Factoring Review

There are a few guidelines to follow when factoring polynomials:

1) Always ask yourself if you can remove a GCF
2)
   - If you can remove a GCF, do so. Then ask yourself if you can factor further using the box method or another strategy. If you can't then you are done. If you can, then continue.
   - If you can't remove a GCF, then ask yourself if you can factor using the box method or another strategy. If you can't then the polynomial is "prime" which means it cannot be factored. If you can factor, then do so.

Example 1 (Level 2):

Factor $x^2 + 12x + 20$

Step 1: Ask yourself if you can remove a GCF ---- the answer is NO!

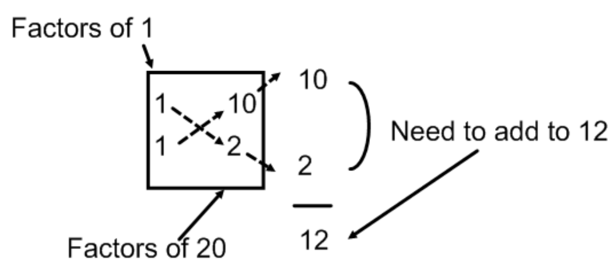
Step 2: There are two strategies for this type of polynomial (the number in front of the x^2 is 1). Choose the strategy you are most comfortable with and factor

Strategy 1: The Box Method

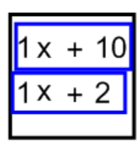
Step 1: Place factors of 1 (from x^2) in Column 1, factors of 20 (constant) In column 2.

Step 2: Multiply diagonally in box Get 10 and 2

Step 3: See if there is a combination of 10 and 2 that will add to 12 (middle Term in polynomial)



Step 4: Factors are now each row of box



Therefore $x^2 + 12x + 20$ in factored form Is $(x + 10)(x + 2)$. You can verify by Multiplying these binomials out.

Strategy 2: Determining two numbers that multiply to "constant" and add to middle "coefficient"

$X^2 + 12x + 20$……. Ask yourself "what two numbers multiply to 20 and add to 12?" The answer is 10 and 2. Therefore your factors are $(x + 10)(x + 2)$. You can verify by multiplying these binomials out.

*** NOTE: if the signs in the trinomial are + and +, the binomials will both be positive

1. Factor the following:

   a) $X^2 + 13x + 36$ b) $x^2 + 13x + 42$

c)$X^2 + 27x + 50$ d) $x^2 + 49x + 48$

e) $x^2 + 8x + 15$ f) $x^2 + 10x + 21$

g) $x^2 + 12x + 27$ h) $x^2 + 8x + 7$

Example 2 (Level 2):

$X^2 – 10x + 24$

Step 1: Ask yourself if you can remove a GCF ---- the answer is NO!

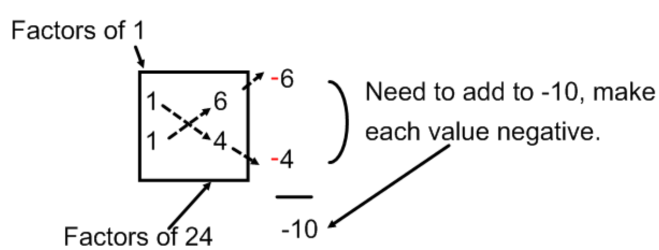
Step 2: There are two strategies for this type of polynomial (the number in front of the x^2 is 1). Choose the strategy you are most comfortable with and factor

Strategy 1: The Box Method

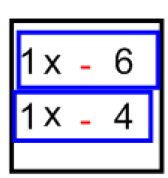
Step 1: Place factors of 1 (from x^2) in Column 1, factors of 24 (constant) In column 2.

Step 2: Multiply diagonally in box Get 6 and 4

Step 3: See if there is a combination of 6 and 4 that will add to -10 (middle Term in polynomial) If you make The two numbers negative this will Occur: -6 + (-4) = -10



Step 4: Since we had to make 6 and 4 negative, You need to make the factors in the Second column of the box negative as well. Factors are now each row of box



Therefore $x^2 - 10x + 24$ in factored form
Is $(x - 6)(x - 4)$. You can verify by
Multiplying these binomials out.

