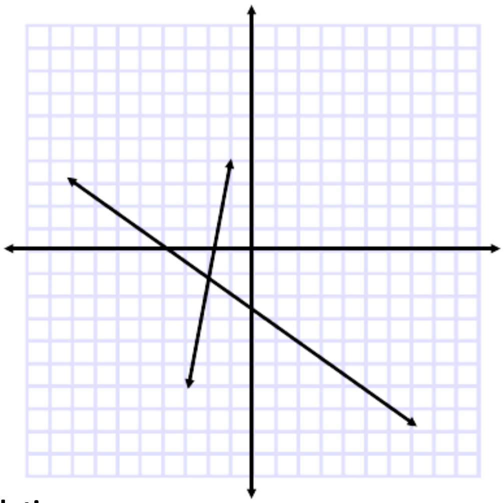
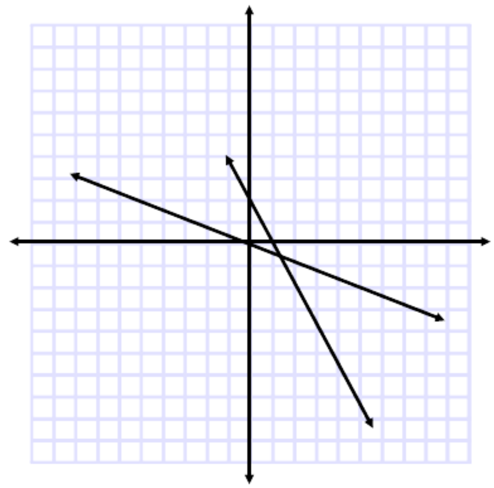
Outcome 10 Review – Foundations and Pre-Calculus 10

Example: Use the graph to determine the solution to the linear system. Is the solution exact or approximate?

a)



b)



Solution:

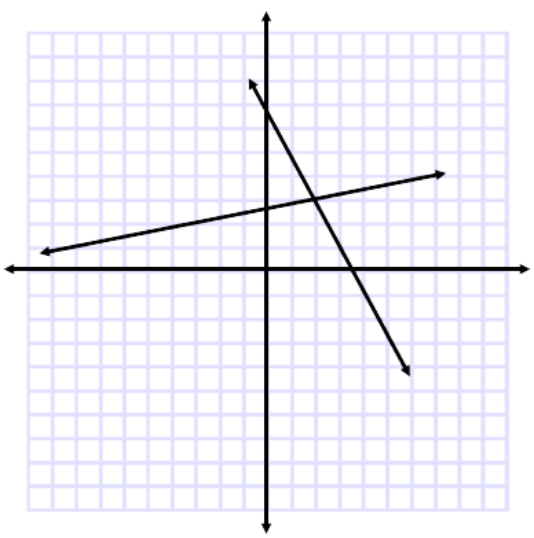
The solultion to a linear system is the point where the two lines intersect.

a) (-2,-1) is exact

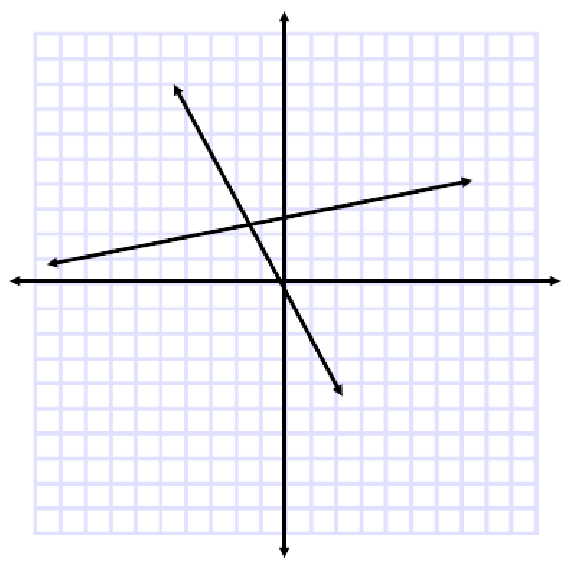
b) (2.1,-.5) is approximate because we estimated the coordinates.

