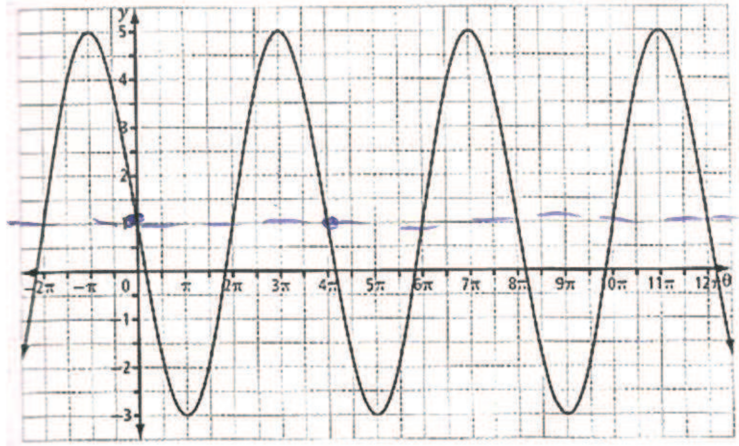
Domain: $x \in \mathbb{R}$

Range: $[-3, 5]$

Period: 4π $b = \frac{1}{2}$

Write the equation of the graph in form *y = a cos b (x - c)+ d*

$$y = 4\cos\tfrac{1}{2}(x - 3\pi) + 1$$



b) Amplitude: 3

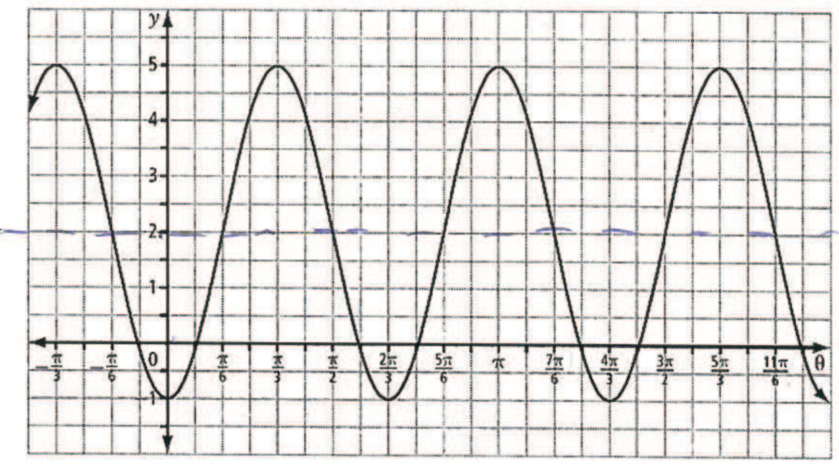
Domain: $x \in \mathbb{R}$

Range: $[-1, 5]$

Period: $\frac{2\pi}{3}$ $b = 3$

Write the equation of the graph in form *y = a sin b (x - c)+ d*

$$y = 3\sin 3\left(x - \frac{\pi}{6}\right) + 2$$



c) Amplitude: 3

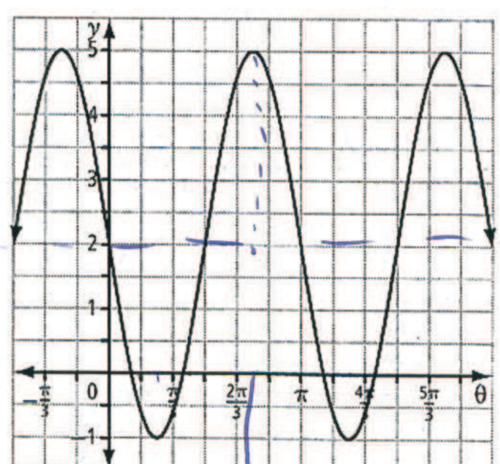
Doman: $x \in \mathbb{R}$

Range: $[-1, 5]$

Period: π $b = 2$

Write the equation of the graph in form *y = a cos b (x - c)+ d*

$$y = 3\cos 2\left(x - \frac{3\pi}{4}\right) + 2$$

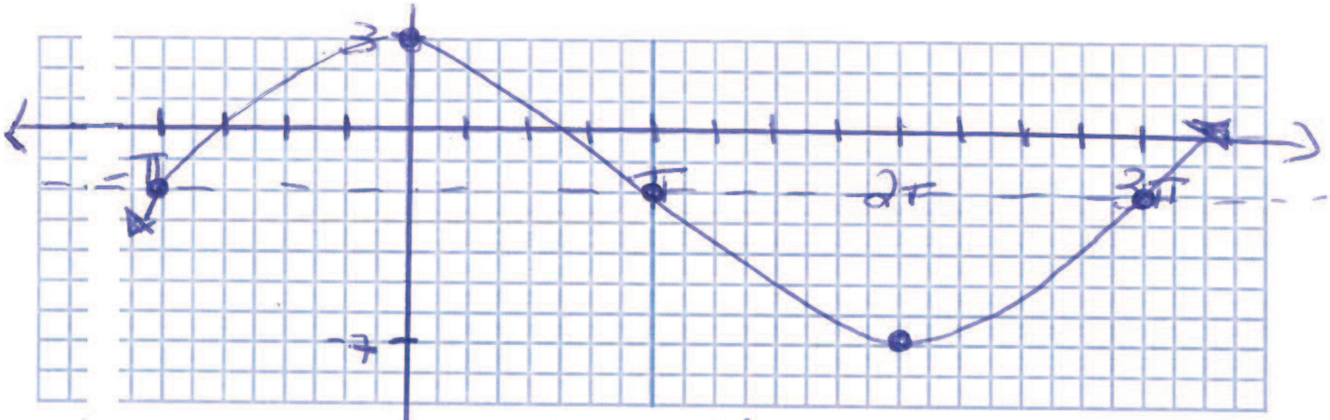