Strategy 2: Determining two numbers that multiply to "constant" and add to middle "coefficient"
$X^2 - 10x + 24$……. Ask yourself "what two numbers multiply to 24 and add to -10?" The answer is -6 and -4. Therefore your factors are $(x – 6)(x – 4)$. You can verify by multiplying these binomials out.

*** NOTE: if the signs in the trinomial are – and + in that order, both signs in the binomials will be negative

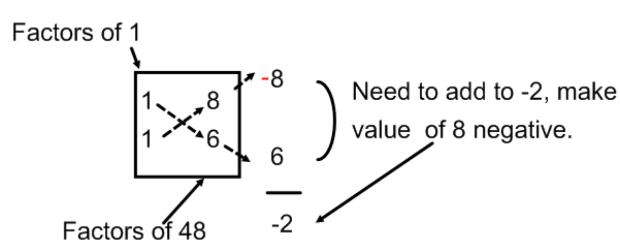
2. Factor:
   a) $X^2 – 14x + 40$ b) $x^2 – 7x + 10$

   c) $X^2 – 14x + 45$ d) $x^2 – 8x + 7$

   e) $X^2 – 9x + 14$ f) $x^2 – 11x + 28$

Example 3 (Level 2):
Factor $x^2 – 2x - 48$
Step 1: Ask yourself if you can remove a GCF ---- the answer is NO!
Step 2: There are two strategies for this type of polynomial (the number in front of the x^2 is 1). Choose the strategy you are most comfortable with and factor

Strategy 1: The Box Method
Step 1: Place factors of 1 (from x^2) in
Column 1, factors of 48 (constant)
In column 2.
Step 2: Multiply diagonally in box
Get 8 and 6
Step 3: See if there is a combination of



8 and 6 that will add to -2 (middle
Term in polynomial) If you make
The 8 negative, this will occur.
-8 + 6 = -2

Step 4: Since we had to make 8 negative,
You need to make the factors of the
Diagonal that 8 was a result of negative as well.
Factors are now each row of box

1x - 8
1x + 6

Therefore x^2 - 2x - 48 in factored form
Is (x - 8)(x + 6). You can verify by
Multiplying these binomials out.

<u>Strategy 2: Determining two numbers that multiply to "constant" and add to middle "coefficient"</u>
X^2 - 2x - 48……. Ask yourself "what two numbers multiply to -2 and add to -48?" The answer is -8 and +6. Therefore your factors are (x – 8)(x + 6). You can verify by multiplying these binomials out.

*** NOTE: If the signs in the trinomial are + and – or – and -, in those order, the signs in the binomial will be one negative and one positive.

3. Factor
   a) X^2 -7x -18 b) x^2 – 13x – 30

   c) x^2 + 3x – 28 d) x^2 + 4x – 45

   e) x^2 – 2x – 24 f) x^2 – 14x -32

g) $x^2 + 6x - 40$

h) $x^2 + 7x - 30$

i) $x^2 - 6x - 27$

j) $x^2 + x - 12$

<u>Example 4 (Level 2)</u>

Factor $6x^2 + 13x - 5$

Step 1: Ask yourself if you can remove a GCF ---- the answer is NO!

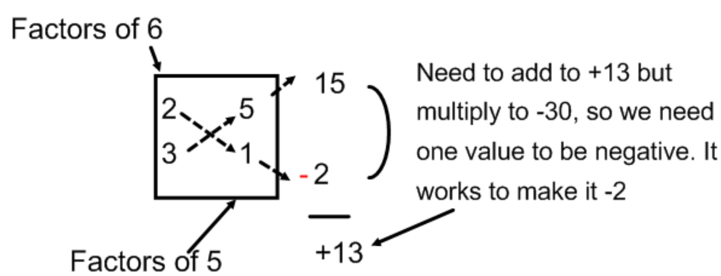
Step 2: Use the box method (or another effective strategy you are comfortable with)

Box Method

Step 1: Place factors of 6 (from x^2) in Column 1, factors of 5(constant) In column 2. You may have to repeat this A couple times until your numbers work.