1. Use the graph to determine the solution to the linear system. Is the solution exact or approximate.

a)



b)



Example:

Is each given point a solution to the system of linear Equations?

$$\boldsymbol{x - 2y = 6}$$

$$\boldsymbol{x + 6y = 22}$$

a) (2,10)

To determine if (2,10) is a solution substitute x=2 and y=10 into both equations. It must work in both in order to be a solution.

$\boldsymbol{x - 2y = 6}$
$\boldsymbol{(2) - 2(10) = 6}$
$\boldsymbol{2 - 20 = 6}$
$\boldsymbol{-18 \neq 6}$

$\boldsymbol{x + 6y = 22}$
$\boldsymbol{(2) + 6(10) = 22}$
$\boldsymbol{2 + 60 = 22}$
$\boldsymbol{62 \neq 22}$

(2,10) is not a solution

b) (10,2)

$\boldsymbol{x - 2y = 6}$
$\boldsymbol{(10) - 2(2) = 6}$
$\boldsymbol{10 - 4 = 6}$
$\boldsymbol{6 = 6}$

$\boldsymbol{x + 6y = 22}$
$\boldsymbol{(10) + 6(2) = 22}$
$\boldsymbol{10 + 12 = 22}$
$\boldsymbol{22 = 22}$

Since (10,2)works in both equations it is the solution.

2. Is each given point a solution to the system of linear equations?

$$x + 4y = 17$$

$$2x - y = 7$$

a) (-5,3)

b)(5,3)

Example:

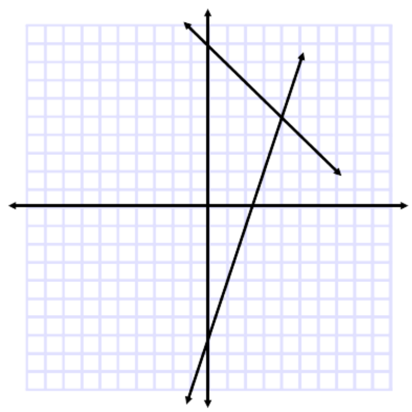
Solve the following system of linear equations graphically.

$$\boldsymbol{y = 3x - 7}$$

$$\boldsymbol{y = -x + 9}$$

Use the skills that you learned for graphing from outcome 8b to graph both lines on the same coordinate plane. Then look for the point of intersection.

The solution is (4,5)



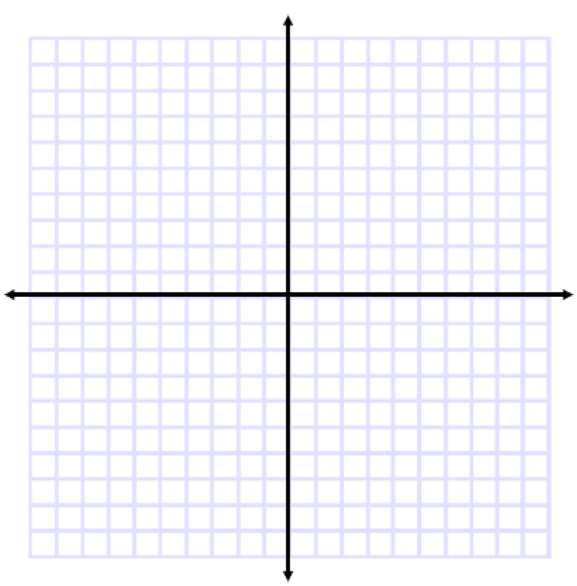
3. Solve the following systems of equations graphically.