$\frac{\pi}{12}$ $\frac{\pi}{6}$

$\frac{9\pi}{12} = \frac{3\pi}{4}$

3. Graph each of the following for at least one cycle. For each state the domain, range, amplitude, and period

a) $y = 5sin\frac{1}{2}(x+\pi) - 2$ Start @ $-\pi$ end $-\pi + 4\pi = 3\pi$



$D = (-\infty, \infty)$ amp = 5

$R = [-7, 3]$ period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

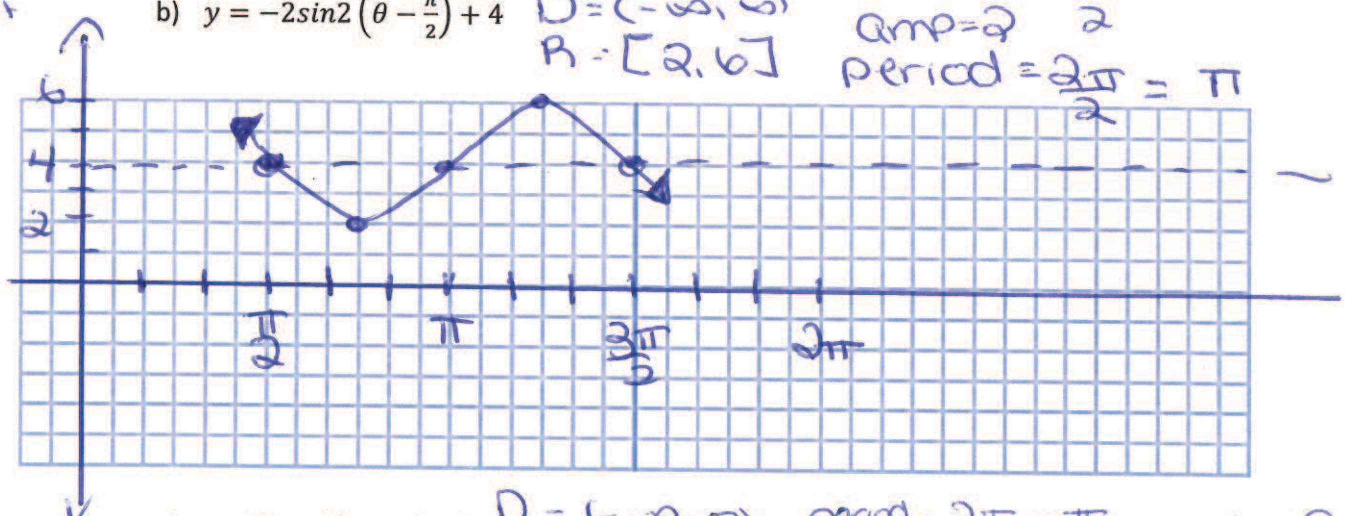
Start $\frac{\pi}{2}$

end $\frac{\pi}{2} + \frac{2\pi}{2}$

$\frac{3\pi}{2}$

b) $y = -2sin2\left(\theta - \frac{\pi}{2}\right) + 4$ $D = (-\infty, \infty)$ amp = 2

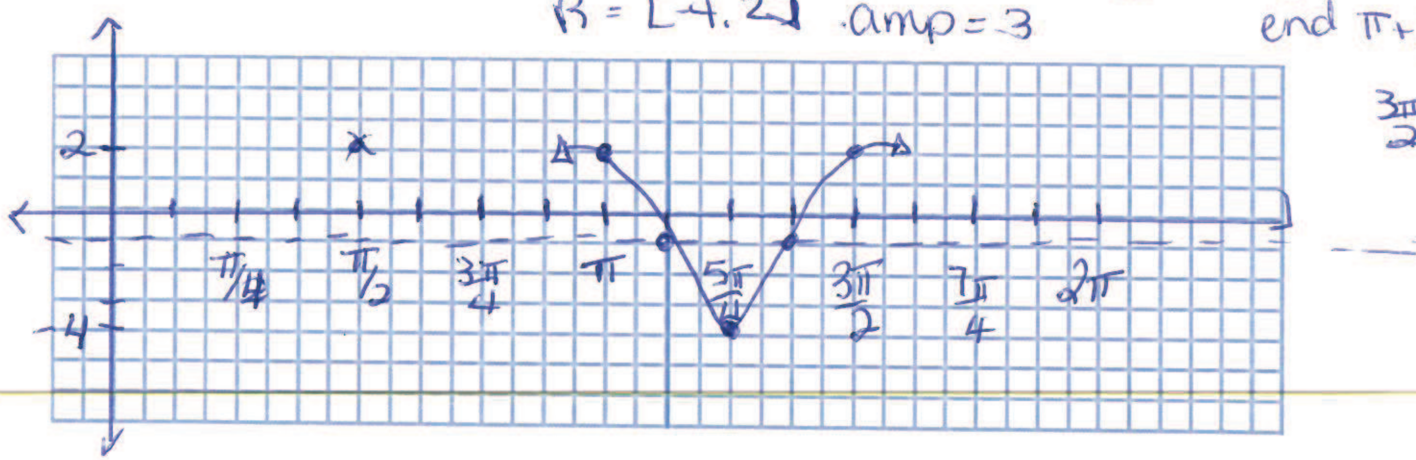
$R = [2, 6]$ period = $\frac{2\pi}{2} = \pi$



c) $y = 3cos4(x-\pi) - 1$ $D = (-\infty, \infty)$ period = $\frac{2\pi}{4} = \frac{\pi}{2}$ Start @ π

