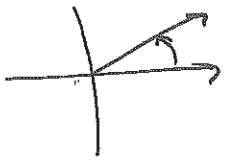
Chapter 2a – Trigonometry - Outcome 20.4

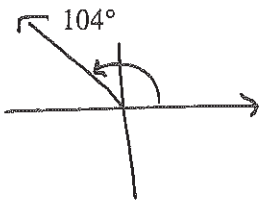
Level 2

1. Sketch an angle in standard position with each given measure.

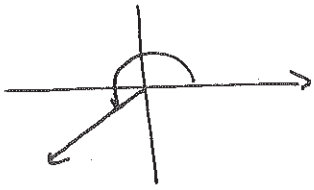
a) 24°



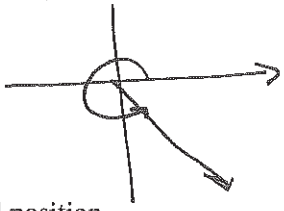
b) 104°



c) 204°



d) 304°



2. State the reference angle for each angle in standard position.

a) 55° b) 155° c) 255° d) 355°

55° 25° 75° 5°

3. Determine the measure of the three other angles in standard position, $0° < \theta < 360°$, that have a reference angle of

a) 40°

140°
220°
320°

b) 72°

108°
252°
288°

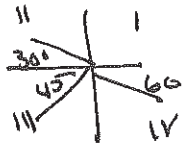
c) 88°

92°
268°
272°

d) 3°

177°
183°
357°

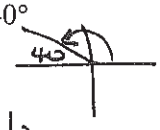
4. Complete the table. Determine the measure of each angle in standard position given its reference angle and the quadrant in which the terminal arm lies.



	Reference Angle	Quadrant	Angle in Standard Position
a)	30°	II	150°
b)	45°	III	225°
c)	60°	IV	300°

5. Determine if the pair of angles have the same reference angle.

a) 50°, 140°



No

b) 200°, 290°

$\theta_R = 20°$ $\theta_R = 70°$

c) 216°, 324°

-180° 360

36° -324

36°

Yes!

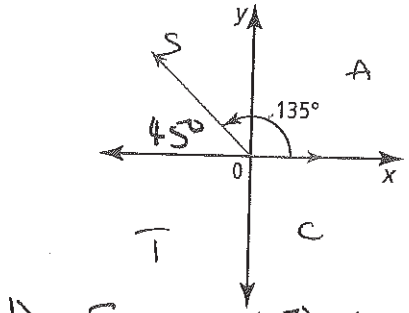
d) 91°, 181°

$\theta_R = 89^\circ$ $\theta_R = 1^\circ$

No

Level 3

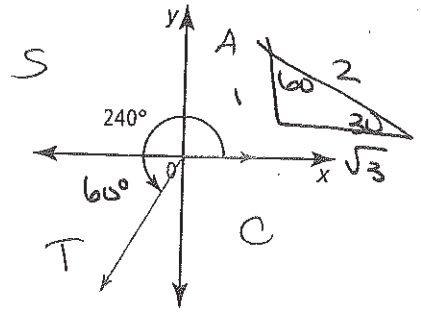
6. Determine the exact values of the sine, cosine, and tangent ratios for each angle.

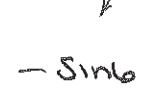


$+\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$-\cos 45^\circ = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$-\tan 45^\circ = \frac{1}{1} = 1$

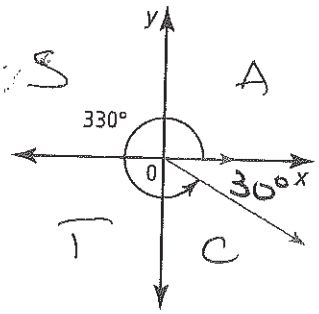




$-\sin 60^\circ = \frac{-\sqrt{3}}{2}$

$-\cos 60^\circ = \frac{-1}{2}$

$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$



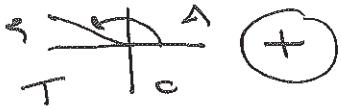
$-\sin 30^\circ = \frac{-1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

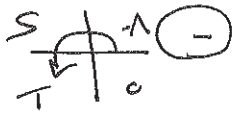
$-\tan 30^\circ = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

7. Without using a calculator, state whether each ratio is positive or negative.

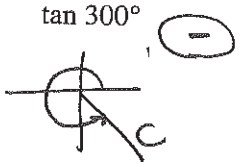
a) sin 100°



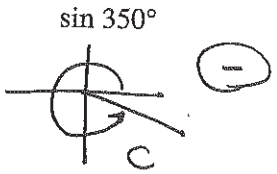
b) cos 200°



c) tan 300°

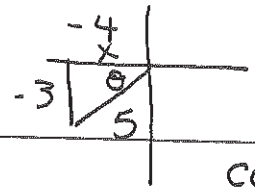


d) sin 350°



8. An angle is in standard position with its terminal arm in the stated quadrant. Determine the exact values for the other two primary trigonometric ratios for each.

a) $\sin\theta = \frac{-3}{5}$; quadrant III



$x^2 + 9 = 25$

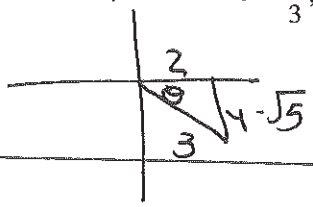
$x^2 = 16$

$x = \pm 4$

$\cos\theta = \frac{-4}{5}$

$\tan\theta = \frac{3}{4}$

b) $\cos\theta = \frac{2}{3}$; quadrant IV



$y^2 + 2^2 = 3^2$

$y + 4 = 9$

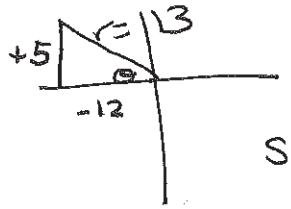
$y = 5$

$y = \pm\sqrt{5}$

$\sin\theta = \frac{-\sqrt{5}}{3}$

$\tan\theta = \frac{-\sqrt{5}}{2}$

c) $\tan\theta = \frac{-5}{12}$; quadrant II



$25 + 144 = r^2$

$169 = r^2$

$13 = r$

$\sin\theta = \frac{+5}{13}$ $\cos\theta = \frac{-12}{13}$

9. Solve each equation, for $0° \le \theta < 360°$. Use a diagram involving a special right triangle.

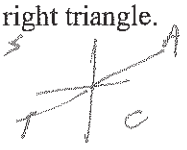
a) $\sin\theta = \frac{-1}{\sqrt{2}}$

$\theta_r = 45$

$\{225, 315°\}$

b) $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta_r = 30$



$\{30, 210\}$

c) $\cos\theta = \frac{\sqrt{3}}{2}$ $\theta_r = 30$

$\{30, 330\}$

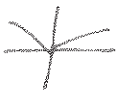
d) $\sin\theta = -1$ $\sin\theta = \frac{y}{r}$