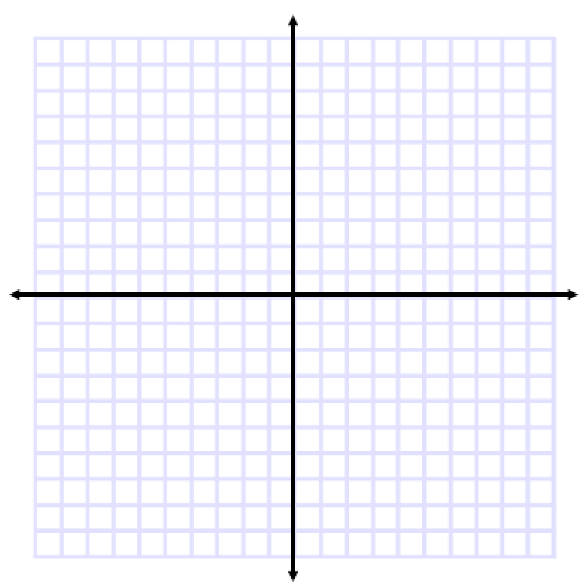
a) $4x - y = 6$

$3x + y = 1$

b) $x - y = -1$

$x + y = 5$





Example

Solve by substitution.

$$\boldsymbol{x + 4y = 17}$$
$$\boldsymbol{2x - y = 7}$$

In order to solve by substitution isolate either x or y in one of your equations.

$$\boldsymbol{x + 4y = 17}$$
$$\boldsymbol{x + 4y - 4y = 17 - 4y}$$
$$\boldsymbol{x = 17 - 4y}$$

Now substitute this into the other equation

$$\boldsymbol{2x - y = 7}$$
$$\boldsymbol{2(17 - 4y) - y = 7}$$
$$\boldsymbol{34 - 8y - y = 7}$$
$$\boldsymbol{34 - 9y = 7}$$
$$\boldsymbol{34 - 34 - 9y = 7 - 34}$$
$$\boldsymbol{-9y = -27}$$
$$\boldsymbol{\frac{-9y}{-9} = \frac{-27}{-9}}$$
$$\boldsymbol{y = 3}$$

Now substitute y=3 in to the equation to find x.

$$\boldsymbol{2x - y = 7}$$
$$\boldsymbol{2x - (3) = 7}$$
$$\boldsymbol{2x - 3 + 3 = 7 + 3}$$
$$\boldsymbol{2x = 10}$$
$$\boldsymbol{x = 5}$$

The solution is (5,3)

4. Solve the following systems by substitution.

a) $y = x + 2$

$3x + 4y = 1$

b) $x - 2y = 10$

$x + 5y = -4$

c) $x + y = -5$

$x + 3y = -15$

d) $4x + y = -5$

$2x + 3y = 5$

Example:

Solve by Elimination.

$$2x + 7y = 13$$
$$3x - 7y = 2$$

Make sure that the coeficients in front of x or y are the same. Then add or subtract to eliminate that variable. So in this case if we add the two equations we will eliminate y.

$$\begin{array}{c} 2x + 7y = 13 \\ 3x - 7y = 2 \\ \hline 5x = 15 \\ x = 3 \end{array}$$

Now substitute into one of the original equations to find y.

$$2x + 7y = 13$$
$$2(3) + 7y = 13$$
$$6 + 7y = 13$$
$$7y = 7$$
$$y = 1$$

The solution is (3,1)

Example:

Solve by elimination.

$$2x + 5y = 16$$
$$x - y = 1$$

When the variables do not have equal coefficients, we need to multply an entire equation by a number to make the coefficients equal. In this question we will multiply the second equation by 2.

$$2 \times (x - y = 1)$$
$$2x - 2y = 2$$

Now rewrite the equations making sure that the variables line up vertically.

$$2x + 5y = 16$$
$$2x - 2y = 2$$

In order to eliminate the variable x, we will need to subtract the two equations.

$$\begin{array}{c} 2x + 5y = 16 \\ 2x - 2y = 2 \\ \hline 7y = 14 \\ y = 2 \end{array}$$

Now substitute $y = 2$ back into either equation to find x.