$R = [-4, 2]$ amp = 3 end $\pi + \frac{\pi}{2}$

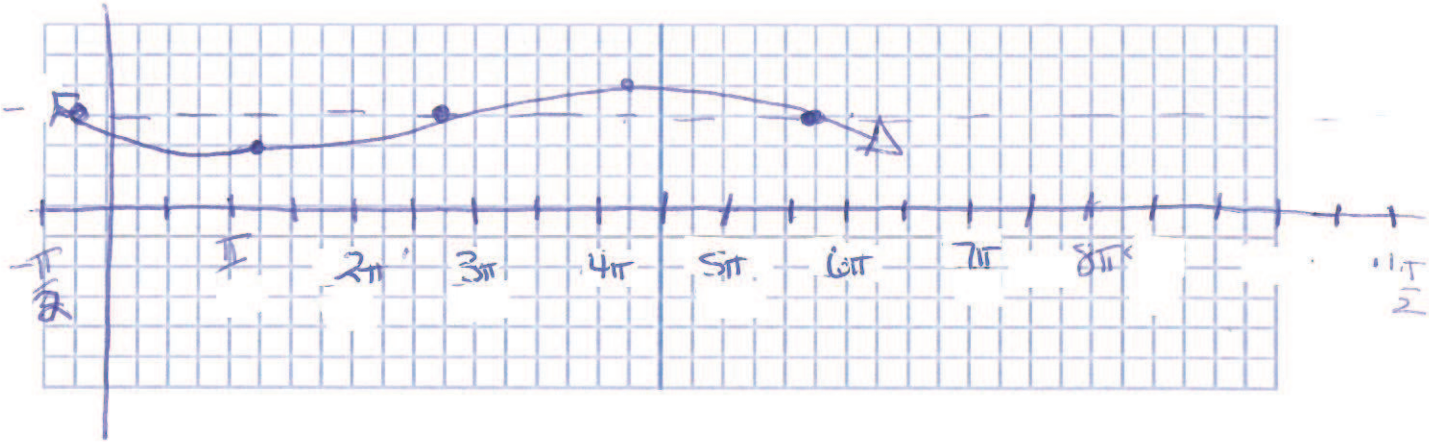
$\frac{3\pi}{2}$



period = 6π

$\frac{\pi}{4} + 6\pi = \frac{25}{4}$

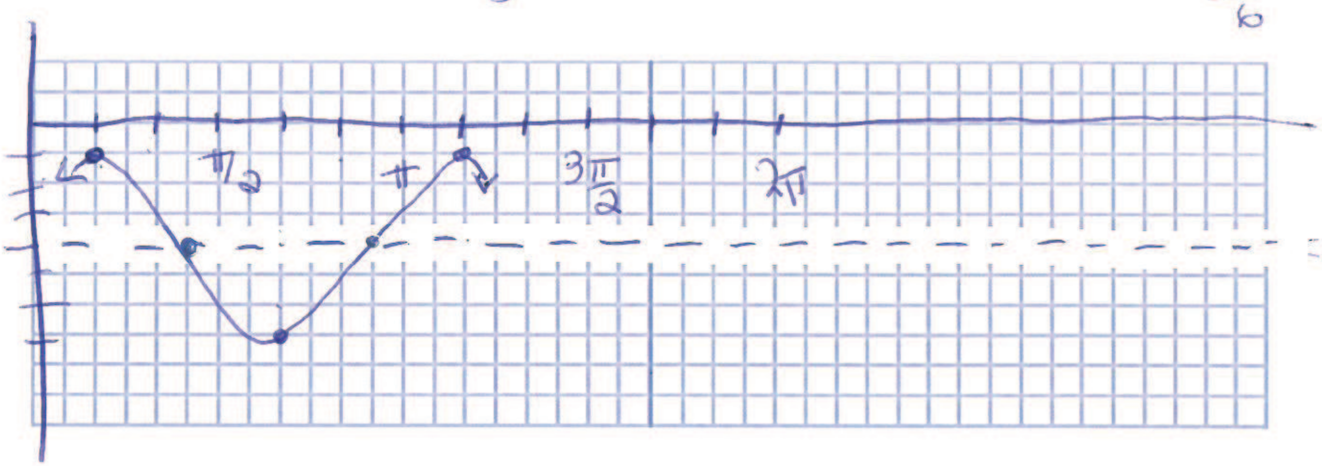
d) $y = -\sin\frac{1}{3}\left(\theta + \frac{\pi}{4}\right) + 3$



$\pi/6 + \pi$

e) $y = 3\cos\left(2\theta - \frac{\pi}{3}\right) - 4$ $y = 3\cos 2(\theta - \pi/6) - 4$

$= \frac{7\pi}{6}$



Chapter 6 - Outcome 30.5

Level 2:

1) Verify that the equation $(\sec x + \tan x)\cos x - 1 = \sin x$ is true for $x = 30°$

$(\sec 30° + \tan 30°)\cos 30° - 1 = \sin 30°$

$\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) - 1 \qquad \frac{1}{2} = \frac{1}{2}$

$\frac{3}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right) - 1 = \frac{1}{2}$

$\frac{3}{2} - 1 = \frac{1}{2}$

$1/2 = 1/2$ ✓

2) Prove the following identities:

a) $\frac{\cos x \csc x}{\sec x \cot x} = \cos x$

LHS = $\frac{\cancel{\cos x}\cos x \cancel{\sin x}}{\cancel{\sin x}\cancel{\cos x}}$

$\cos x$

LHS = RHS □

b) $\cot x \sin x = cosx$

LHS = $\frac{\cos x \cdot \cancel{\sin x}}{\cancel{\sin x}}$

$= \cos x$

LHS = RHS □

c) $\csc x \tan x \sec x \cos x = secx$

LHS = $\frac{\cancel{\sin x} \cdot \cancel{\cos x}}{\cancel{\sin x} \cdot \cancel{\cos x} \cdot \cos x}$

$= \frac{1}{\cos x}$

$= \sec x$

LHS = RHS □

3) Determine the <u>exact</u> value of each trigonometric expression

a) $\sin 105°$

$\sin(45+60)$

$\sin 45 \cos 60 + \cos 45 \sin 60$

$\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

b) $cos\frac{\pi}{9}cos\frac{2\pi}{9} - sin\frac{\pi}{9}sin\frac{2\pi}{9}$

$\cos(\pi/9 + 2\pi/9)$

$\cos\left(\frac{3\pi}{9}\right)$

$\cos(\pi/3)$

$1/2$

Level 3:

4) Prove the following identities.

a) $sin\theta(cot\theta + 1) = sin\theta + cos\theta$

$\sin\theta \cot\theta + \sin\theta$

$\cancel{\sin\theta} \cdot \frac{\cos\theta}{\cancel{\sin\theta}} + \sin\theta$

$\cos\theta + \sin\theta$ □

LHS = RHS

b) $\frac{\sin x + \sin^2 x}{\cos x + \sin x \cos x} = tanx$

$\frac{\sin\theta(\cancel{1+\sin\theta})}{\cos\theta(\cancel{1+\sin\theta})}$

$\frac{\sin\theta}{\cos\theta}$

$\tan\theta$

LHS = RHS □

c) $\sin 2x = \tan x + \tan x \cos 2x$

RHS $= \tan x(1+\cos 2x)$

$= \tan x(1 + 2\cos^2 x - 1)$

$= \tan x(2\cos^2 x)$

$= \frac{\sin x(2\cos^2 x)}{\cancel{\cos x}}$

$= 2\sin x\cos x$

$= \sin 2x$

d) $\frac{tan\theta}{cos\theta + cos\theta tan^2\theta} = sinx$

$\frac{\tan}{\cos\theta(1+\tan^2\theta)}$

$\frac{\tan\theta}{\cos\theta \cdot \sec^2\theta}$

$\frac{\sin\theta \cdot \cos^2\theta}{\cos\theta \cdot \cos\theta}$

$\to \sin x$

LHS = RHS $\square$

e) $\frac{1(1+\cos\theta)}{(1+\cos\theta)(1-cosx)} - \frac{1(1-\cos\theta)}{(1+cosx)(1-\cos\theta)} = 2\cot x\, csc\, x$