$\{270\}$

10. Solve each equation, for $0° \le \theta < 360°$.

a) $\sin\theta = 0.7760$

$\theta_r = 50.9$



$\{50.9, 129.1\}$

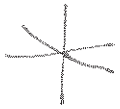
b) $\cos\theta = -0.8090$

$\theta_r = 36$

$\{144, 216\}$

c) $\tan\theta = -0.9004$

$\theta_r = 42$



$\{138, 318\}$

d) $\sin\theta = -0.9848$

$\theta_r = 80$

$\{260, 280\}$

11. Determine the exact values for sine, cosine and tangent for each angle below. Show your work

a) $225°$

$\sin 225 = -\frac{\sqrt{2}}{2}$

$\cos 225 = -\frac{\sqrt{2}}{2}$

$\tan 225 = 1$

b) $150°$

$\sin 150 = \frac{1}{2}$

$\cos 150 = -\frac{\sqrt{3}}{2}$

$\tan 30 = -\frac{\sqrt{3}}{3}$

c) $300°$

$\sin 300 = -\frac{\sqrt{3}}{2}$

$\cos 300 = \frac{1}{2}$

$\tan 300 = -\sqrt{3}$

d) $30°$

$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

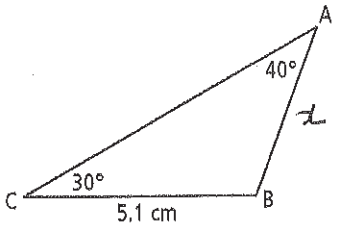
$\tan 30 = \frac{\sqrt{3}}{3}$

Chapter 2b – Sine and Cosine Law – Outcome 20.5

Level 2

1 .Determine the length of AB in each triangle.

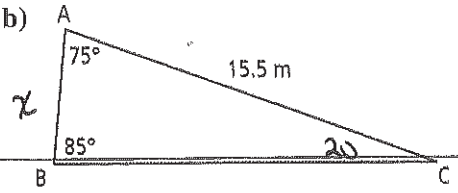
a)



$$\frac{x}{\sin 30^\circ} = \frac{5.1}{\sin 40}$$

$$x = 3.97 \text{ cm}$$

b)

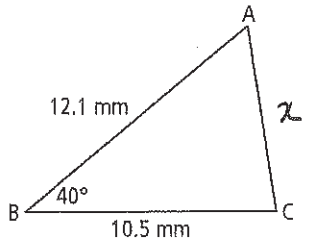


$$\frac{x}{\sin 20^\circ} = \frac{15.5}{\sin 85}$$

$$x = 5.32 \text{ m}$$

2. Determine the length of the unknown side of each triangle.

a)

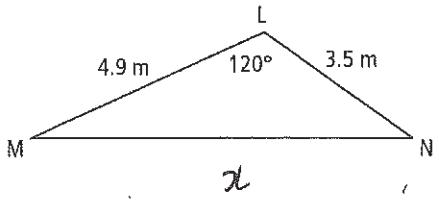


$$x^2 = 12.1^2 + 10.5^2 - 2(12.1)(10.5)\cos 40^\circ$$

$$x^2 = 62.00\ldots$$

$$x = 7.87 \text{ mm}$$

b)



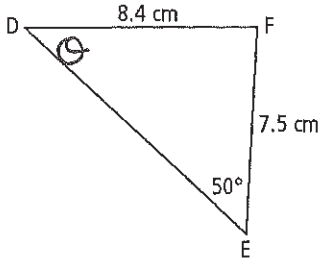
$$x^2 = 4.9^2 + 3.5^2 - 2(4.9)(3.5)\cos 120^\circ$$

$$x^2 = 53.41$$

$$x = 7.31 \text{ m}$$

3. Determine the measure of the indicated angle.

a) ∠D

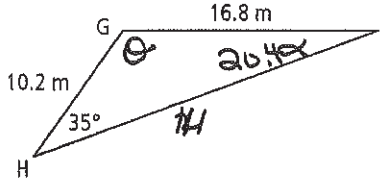


$$\frac{\sin\theta}{7.5} = \frac{\sin 50}{8.4}$$

$$\sin\theta = 0.6839\ldots$$

$$\theta = 43.2^\circ$$

b) ∠G



Find ∠I first (α)

$$\frac{\sin\alpha}{10.2} = \frac{\sin 35}{16.8}$$

$$\sin\alpha = 0.3482$$

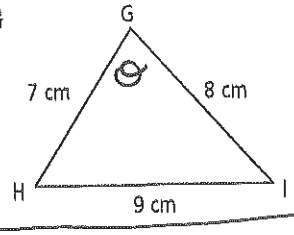
$$\alpha = 20.4^\circ$$

$$\theta = 180^\circ - 20.4^\circ - 35^\circ$$

$$\theta = 124.6^\circ$$

4. Determine the measure of the indicated angle.

a) ∠G



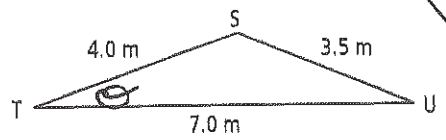
$9^2 = 7^2 + 8^2 - 2(7)(8)\cos\theta$

$-32 = -112\cos\theta$

$0.285\ldots = \cos\theta$

$\theta = 73.4°$

b) ∠T



$3.5^2 = 4^2 + 7^2 - 2(4)(7)\cos\theta$

$-52.75 = -56\cos\theta$

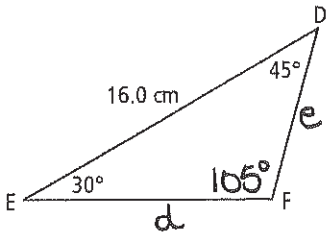
$0.9419\ldots = \cos\theta$

$19.6° = \theta$

Level 3

5. Solve each triangle by determining the unknown sides and angles.

a)



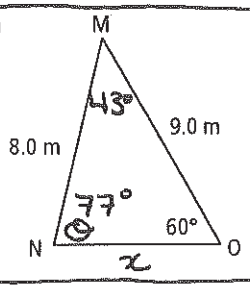
$\frac{e}{\sin 30°} = \frac{16.0}{\sin 105}$

$e = 8.3\text{cm}$

$\frac{d}{\sin 45°} = \frac{16}{\sin 105}$

$d = 11.7\text{ cm}$

b)



$\frac{\sin\theta}{9} = \frac{\sin 60}{8}$

$\theta = 77°$

$\angle N = 77°$

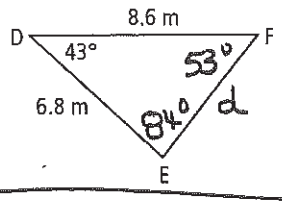
$\angle M = 43°$

$x^2 = 8^2 + 9^2 - 2(8)(9)\cos 43$

$x^2 = 39.68\ldots$

$m = x = 6.3$

c)



$d^2 = 6.8^2 + 8.6^2 - 2(6.8)(8.6)\cos 43$

$d^2 = 34.6\ldots$

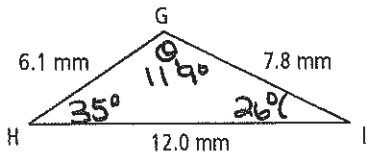
$d = 5.9\text{m}$

$\frac{\sin E}{8.6} = \frac{\sin 43}{5.9}$

$E = 84°$

$\angle F = 53°$

d)