Step 2: Multiply diagonally in box Get 15 and 2

Step 3: See if there is a combination of 15 and 2 that will add to +13 (middle Term in polynomial) If you make The 2 negative, this will occur. $15 + (-2) = 13$



4. Factor

a) $2x^2 + 11x + 12$

b) $6x^2 + 11x + 4$

c) $3x^2 + 11x + 6$

d) $10x^2 + 19x + 6$

e) $12x^2 – 13x + 3$

f) $2x^2 – 13x + 15$

g) $15x^2 – 26x + 8$

h) $14x^2 -39x + 10$

i) $4x^2 + 4x – 3$

j) $3x^2 – 11x – 20$

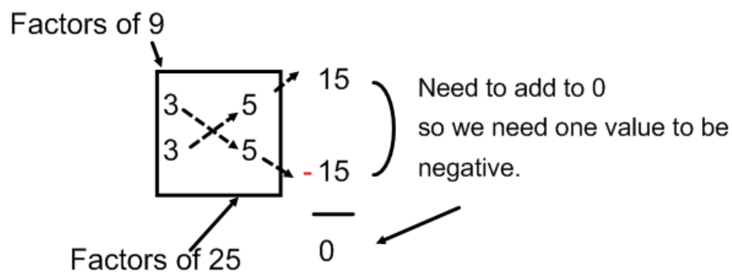
k) $15x^2 + x – 6$

l) $4x^2 + 4x – 35$

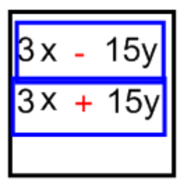
Example 5 (Level 2)
Factor $9x^2 - 25y^2$

Step 1: Ask yourself if you can remove a GCF ---- the answer is NO!
Step 2: There are two strategies for this type of polynomial. Choose the strategy you are most comfortable with and factor

Strategy 1: The Box Method
Step 1: Place factors of 9 (from x^2) in Column 1, factors of 25 (from y^2) In column 2.
Step 2: Multiply diagonally in box Get 15 and 15
Step 3: The middle term is missing, so you Want the two numbers to add to 0. Make one of them negative.



Step 4: Factors are now each row of box



Therefore $9x^2 - 25y^2$ in factored form Is $(3x - 5y)(3x + 5y)$. You can verify by Multiplying these binomials out.

Strategy 2: Determining the "difference of squares"
$9x^2 - 25y^2$…. Since both terms are perfect squares and they are separated by a minus sign, you have a difference of squares. Simply find the square root of each term and make one binomial negative, the other positive. $(3x - 5y)(3x + 5y)$

5. Factor
   a) $16x^2 - y^2$
   b) $a^2 - 4$
   c) $4m^2 - 49$
   d) $m^2 - 64$

e) $25h^2 - 1$

f) $k^2 - m^2$

g) $9r^2 - 64y^2$

h) $36k^2 - 1$

Example 6 (Level 2)

a) **Factor $9a^2 + 12a$**

Step 1: Ask yourself if you can remove a GCF ---- the answer is YES!
Step 2: Remove the GCF

"3a" is the GCF. Divide 3a into both terms. Write 3a as a factor in front of a bracket. Write what is left after dividing each term by 3a in the bracket. Because we started with two Terms, there must be two terms in the bracket!!!!

$3a(3a + 4)$ ****Notice if we multiplied 3a into the bracket we would have our question again!

Step 3: Ask yourself if you can continue to factor using a method from above…. In this example – NO!

b) **Factor $5x^2 + 10x - 5$**

Step 1: Ask yourself if you can remove a GCF ---- the answer is YES!
Step 2: Remove the GCF

"5" is the GCF. Divide 5 into all terms. Write 5 as a factor in front of a bracket. Write what is left after dividing each term by 5 in the bracket. Because we started with three Terms, there must be three terms in the bracket!!!!

$5(x^2 + 2x - 1)$ ****Notice if we multiplied 5 into the bracket we would have our question again!

Step 3: Ask yourself if you can continue to factor using a method from above…. In this example – NO!

c) **Factor $14a^5b^3 + 21a^3b^7 - 42a^7b^4$**

Step 1: Ask yourself if you can remove a GCF ---- the answer is YES!
Step 2: Remove the GCF

"$7a^3b^3$" is the GCF. 7 goes into 14, 21 and 42. When finding the GCF for variables, you need to make sure the variable is in <u>every</u> term and then you take the <u>lowest</u> exponent you see on the variable. Divide $7a^3b^3$ into all terms. Write $7a^3b^3$ as a factor in front of a bracket.
Write what is left after dividing each term by $7a^3b^3$ in the bracket. Because we started with three Terms, there must be three terms in the bracket!!!!