$\frac{1+\cos\theta - (1-\cos\theta)}{1-\cos^2\theta}$

$\frac{2\cos\theta}{\sin^2\theta}$

$\to \frac{2\cos\theta}{\sin\theta\sin\theta}$

$2\cot\theta\csc\theta$

LHS = RHS $\square$

Level 4:

5) Prove:

$$\frac{1-cos2\theta}{sin2\theta} = tan\theta$$

$\frac{1-(1-2\sin^2\theta)}{2\sin x\cos x}$

$\frac{\cancel{2}\sin^2\theta}{\cancel{2}\sin\theta\cos\theta}$

$\frac{\sin\theta}{\cos\theta}$

$\to \tan\theta$

LHS = RHS $\square$

6) State the non-permissible values for questions 4a, 4d and 5.

4a) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin\theta \neq 0$

$\theta \neq 0, \pi, 2\pi$

d) $\cos\theta(1+\tan^2\theta) \neq 0$

$\cos\theta \neq 0$ $\quad \sqrt{\tan^2\theta} \neq \sqrt{-1}$

$\theta \neq \pi/2, 3\pi/2$ $\quad \tan\theta \neq \cancel{\pm 1}$

no restriction

$\theta \neq \pi/2 \pm \pi n, n \in N$

$\tan\theta \neq 0$

$\frac{\sin\theta}{\cos\theta}$ $\quad \cos\theta \neq 0$

5. $\sin x \neq 0$

$\theta \neq 0 \pm \pi n, n \in N$

$\cos x \neq 0$

$\theta \neq \pi/2 \pm \pi n, n \in$

Chapter 7 – Outcome30.9c

Level 2

1. Solve

a) $2^x = 64$

$2^x = 2^6$ $\{6\}$

$x = 6$

b) $3^x = 27^{x-2}$

$3^x = 3^{3(x-2)}$

$x = 3x - 6$ $x = 3$ $\{3\}$

$-2x = -6$

c) $8^{2x} = 16^{x+3}$

$2^{3(2x)} = 2^{4(x+3)}$

$6x = 4x + 12$ $x = 6$

$2x = 12$ $\{6\}$

d) $9^{2x-5} = 27^{x+6}$

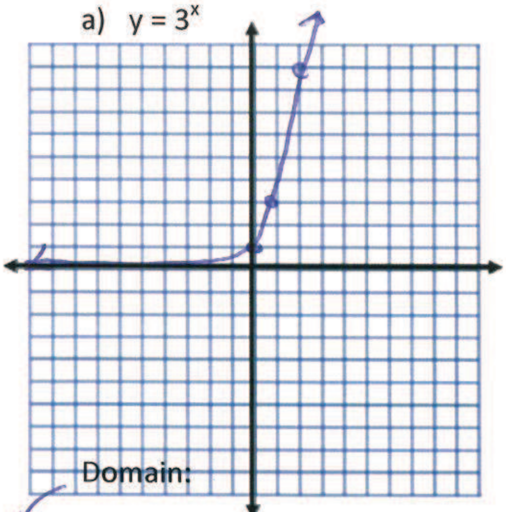
$3^{2(2x-5)} = 3^{3(x+6)}$

$4x - 10 = 3x + 18$

$x = 28$ $\{28\}$

2. Graph each of the following, and then determine the:

a) $y = 3^x$



Domain: $(-\infty, \infty)$

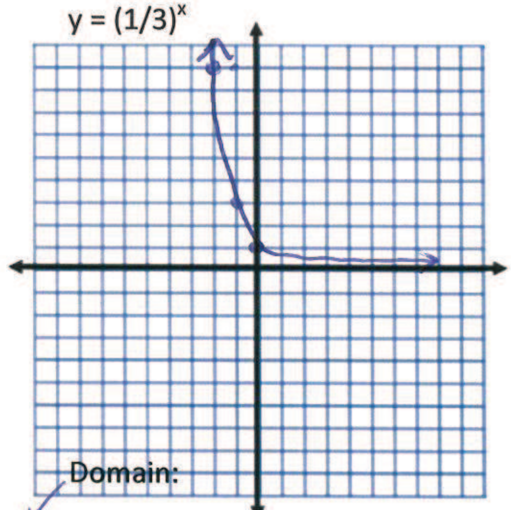
Range: $(0, \infty)$

Horizontal asymptote $y = 0$

Y intercept: $(0, 1)$

Increasing or Decreasing:

$y = (1/3)^x$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote $y = 0$

Y intercept: $(0, 1)$

Increasing or Decreasing:

2. . Identify all of the transformations of the following: (ie vertical translation up 2)