$12^2 = 6.1^2 + 7.8^2 - 2(6.1)(7.8)\cos G$

$45.95 = -95.16$

$-0.4828\ldots = \cos G$

$G = 119°$

$\frac{\sin I}{6.1} = \frac{\sin 119}{12}$

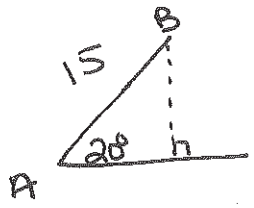
$\angle I = 26°$

$\angle H = 35°$

6. Sketch each triangle. Then, determine the unknown side and angles. If two solutions are possible, give both.

a) In $\triangle ABC$, AB = 15 m, BC = 5 m, and $\angle A = 20°$. SSA

$h = 15\sin 20 = 5.1$

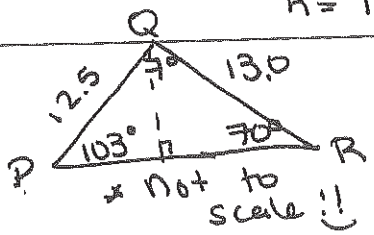


No triangle formed.

b) In $\triangle PQR$, PQ = 12.5 cm, QR = 13.0 cm, and $\angle P = 103°$.

$h = 12.5\sin 103 = 12.17$

1 triangle



$\frac{\sin R}{12.5} = \frac{\sin 103}{13.0}$

$R = 70°$

$x^2 = 12.5^2 + 13^2 - 2(12.5)(13)\cos 37°$

$x^2 = 2.67$

$x = 1.63$ cm

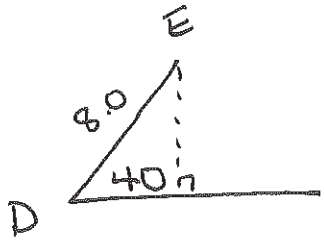
c) In $\triangle DEF$, DE = 8.0 cm, EF = 6.0 cm, and $\angle D = 40°$.

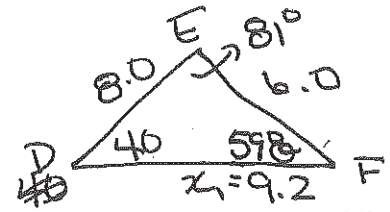
$\frac{\sin F}{8} = \frac{\sin 40}{6}$

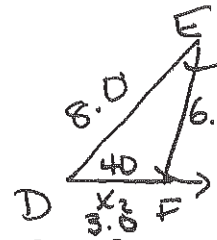
$F = 59°$

$h = 8.0\sin 40 = 5.14$

2 triangles







$180 - 59 = 121°$

$x_1^2 = 8^2 + 6^2 - 2(8)(6)\cos 81$

$x_1^2 = 84.96...$

$x_1 = 9.2$

$x_2^2 = 8^2 + 6^2 - 2(8)(6)\cos 19$

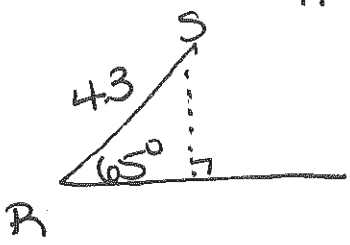
$x_2^2 = 9.2...$

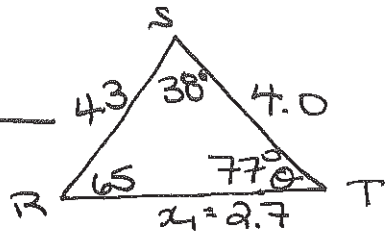
$x_2 = 3.0$

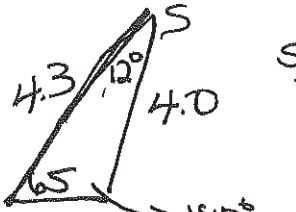
d) In $\triangle RST$, RS = 4.3 mm, ST = 4.0 mm, and $\angle R = 65°$.

$h = 4.3\sin 65 = 3.9$

2 triangles







$\frac{\sin T}{4.3} = \frac{\sin 65}{4}$

$T = 77°$

$x_1^2 = 4.3^2 + 4.0^2 - 2(4.3)(4.0)\cos 38$

$x_1 = 2.7$

$x_2^2 = 4.3^2 + 4^2 - 2(4.3)(4)\cos 12$

$x_2 = 0.9$

AK

Final Exam Review - Outcomes 20.7a, 20.7b,20.8b, 20.9b

Chapters 3a and 3b (Quadratic functions in Vertex and Standard Form) Chapter 4b (Solving Quadratic Equations) and Chapter 9b (Solving Quadratic Inequalities in One Variable)

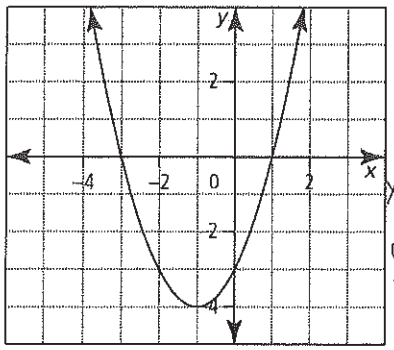
Chapter 3a – Outcome 20.7a

Level 2

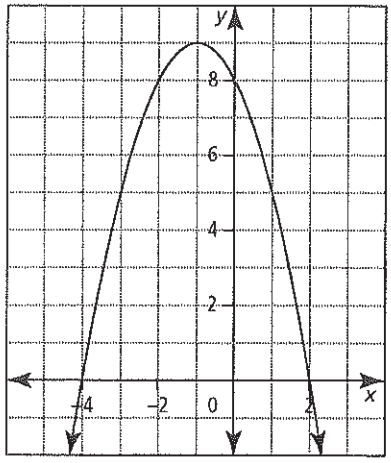
1. Complete the following table

Function	Vertex	Width (normal, narrow or wide)	Direction (Concave up or Concave down)	Axis of Symmetry
$y = -(x+4)^2 + 5$	(-4, 5)	norm	down	$x = -4$
$y = \frac{2}{3}(x-8)^2 + 4$	(8, 4)	~~nar~~ wide	up	$x = 8$
$y = 3(x-2)^2$	(2, 0)	narrow	up	$x = 2$
$y = -\frac{5}{9}(x+6)^2$	(-6, 0)	wide	down	$x = -6$
$y = -2.5(x)^2 - 3$	(0, -3)	narrow	down	$x = 0$

2. For each graph, identify the following:
   - the coordinates of the vertex
   - the equation of the axis of symmetry
   - the x-intercepts and y-intercept
   - the direction of opening
   - the maximum or minimum value
   - the domain and range



V (-1, -4)
x = -1
x: (-3, 0) (1, 0)
y: (0, -3)
up
min @ -4
D: (-∞, ∞)
R: [-4, ∞)



(-1, 9)
x = -1
x: (-4, 0) (2, 0)
y: (0, 8)
down
max @ 9
D: (-∞, ∞)
R: (-∞, 9]