$$\mathbf{2x + 5y = 16}$$
$$\mathbf{2x + 5(2) = 16}$$
$$\mathbf{2x + 10 = 16}$$
$$\mathbf{2x = 6}$$
$$\mathbf{x = 3}$$

The solution is (3,2)

5. Solve by elimination.

a) 2x – 3y = 7
2x + y = 3

b) -3x + y = 5
4x – y = -8

c) $4x - 3y = -13$
$-2x + 2y = 8$

d) $4x - 3y = 9$
$2x - 5y = 1$

6. Solve using either substitution or elimination.

a) x + 2y = 9
x = 2y – 3

b) 2x + 3y = 11
x – 3y = -17

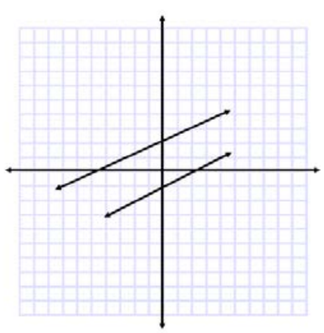
c) -2x + y = -14
4x + 3y = -2

d) 4x – y = -7
2x + 3y = -21

Example:

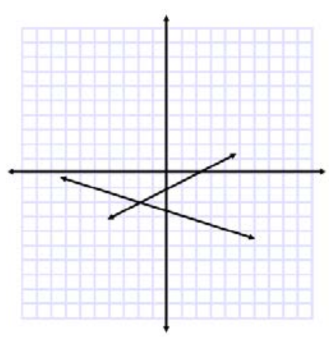
A system of linear equations can have zero, one, or an infinite number of solutions. In order to know how many solutions there are, we need to compare the slopes and y intercepts of each equation.

Zero solutions



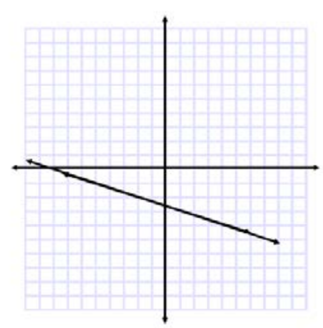
slopes equal
y intercepts different

One Solution



slopes different

Infinite solutions



slopes equal
y intercepts equal

How many solutions does the follow system have?

a) $\boldsymbol{6x - y = 1}$
$\boldsymbol{y = 6x + 1}$

b) $\boldsymbol{8x - y = 13}$
$\boldsymbol{x - 8y = 13}$

c)$\boldsymbol{5y + x - 10 = 0}$
$\boldsymbol{y = -\frac{1}{5}x + 2}$

In order to compare slopes and y intercepts, rewrite each equation in slope intercept form.

a) $\boldsymbol{6x - y = 1}$ **rewrite as** $\boldsymbol{y = 6x - 1}$ **compare to** $\boldsymbol{y = 6x + 1}$

Slopes are the same, but y intercepts are different. This means the lines are parallel and the system has no solutions.

b) $\boldsymbol{8x - y = 13}$ **rewrite as** $\boldsymbol{y = 8x - 13}$**.**

$\boldsymbol{x - 8y = 13}$ **rewrite as** $\boldsymbol{y = \frac{1}{8}x - \frac{13}{8}}$**. Slopes are different, so there is one solution.**

c) $\boldsymbol{5y + x - 10 = 0}$ **rewrite as** $\boldsymbol{y = -\frac{1}{5}x + 2}$**. Compare to** $\boldsymbol{y = -\frac{1}{5}x + 2}$**. Slopes and y intercept are the same, so there are an infinite number of solutions.**

7. Determine whether the system has zero, one or an infinite number of solutions.

a) $x = 2y - 5$
$\mathrm{y} = \frac{1}{2}\mathrm{x} + \frac{5}{2}$

b) $6x - y = 5$
$\mathrm{y} = 6\mathrm{x} + 7$

c) $2x - 5y = 10$
$3\mathrm{x} - 4\mathrm{y} = 24$