Base $y = 3^x$

a) $f(x) = 3^{-x} + 5$

$b = -1$ h-reflection about y-axis

$K = 5$ v. trans up 5

Base $y = \left(\frac{1}{3}\right)^x$

b) $h(x) = -2\left(\frac{1}{3}\right)^{x+1}$

$a = -2$ v. stretch x2

v. reflection about x-axis

$h = -1$ move 1 unit left

Level 3

4. Sketch the graph of

$y = -3^{x-2}$

Base $y = 3^x$

$(x, y) \rightarrow (x+2, -y)$

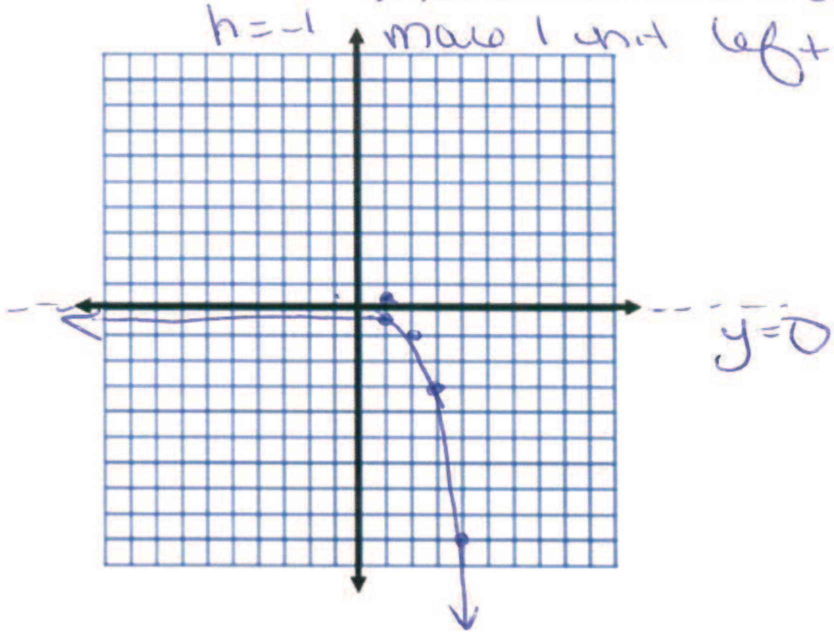
$(-1, +\frac{1}{3})$ $(1, -\frac{1}{3})$

$(0, 1)$ $(2, -1)$

$(1, 3)$ $(3, -3)$

$(2, 9)$ $(4, -9)$

$(3, 27)$ $(5, -27)$



$y = 3(2^{-x}) - 5$

Base $y = 2^x$

$(x, y) \rightarrow (-x, 3y-5)$

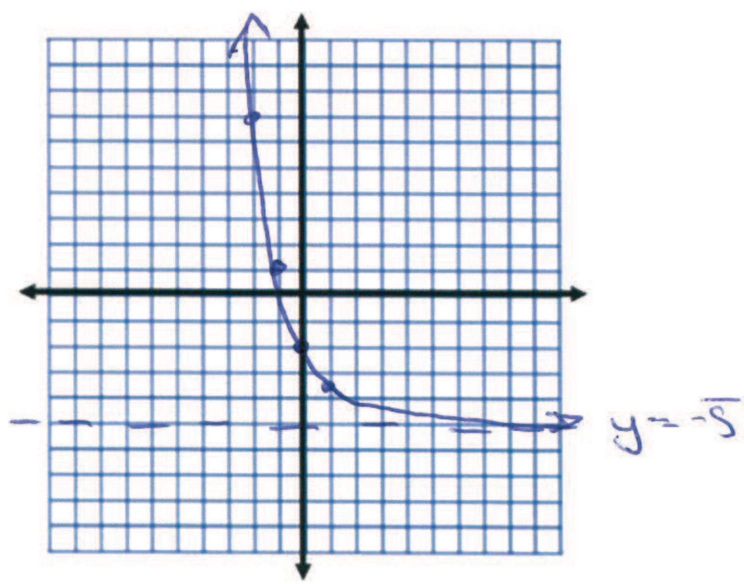
$(-1, \frac{1}{2}) \rightarrow (1, -3.5)$

$(0, 1) \rightarrow (0, -2)$

$(1, 2) \rightarrow (-1, 1)$

$(2, 4) \rightarrow (-2, 7)$

$(3, 8) \rightarrow (-3, 19)$



$y = 2^{2x+4} - 1$

$y = 2^{2(x+2)} - 1$

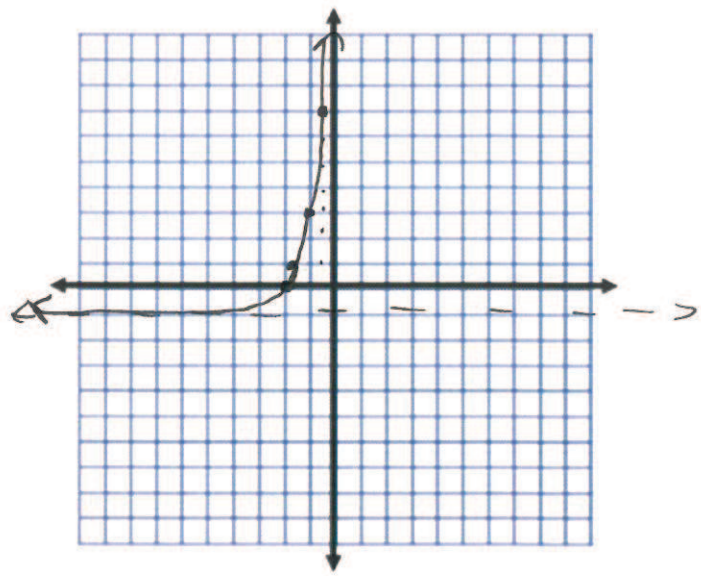
$(x, y) \to (\frac{1}{2}x - 2, y - 1)$

$(0, 1) \to (-2, 0)$

$(1, 2) \to (-1.5, 1)$

$(2, 4 \to (-1, 3)$

$(3, 8) \to (-0.5, 7)$



Chapter 8 – Part 1

Level 2

1. Express as a logarithmic statement.

$$2^3 = 8$$

$\log_2 8 = 3$

2. Express as an exponential statement.

$$\log_3 81 = 4$$

$3^4 = 8$

3. Determine the value of each logarithm.

a) $\log_5 25$

2

c) $\log_9 1$

0

b) $\log_2 \frac{1}{8}$

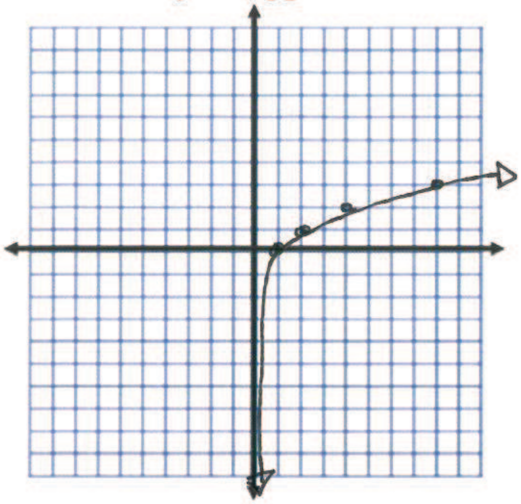
-3

d) $\log_6 6$

1

4. Graph each of the following and determine

b) $y = log_2 x$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical asymptote $x = 0$

x intercept: $(1, 0)$

y intercept: NA

5. Identify all of the transformations of the following: (state all stretches/reflections/translations up, down left or right)