Level 3

3. Determine a quadratic function in vertex form that has the given characteristics.

a) its vertex at (–2, 3) and passes through the point (–1, 1)

$y = a(x+2)^2 + 3$

$1 = a(-1+2)^2 + 3$

$-2 = a(1)^2$

$a = -2$

$y = -2(x+2)^2 + 3$

b) its vertex at (3, –2) and has an x-intercept of (5, 0)

$y = a(x-3)^2 + 2$

$0 = a(5-3)^2 + 2$

$-2 = a(2)^2$

$-2 = 4a$

$a = -\frac{2}{4} = -\frac{1}{2}$

$y = -\frac{1}{2}(x-3)^2 + 2$

c) its vertex at (4, 1) and has a y-intercept of (0, –15)

$y = a(x-4)^2 + 1$

$-15 = a(0-4)^2 + 1$

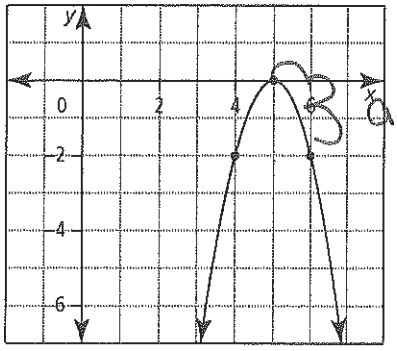
$-16 = a(16)$

$a = -1$

$y = -(x-4)^2 + 1$

4. Determine a quadratic function in vertex form for each parabola.

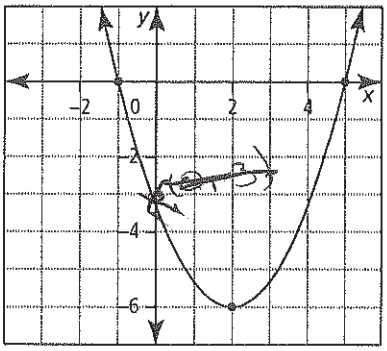
a)



$a = -2$

~~$y = a$~~

$y = -2(x-5)^2$



(5,0)

$y = a(x-2)^2 - 6$

$0 = a(5-2)^2 - 6$

$6 = a(3)^2$

$6 = 9a$

$a = \frac{6}{9} = \frac{2}{3}$

$y = \frac{2}{3}(x-2)^2 - 6$

5. For each function below, state the domain, range, vertex, direction of opening, max or min value, number of x-intercepts and the width (narrow, normal or wide) Then sketch the graph.

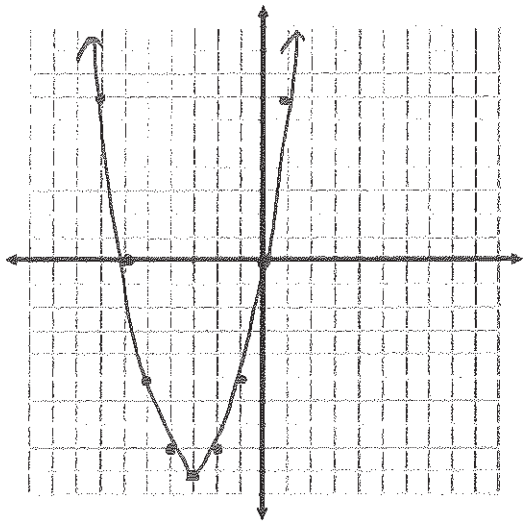
a) $y = (x + 3)^2 - 9$.

V(-3, -9)

a = 1

1, 3, 5

D: (-∞, ∞)
R: [-9, ∞)
up
min @ -9
2 x ints
normal

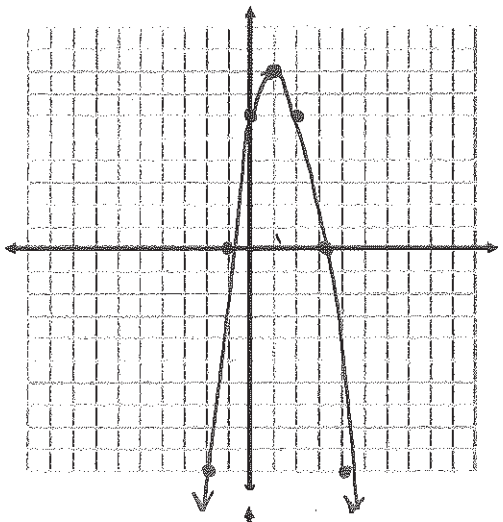


b) $y = -2(x - 1)^2 + 8$. Sketch the graph.

V(1, 8)

-2, -6, -10

D: (-∞, ∞)
R: (-∞, 8]
down
max @ 8
2 x ints
narrow

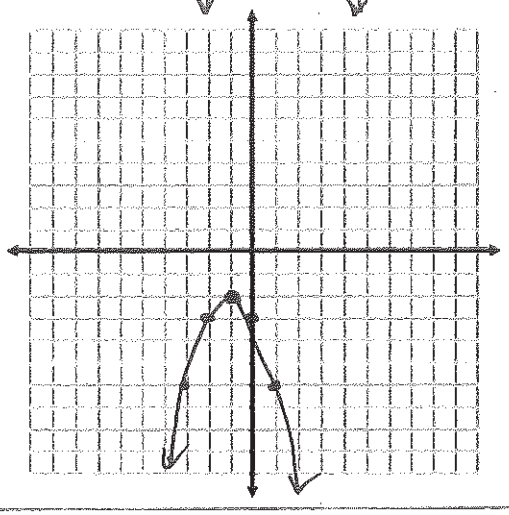


c) $y = -(x + 1)^2 - 2$.

V(-1, -2)

-1, -3, -5

D: (-∞, ∞)
R: (-∞, -2]
down
max @ -2
0 x ints
normal.



d) $y = \frac{2}{3}(x-2)^2 - 4$. .

D: $(-\infty, \infty)$

R: $[2, \infty)$

V(2, -4)

$\frac{a}{3}$, $\frac{2a}{3}$, $\frac{5a}{3}$

2, $\frac{10}{3}$

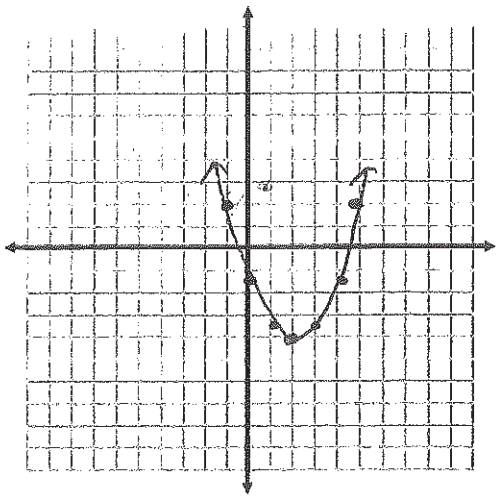
$(\frac{8}{3}, -\frac{1}{3})$

up

min @ -4

2 x ints

wide



Chapter 3b – Outcome 20.7b – Graphing in standard form

Level 2

1. 1. Determine the characteristics of each quadratic function below. State the vertex, domain, range, axis of symmetry, number of x-intercepts, the y-intercept point, and write the equation in vertex form. Then sketch the function.