$7a^3b^3(2a^2 + 3b^4 - 6a^4b)$

****Notice if we multiplied 5 into the bracket we would have our question again!

Step 3: Ask yourself if you can continue to factor using a method from above…. In this example – NO!

6. Factor

a) $5m^2 + 10m$

b) $3h^2 - 9h$

c) $8k^2 + 4k$

d) $10x^2 - 2x$

e) $6x^2 + 4x - 10$

f) $9h^2 - 12h + 15$

g) $36x^4 + 18x$

h) $20m^5 + 15m$

i) $15x^4y^4 - 25x^2y^3 + 10x^5y^2$

j) $8m^5n^4 - 12m^7n^3 + 4m^2n^6$

Example 7 (Level 3)

a) **Factor completely $2x^2 + 26x + 84$**

Step 1: Ask yourself if you can remove a GCF ---- the answer is YES!
Step 2: Remove the GCF:

"2" is the GCF. Divide 2 into all terms. Write 2 as a factor in front of a bracket.
Write what is left after dividing each term by 2 in the bracket. Because we started with three Terms, there must be three terms in the bracket!!!!

$2(x^2 + 13x + 42)$

Step 3: Ask yourself if what is left in the brackets can be factored again….. YES!!!!

Step 4: Determine which factoring strategy can be used (after removing a GCF the types of factoring left can all be done with the box method, although there may be a quicker strategy you can use – use what you are comfortable with)

This one can be done with the box method if you would like. However, there is a quicker strategy:

$2(x^2 + 13x + 42)$……. Ask yourself "what two numbers multiply to 13 and add to 42?" The answer is 6 and 7. Therefore your factors are $2(x + 6)(x + 7)$. You can verify by multiplying these binomials out.

b) **Factor completely: $12a^2 – 3b^2$**

Step 1: Ask yourself if you can remove a GCF ---- the answer is YES!
Step 2: Remove the GCF:

"3" is the GCF. Divide 3 into all terms. Write 3 as a factor in front of a bracket.
Write what is left after dividing each term by 3 in the bracket. Because we started with two Terms, there must be two terms in the bracket!!!!

$3(4a^2 – b^2)$

Step 3: Ask yourself if what is left in the brackets can be factored again….. YES!!!!

Step 4: Determine which factoring strategy can be used (after removing a GCF the types of factoring left can all be done with the box method, although there may be a quicker strategy you can use – use what you are comfortable with)

This one can be done with the box method if you would like. However, there is a quicker strategy called difference of squares:

$3(4a^2 – b^2)$…. Two terms, both are perfect squares, separated by a minus sign…. Take the square root of both terms, and then one factor has a minus, one has a positive, don't forget to put the '3' in front.
The factored form is: $3(2a – b)(2a + b)$

c) **Factor completely: $12x^2 – 10x – 8$**

Step 1: Ask yourself if you can remove a GCF ---- the answer is YES!
Step 2: Remove the GCF:

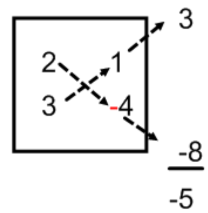
“2” is the GCF. Divide 2 into all terms. Write 2 as a factor in front of a bracket.
Write what is left after dividing each term by 2 in the bracket. Because we started with three Terms, there must be three terms in the bracket!!!!

$2(6x^2 – 5x – 4)$

Step 3:   Ask yourself if what is left in the brackets can be factored again….. YES!!!!

Step 4: Determine which factoring strategy can be used (after removing a GCF the types of factoring left can all be done with the box method, although there may be a quicker strategy you can use – use what you are comfortable with)

This one can only be done with the box method.



The answer in factored form is:
$2(2x + 1)(3x – 4)$

7. Factor completely
   a) $3x^2 + 3x – 36$
   b) $5x^2 + 40x + 75$
   b) $27m^2 – 48$
   d) $5x^2 – 125y^2$
   d) $4x^2 + 2x – 30$
   f) $30x^2 – 27x + 6$

g) $4x^2 - 16x - 84$

h) $3m^2 - 300$

i) $42x^2 - 35x - 28$

j) $3x^2 - 12x - 15$

k) $4x^2 - 16y^2$

l) $6x^2 + 39x - 21$

m) $10x^2 + 20x + 10$

n) $5x^2 - 20x + 20$