a) $y = -2log_3(x-5)+2$

a = -2 v. stretch x2

b = 1 v. ref. about x axis

h = 5 right 5

k = 2 up 2

$y = 2\log_3(-x)+1$

a = 2 v. stretch x2

b = -1 h. reflect. about y

h = 0

k = 1 up 1

Level 3

6. Sketch

$y = -\log_2(x+1) - 2$

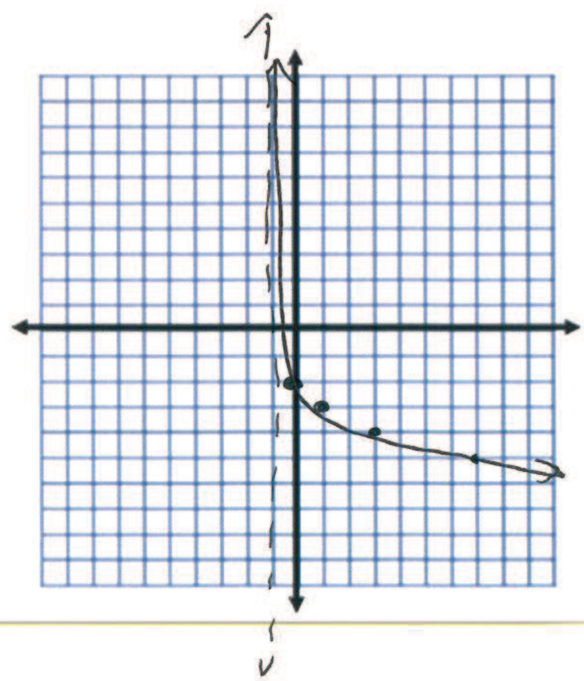
$(x, y) \rightarrow (x-1, -y-2)$

$(1, 0) \rightarrow (0, -2)$

$(2, 1) \rightarrow (1, -3)$

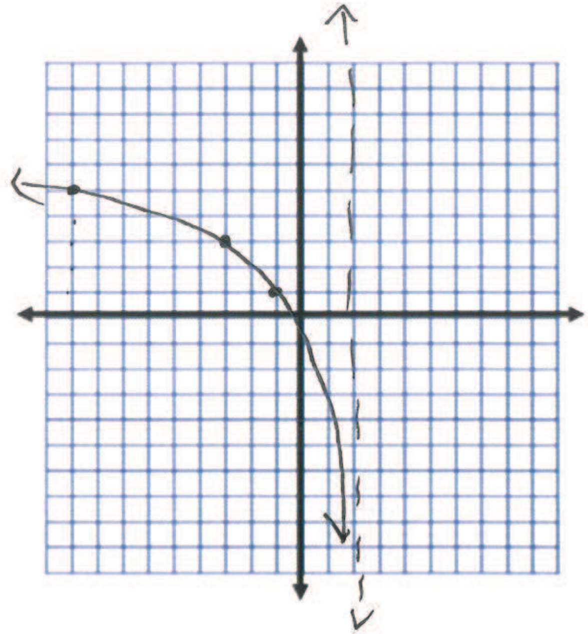
$(4, 2) \rightarrow (3, -4)$

$(8, 3) \rightarrow (7, -5)$



$y = 2\log_3(x-2)+1$

$(x,y) \to (-x, 2y+1)$
$(1,0) \to (-1,1)$
$(3,1) \to (-3,3)$
$(9,2) \to (-9,5)$



Chapter 8 Part 2

Level 2

1. Use your laws of logarithms to expand each of the following:
   a) $\log_4 \frac{x}{3}$ b) $\log_4 x^5$ c) $\log_2 yx^5$

   $\log_4 x - \log_4 3$ $5\log_4 x$ $\log_2 y + 5\log_2 x$

2. Use the laws of logarithms to simplify each of the following:
   a) $\log 2 + \log 7$ b) $4\log_3 5$ c) $\log_2 42 - \log_2 6$