a) $y = x^2 - 6x + 10$

$y = x^2 - 6x + 9 - 9 + 10$

$= (x-3)^2 + 1$

$a = 1$

1, 3, 5

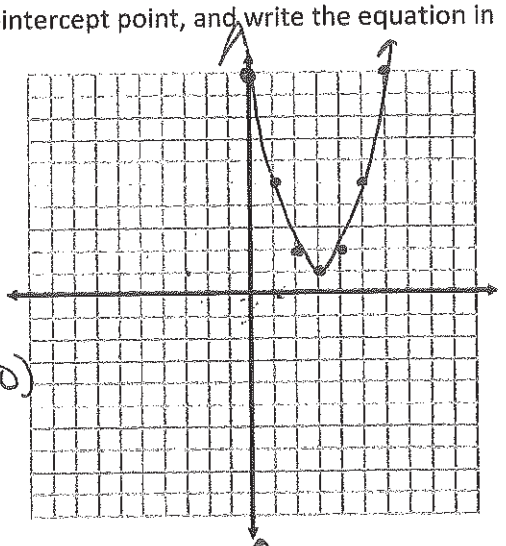
V(3, 1)

D: $(-\infty, \infty)$

R: $[1, \infty)$

x = none

y int = (0, 10)



b) $y = x^2 + 8x + 6$

$y = x^2 + 8x + 16 - 16 + 6$

$= (x+4)^2 - 10$

$a = 1$

1, 3, 5, 7

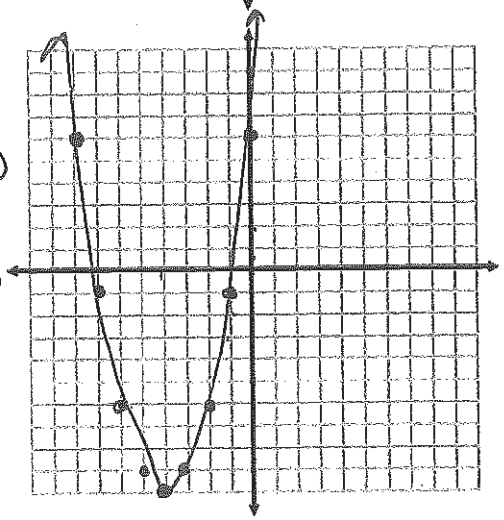
V(-4, -10)

D: $(-\infty, \infty)$

R: $[-10, \infty)$

2 x ints

y int (0, 6)



1a 3a 5a

$a = 3$

3, 9

c) $y = 3x^2 - 12x + 9$

$y = 3(x^2 - 4x + 4 - 4) + 9$

$y = 3(x-2)^2 - 12 + 9$

$y = 3(x-2)^2 - 3$

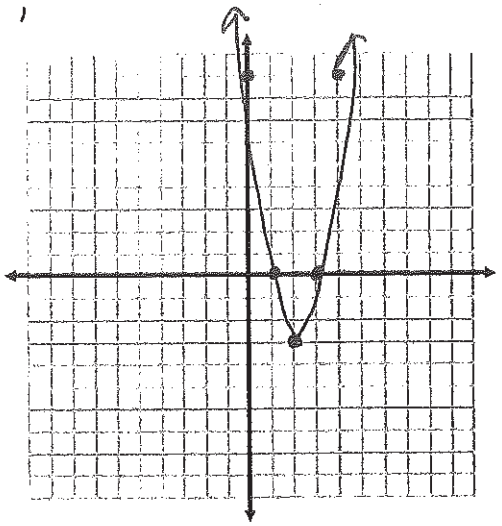
V(2, -3)

D: $(-\infty, \infty)$

R: $[-3, \infty)$

2 x ints

y int (0, 9)



Level 3

Write each function in vertex form by completing the square. State the vertex of the funciton

a) $y = x^2 + 2x - 4$

$y = x^2 + 2x + 1 - 1 - 4$

$y = (x+1)^2 - 5$

$V(-1, -5)$

b) $y = x^2 + 24x + 54$

$y = x^2 + 24x + 144 - 144 + 54$

$y = (x+12)^2 - 90$

$V(-12, -90)$

c) $y = -2x^2 - 20x - 56$

$y = -2(x^2 + 10x + 25 - 25) - 56$

$y = -2(x+5)^2 + 50 - 56$

$y = -2(x+5)^2 - 6$

$V(-5, -6)$

d) $y = 6x^2 - 48x$

$y = 6(x^2 - 8x + 16 - 16)$

$y = 6(x-4)^2 - 96$

$V(4, -96)$

e) $y = 2x^2 - 16x + 31$

$y = 2(x^2 - 8x + 16 - 16) + 31$

$y = 2(x-4)^2 - 32 + 31$

$y = 2(x-4)^2 - 1$

$V(4, -1)$

Chapter 4a – Outcome 20.6 – Factoring

Level 2

1. Factor the following

a) $x^2 - x - 20$

$(x-5)(x+4)$

b) $3x^2 - 30x + 63$

$3(x^2-10x+21)$

$3(x-7)(x-3)$

c) $-4x^2 - 12x - 8$

$-4(x^2+3x+2)$

$-4(x+2)(x+1)$

d) $14x^2 + 3x - 5$

$(2x-1)(7x+5)$

e) $3x^2 + 11x - 20$

$(3x-4)(x+5)$

f) $4x^2 + 7xy + 3y^2$

$(x+y)(4x+3y)$

g) $12x^2 - 4xy - 8y^2$

$4(3x^2-xy-2y^2)$

$4(3x-2y)(x+y)$

h) $x^2 - 49y^2$

$(x+7y)(x-7y)$

i) $25x^2 - 9$

$(5x+3)(5x-3)$

j) $x^2 - \frac{25}{4}y^2$

$(x+\frac{5}{2}y)(x-\frac{5}{2}y)$

Level 3

2. Factor the following

a) $(x+1)^2 - (x-7)^2$

$a^2 - b^2$

$(a+b)(a-b)$

$(x+1+x-7)(x+1-(x-7))$

$(2x-6)(8)$

$16(x-3)$

b) $(x-1)^2 - 2(x-1) - 35$

$a^2 - 2a - 35$

$(a-7)(a+5)$

$(x-1-7)(x-1+5)$

$(x-8)(x+4)$

c) $6(2x+1)^2 - 7(2x+1) - 20$

$6a^2 - 7a - 20$

$(2a-5)(3a+4)$

$(2(2x+1)-5)(3(2x+1)+4)$

$(4x-3)(6x+7)$

d) $(x^2-2x)^2 - 11(x^2-2x) + 24$

$a^2 - 11a + 24$

$(a-3)(a-8)$

$(x^2-2x-3)(x^2-2x-8)$

$(x-3)(x+1)(x-4)(x+2)$

e) $(2x^2+2x-4)^2 - (x^2+2x+5)^2$

$a^2 - b^2$

$(a+b)(a-b)$

$(2x^2+2x-4+x^2+2x+5)(2x^2+2x-4-(x^2+2x+5))$

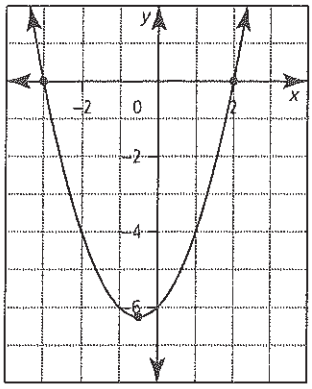
$(3x^2+4x+1)(x^2-9)$

$(3x+1)(x+1)(x+3)(x-3)$

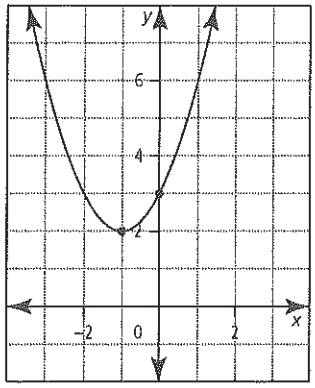
Chapter 4b – Outcome 20.8b – Solving quadratic equation

Level 2

1. Determine the solutions to each equation graphically. * Find x-ints



$\{-3, 2\}$



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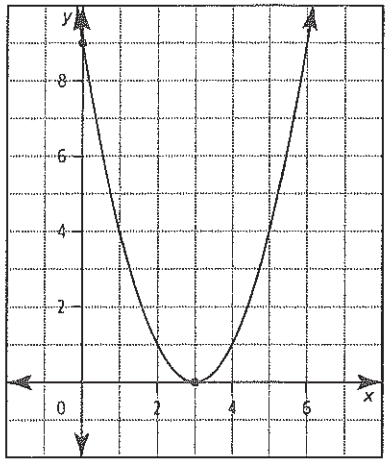


$\{3\}$

2. Solve each of the following algebraically. * make = 0

a. $x^2 - 2x - 15 = 0$

$(x-5)(x+3)=0$

$x=5 \quad x=-3$

$\{-3, 5\}$

* Try to factor

e) $7x^2 = 34x + 5$

$7x^2 - 34x - 5 = 0$

$(7x+1)(x-5)=0$

$x = -\frac{1}{7} \quad x=5$

$\{-\frac{1}{7}, 5\}$

b. $2x^2 + 8x = 64$

$2x^2 + 8x - 64 = 0$

$2(x^2 + 4x - 32) = 0$

$2(x+8)(x-4)=0$

$x=-8 \quad x=4$

$\{4, -8\}$

f) $5x^2 = 9x + 2$

$5x^2 - 9x - 2 = 0$

$(5x+1)(x-2)=0$

$x=-\frac{1}{5} \quad x=2$

$\{-\frac{1}{5}, 2\}$

c. $7x^2 - 28 = 0$

$7(x^2 - 4) = 0$

$(x+2)(x-2)=0$

$x=-2 \quad x=2$

$\{-2, 2\}$

g) $2x^2 + 9x = 18$

$2x^2 + 9x - 18 = 0$

$(2x-3)(x+6)=0$

$x=\frac{3}{2} \quad x=-6$

$\{-6, 3/2\}$

d. $6x^2 - 5x = 4$

$6x^2 - 5x - 4 = 0$

$(3x-4)(2x+1)=0$

$x=\frac{4}{3} \quad x=-\frac{1}{2}$

$\{-\frac{1}{2}, \frac{4}{3}\}$

Level 3

* make = 0
factor or quad form or comp. □

3. Solve the following equations algebraically.

a) $x^2 + 2x - 2 = 0$

$x^2+2x+1-1-2=0$

$(x+1)^2-3=0$

$\sqrt{(x+1)^2}=\pm\sqrt{3}$

$x+1=\pm\sqrt{3}$

$x=-1\pm\sqrt{3}$

$\{-1\pm\sqrt{3}\}$

b) $x^2 - 5x + 3 = 0$

$x=\frac{5\pm\sqrt{25-12}}{2}$

$x=\left\{\frac{5\pm\sqrt{13}}{2}\right\}$

c) $4x^2 + x - 3 = 0$

$x=\frac{-1\pm\sqrt{(1)^2-4(4)(-3)}}{2(4)}$

$x=\frac{-1\pm\sqrt{49}}{8}$

$x=\frac{-1\pm7}{8}$

$x=\frac{6}{8}$ $x=\frac{-8}{8}$

$\frac{3}{4}$ $x=-1$

$\{-1,\frac{3}{4}\}$

d) $2x^2 - 16x + 10 = 0$

$2(x^2-8x+16-16)+10=0$

$2(x-4)^2-32+10=0$

$2(x-4)^2=22$

$(x-4)^2=11$

$x-4=\pm\sqrt{11}$

$x=4\pm\sqrt{11}$

$\{4\pm\sqrt{11}\}$

e) $10x^2 - 15x = 0$

$5x(2x-3)=0$

$x=0$ $x=3/2$

$\{0,3/2\}$

f) $x^2 - 10x + 23 = 0$

$x^2-10x+25-25+23=0$

$(x-5)^2-2=0$

$(x-5)^2=2$

$x-5=\pm\sqrt{2}$

$x=5\pm\sqrt{2}$

$\{5\pm\sqrt{2}\}$

4. Use the discriminant to determine the number of solutions for each equation below.

a) $7x^2 + x - 1 = 0$

dis: b^2-4ac

$1^2-4(7)(-1)$

$1+28$

29

2 roots

b) $3x^2 - 4x + 5 = 0$

$16-60$

-44

no roots

c) $8y^2 - 8y + 2 = 0$

$(-8)^2-4(8)(2)$

0

one root.

d) $3x^2 + 6 = 0$

$0^2-4(3)(6)$

-72

no roots

Outcomes 20.7a, 20.7b and 20.8b

Accurately graph the following quadratic functions. Please include the roots, vertex and y-intercept on your graph

a) $f(x) = (x+7)^2 - 3$

Root $0 = (x+7)^2 - 3$

$3 = (x+7)^2$

$\pm\sqrt{3} = x+7$

$-7 \pm \sqrt{3} = x$

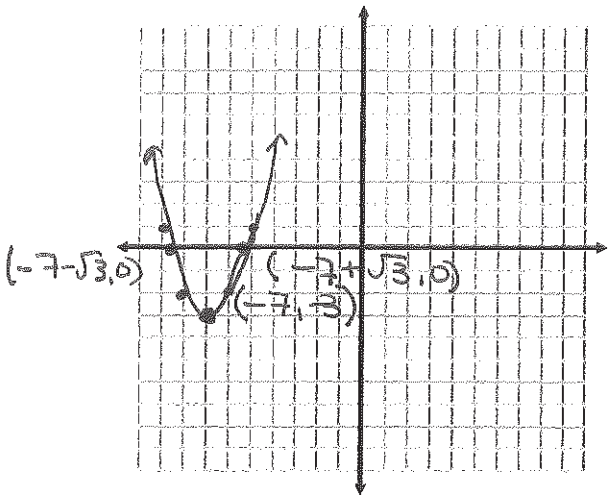
V(-7, -3)