   $\log 14$ $\log_3 625$ $\log_2 7$

3. Determine the value of x.
   a) $\log_2 x = 3$ b) $3\log_5 x = \log_5 125$

   $2^3 = x$
   $8 = x$

   $\log_5 x^3 = \log_5 5^3$
   $x = 5$

   c) $6^x = 216$ d) $4^{x+1} = 64$

   $6^x = 6^3$
   $x = 3$

   $4^{x+1} = 4^3$
   $x = 2$

Level 3

4. Use the laws of logarithms to simplify and then evaluate each of the following:

a) $\log_3 270 - (\log_3 2 + \log_3 5)$

$\log_3 \frac{270}{2 \cdot 5}$

$\log_3 \frac{270}{10}$

$\log_3 27$

3

b) $3\log_2 6 - 3\log_2 3$

$\log_2 \frac{\cancel{216}}{27}$

$\log_2 8$

3

5. Write each expression in terms of individual logarithms.

a) $\log_2 \frac{x^5 \sqrt[3]{y}}{7z}$

$5\log_2 x + \frac{1}{3}\log_2 y - \log_2 7 - \log_2 z$

b) $\log_5 \sqrt{xy^3}$

$\frac{1}{2}\log_5 x + \frac{3}{2}\log_5 y$

6. Write each expression as a single logarithm.

a) $3\log w \quad + \log\sqrt{w} - 2\log w$

$\log w^3 + \log w^{1/2} - \log w^2$

$\log \frac{w^3 \cdot w^{1/2}}{w^2}$

$\log w^{3/2}$

$\frac{3}{2}\log w$

$3 + \frac{1}{2} - 2$

$1\frac{1}{2}$

$\frac{3}{2}$

b) $\log_2(x+6) + \log_2(x-1)$

$\log_2 (x+6)(x-1)$

$\log_2 (x^2+5x-6)$

7. Solve for x.

a) $\log_5 x + 6 = 8$

$\log_5 x = 2$

$5^2 = x$

$\boxed{25 = x}$

b) $log_4 x + \cancel{2}log_4 x^2 = 6$

$\log_4 x^3 = 6$

$\sqrt[3]{4^6} = \sqrt[3]{x^3}$

$4^2 = x$

$\boxed{16 = x.}$

c) $\log_2 x^2 - \log_2 5 = \log_2 20$

$\log_2 \frac{x^2}{5} = \log_2 20$

$\frac{x^2}{5} = 20$

$\sqrt{x^2} = \pm\sqrt{100}$

$x = \pm 10$

* -10 works b/c $\log_2 (-10)^2 = \log_2 100$ ✓

d) $log_3(x+7) - log_3(x-3) = 2$

$\log_3 \frac{(x+7)}{(x-3)} = 2$

$3^2 = \frac{x+7}{x-3}$

$9 = \frac{x+7}{x-3}$

$9x - 27 = x + 7$

$8x = 34$

$\boxed{x = \frac{34}{8} = \frac{17}{4} = 4.25}$

e) $3^x = 100$

$\log 3^x = \log 100$

$x \log 3 = \log 100$

$x = \frac{\log 100}{\log 3}$

$x \approx 4.1918$

f) $7^{x-3} = 517$

$\log 7^{x-3} = \log 517$

$\frac{(x-3)\log 7 = \log 517}{\log 7}$

$x - 3 = \frac{\log 517}{\log 7} + 3$
$+3$

$x = \frac{\log 517}{\log 7} + 3$

$x \approx 6.21086$

Level 4

8. Solve the following. State any restrictions

$log_6(x+3) - 2 = -log_6(x-2)$

$\log_6(x+3) + \log_6(x-2) = 2$

$\log_6 (x+3)(x-2) = 2$

$6^2 = (x+3)(x-2)$

$36 = x^2 + 3x - 2x - 6$
$-36 \qquad -36$

$0 = x^2 + x - 42$

$0 = (x+7)(x-6)$

$\cancel{x = -7}$ $\boxed{x = 6}$

$x + 3 > 0$

$\cancel{x > -3}$

$x - 2 > 0$

$x > 2$

9. Use what you have learned about logarithms to show how you could use two different transformations to graph the logarithmic function $y = log_2 8x$

① $y = \log_2 8x \rightarrow$ h. stretch of $\frac{1}{8}$

2. $y = \log_2 8 + \log_2 x$

$y = 3 + \log_2 x$ v. trans up 3

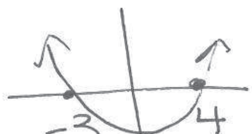
10. Simplify the following logarithm. State the restrictions

log $(x^2 - x - 12) - log\ (x^2 - 9)$

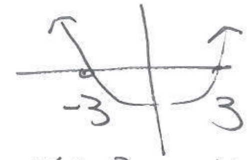
$\log\left(\frac{x-4}{x-3}\right)$, $x<-3$, $x>4$

$\log\left(\frac{x^2-x-12}{x^2-9}\right)$

$\log \frac{(x-4)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$



$\boxed{x<-3}$ $\boxed{x>4}$



$x<-3$ $x>3$

Chapter 9 Review

Level 2

1. Determine the characteristics of the following functions:

a) $y = \frac{2x-1}{x-4}$

Equation of Vertical Asymptotes: $x = 4$

Points of Discontinuity (holes): NA

Equation of Horizontal Asymptote: $y = 2$

b) $y = \frac{\cancel{x+5}}{\cancel{(x+5)}(x-3)} = \frac{1}{x-3}$

Equation of Vertical Asymptotes: $x = 3$

Points of Discontinuity (holes): $(-5, -1/8)$

Equation of Horizontal Asymptote:

$y = 0$

c) $y = \frac{x^2-4}{x^2+3x+2} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+1)} = \frac{x-2}{x+1}$

Equation of Vertical Asymptotes: $x = -1$

Points of Discontinuity (holes): $(-2, 4)$

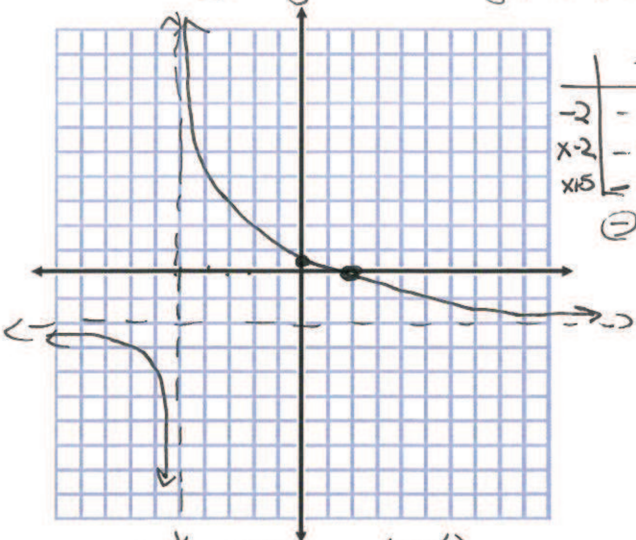
Equation of Horizontal Asymptote: $y = 1$

Level 3/Level 4 (Level 4 Questions will have an oblique asymptote. You will need to determine that on your own.)

2. Graph the following functions. Be sure to give the equations of all asymptotes.

a) $y=\frac{-2x+4}{x+5} = \frac{-2(x-2)}{x+5}$

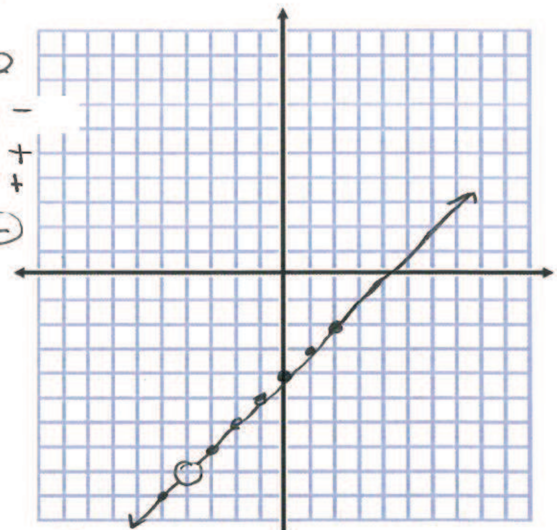
V.A @ x = -5
HA @ y = -2
x int (2,0)
y int (0, 4/5)