$f(0) = (7)^2 - 3$

$= 49 - 3$

$= 46$

y int (0, 46)



b) $f(x) = 4(x+2)^2 - 1$

Roots

$0 = 4(x+2)^2 - 1$

$1 = 4(x+2)^2$

$\pm\frac{1}{4} = \sqrt{(x+2)^2}$

V(-2, -1)

$f(0) = 4(2)^2 - 1$

$= 16 - 1$

$= 15$

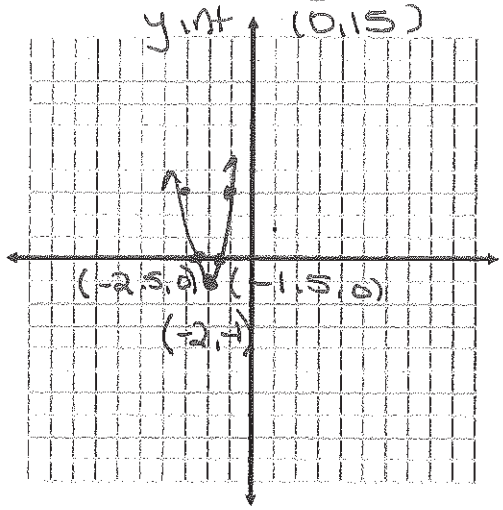
$\pm\frac{1}{2} = x+2$

$x = -2 \pm \frac{1}{2}$

$x = -2.5$

$x = -1.5$

y int (0, 15)



c) $y = x^2 - 8x + 7$

$y = (x-1)(x-7)$

$x = 1 \quad x = 7$

(1, 0) (7, 0)

$f(4) = 4^2 - 8(4) + 7$

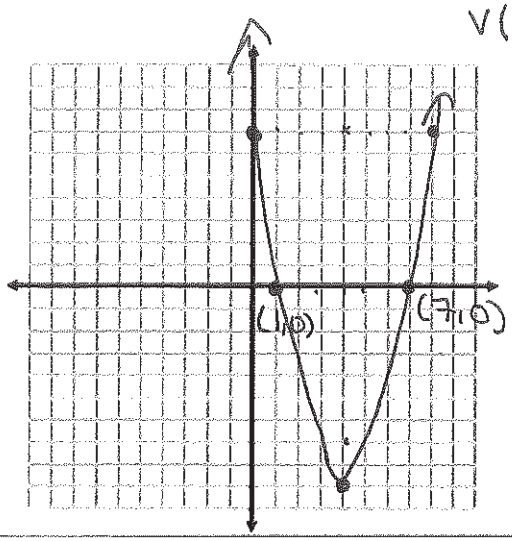
$= 16 - 32 + 7$

$= -16 + 7$

$= -9$

V(4, -9)

y



d) $y = x^2 - 4x$

$y = x(x-4)$

$x = 0 \quad x = 4$

(0, 0) (4, 0)

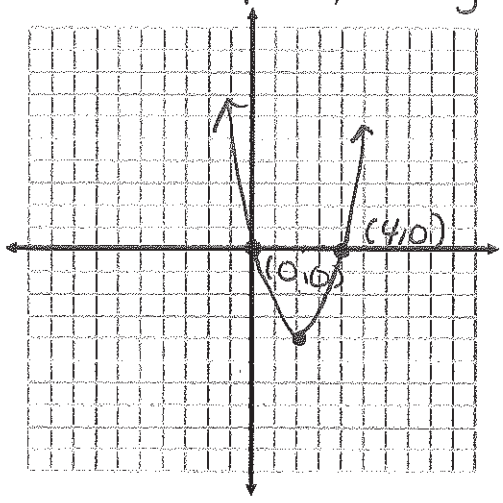
V = (2, -4)

$f(2) = 2^2 - 4(2)$

$= 4 - 8$

$= -4$

y int also (0, 0)