	-5		2	
-2	-	-		-
x-2	-	-		+
x+5	-	+		+
	⊖	⊕		⊖

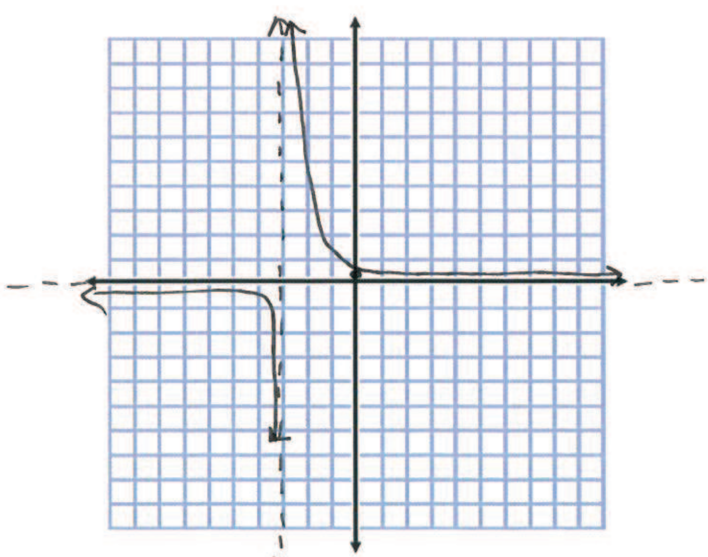
b) $y=\frac{x^2-16}{x+4} = \frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}} = x-4$

hole (-4, -8



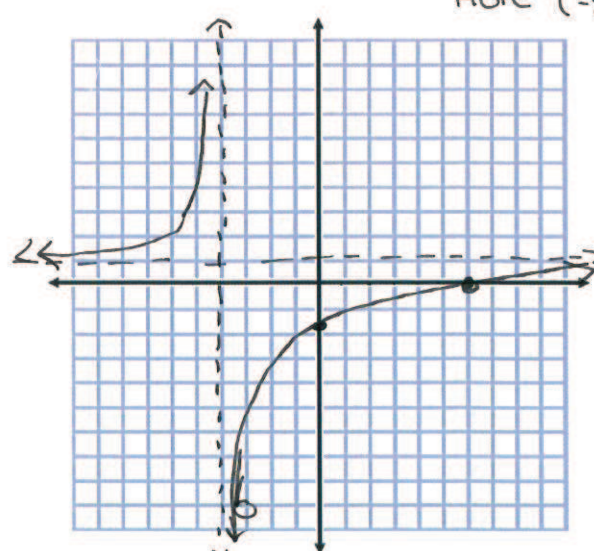
c) $y=\frac{x-5}{x^2-2x-15} = \frac{\cancel{(x-5)}}{\cancel{(x-5)}(x+3)} = \frac{1}{x+3}$

VA @ x = -3
HA @ y = 0
y int (0, 1/3)
no x int



d) $y=\frac{x^2-3x-18}{x^2+7x+12} = \frac{(x-6)\cancel{(x+3)}}{\cancel{(x+3)}(x+4)} = \frac{x-6}{x+4}$

V.A @ x = -4
H.A @ y = 1
X int (6,0)
y int (0, -1.5)
Hole (-3, -9)



		-3	
X+3	-		+
	⊖		⊕

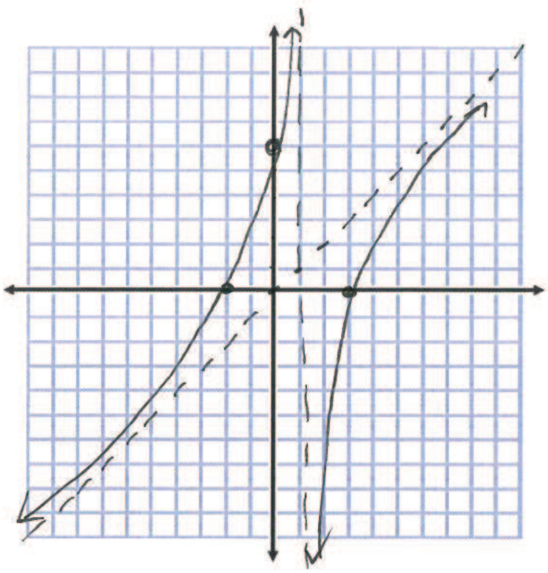
	-4		6	
X-6	-	-		+
X+4	-	+		+
	⊕	⊖		⊕

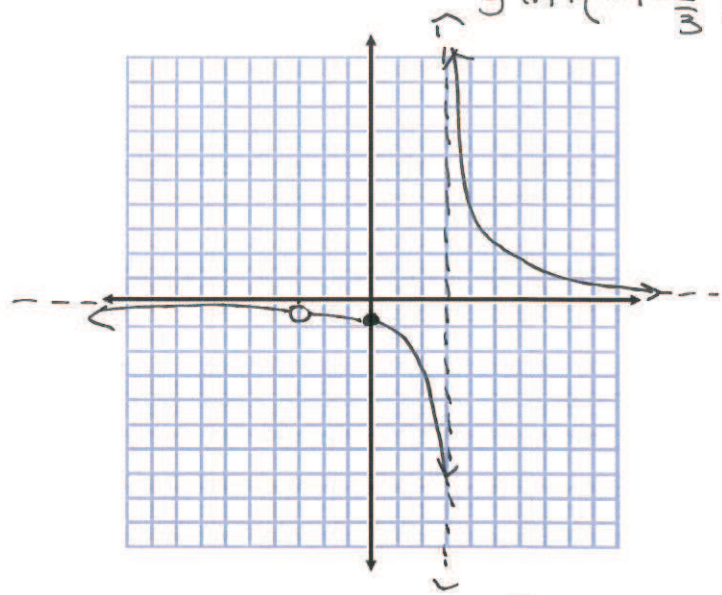
$y=\frac{x^2-x-6}{x-1} = \frac{(x-3)(x+2)}{(x-1)}$

$y=\frac{2x+6}{x^2-9} = \frac{2(\cancel{x+3})}{(\cancel{x+3})(x-3)} = \frac{2}{x-3}$

Hole $(-3, -\frac{1}{3})$

y int $(0, -\frac{2}{3})$





O.A.

$$\begin{array}{r|rrr} -1 & 1 & -1 & -6 \\ & \downarrow & -1 & 0 \\ \hline & 1x & 0 & \boxed{-6} \end{array}$$

xint $(-2,0)$ $(3,0)$

yint $(0,6)$

OA @ $y=x$

	-2	1	3	
$x-3$	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$+$	$+$
	$-$	$+$	$-$	$+$

	-3	
2	$+$	$+$
$x-3$	$-$	$+$
	$-$	$+$