e) $y = x^2 + 2x - 4$ Doesn't factor

$y = x^2 + 2x + 1 - 1 - 4$

$y = (x+1)^2 - 5$

$V(-1, -5)$

Roots

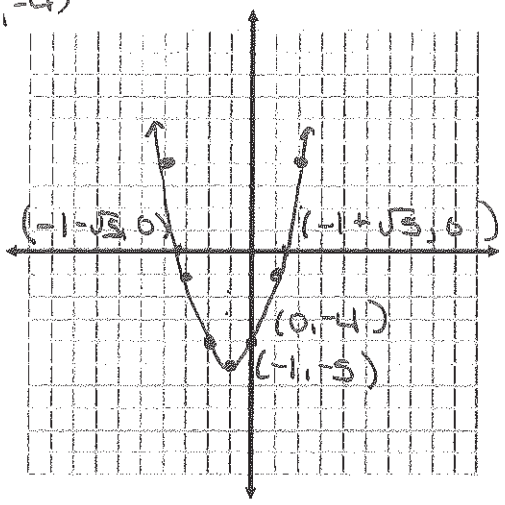
$0 = (x+1)^2 - 5$

$5 = (x+1)^2$

$\pm\sqrt{5} = x+1$

$-1 \pm \sqrt{5} = x$

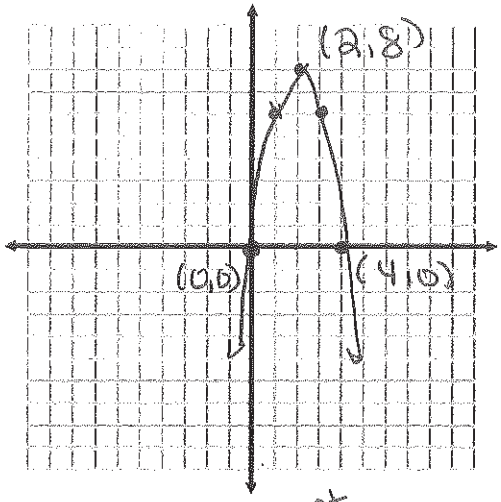
y int $(0, -4)$



f) $f(x) = -2(x-2)^2 + 8$ 2, 6, 10

$V(2, 8)$

y-int also $(0, 0)$



g) $y = 3x^2 - 12x + 9$ y int $(0, 9)$

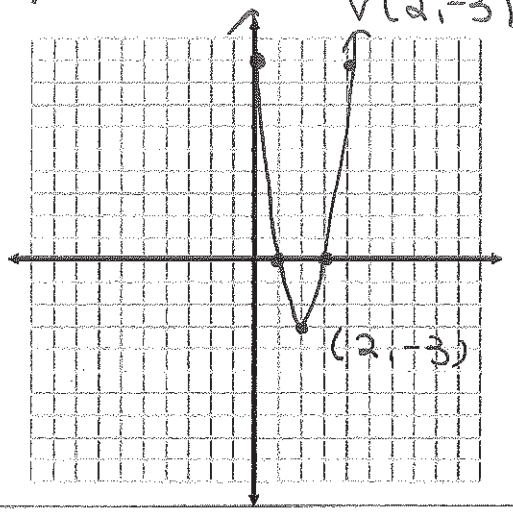
$y = 3(x^2 - 4x + 3)$

$y = 3(x-1)(x-3)$

$x = 1 \quad x = 3$

$f(2) = 3(1)(-1) = -3$

$V(2, -3)$



h) $y = \frac{1}{2}x^2 - 6x + 8$ y int

$\frac{1}{2}(x^2 - 12x + 16)$

$\frac{1}{2}(x -$

$y = \frac{1}{2}(x^2 - 12x + 36 - 36) + 8$

$y = \frac{1}{2}(x-6)^2 - 18 + 8$

$y = \frac{1}{2}(x-6)^2 - 10$ $V(+6, -10)$

Roots

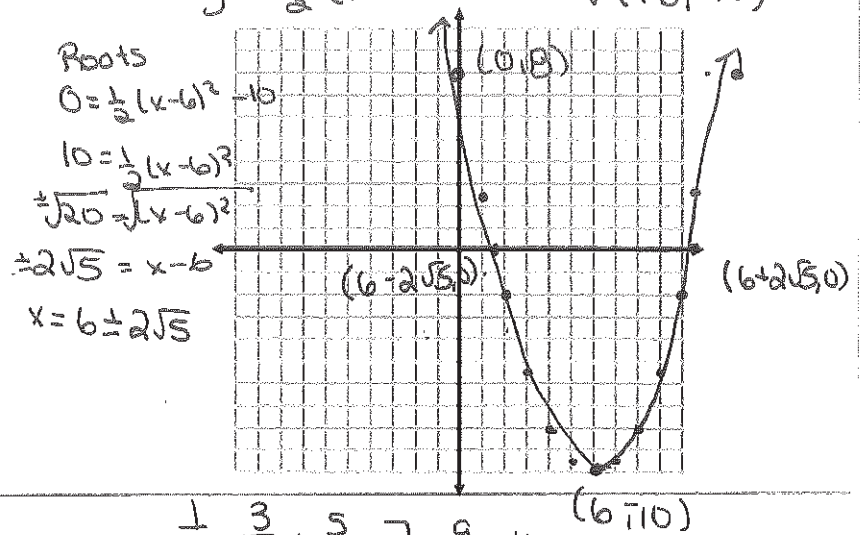
$0 = \frac{1}{2}(x-6)^2 - 10$

$10 = \frac{1}{2}(x-6)^2$

$\pm\sqrt{20} = \sqrt{(x-6)^2}$

$\pm 2\sqrt{5} = x - 6$

$x = 6 \pm 2\sqrt{5}$



$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$

0.5 1.5 2.5 3.5 4.5 5.5

Chapter 5a – Outcome 20.2a – Radicals

1. Express each radical as a simplified mixed radical.

a) $\sqrt{54}$ $\sqrt{9}\cdot\sqrt{6}$

$3\sqrt{6}$

b) $\sqrt{350}$

35 10

7 5 5 2

$5\sqrt{14}$

c) $\sqrt{98x^2}, x \geq 0$

$\sqrt{49}\cdot\sqrt{2}\cdot\sqrt{x^2}$

$7x\sqrt{2}$

d) $\sqrt{363x^5y^3}, x \geq 0, y \geq 0$

11 33

11 3

$11x^2y\sqrt{3xy}$

2. Express each mixed radical as an equivalent entire radical.

a) $4\sqrt{5}$

$\sqrt{4^2\cdot 5}$

$\sqrt{80}$

b) $23\sqrt{13}$

$\sqrt{23^2\cdot 13}$

$\sqrt{6877}$

c) $9x^2\sqrt{x}, x \geq 0$

$\sqrt{9^2(x^2)^2 x}$

$\sqrt{81x^5}$

d) $5xy\sqrt{7y}, x \geq 0, y \geq 0$

$\sqrt{5^2x^2y^2 7y}$

$\sqrt{175x^2y^3}$

3. Order each set of numbers from least to greatest.

a) $-2\sqrt{3}, \sqrt{50}, -\sqrt{14}, 3\sqrt{5}$

$-\sqrt{12}$ $\sqrt{45}$

$-\sqrt{14}, -2\sqrt{3}, 3\sqrt{5}, \sqrt{50}$

b) $4\sqrt{3}, \sqrt{12}, 2\sqrt{6}, \sqrt{20}$

$\sqrt{48}$ $\sqrt{24}$

$\sqrt{12}, \sqrt{20}, \sqrt{24}, 4\sqrt{3}$

4. Simplify each expression.

a) $7\sqrt{11} - 3\sqrt{11} + 8\sqrt{11}$

$12\sqrt{11}$

b) $4\sqrt{3x} - 4\sqrt{2} + \sqrt{3x} - \sqrt{2}$

$5\sqrt{3x} - 5\sqrt{2}$

c) $3\sqrt{20d} + 5\sqrt{45d}$

4·5 9·5

$6\sqrt{5d} + 15\sqrt{5d}$

$21\sqrt{5d}$

d) $\sqrt{10e} - \sqrt{90e} + 4\sqrt{40e}$

9·10 4·10

$\sqrt{10e} - 3\sqrt{10e} + 8\sqrt{10e}$

$6\sqrt{10e}$

e) $5\sqrt{3} + \sqrt{12} - \sqrt{48} + 2\sqrt{75}$

$4 \cdot 3 \quad 16 \cdot 3 \quad 25\,3$

$5\sqrt{3} + 2\sqrt{3} - 4\sqrt{3} + 10\sqrt{3}$

$13\sqrt{3}$

f) $\sqrt{63} + \sqrt{75} - 2\sqrt{28} - 3\sqrt{27}$

$9 \cdot 7 \quad 25\,3 \quad 4 \cdot 7 \quad 9\,3$

$3\sqrt{7} + 5\sqrt{3} - 4\sqrt{7} - 9\sqrt{3}$

$-\sqrt{7} \quad -4\sqrt{3}$

5. Multiply. Express each answer as a mixed radical in simplest form.

a) $(6\sqrt{5})(2\sqrt{3})$

$12\sqrt{15}$

b) $(\sqrt{27x^5})(\sqrt{3x^7})$

$\sqrt{27 \cdot 3 \cdot x^5 \cdot x^7}$

$3 \quad 9 \quad x^{12}$

$3 \quad 3$

$9x^6$

6. Simplify each expression.

a) $\sqrt{10}(2\sqrt{10} + \sqrt{5})$

$2(10) + 5\sqrt{2}$

$20 + 5\sqrt{2}$

b) $\sqrt{15}(3\sqrt{5} - \sqrt{3})$

$15\sqrt{3} - 3\sqrt{5}$

c) $2\sqrt{2}(1 + \sqrt{2})$

$2\sqrt{2} + 2(2)$

$2\sqrt{2} + 4$

7. Rationalize each denominator. Express each radical in simplest form.

a) $\frac{\sqrt{10}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{\sqrt{30}}{3}$

b) $\frac{8\sqrt{15}}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{8\sqrt{30}}{5 \cdot 10}$

$\frac{4\sqrt{30}}{5}$

c) $\frac{25}{\sqrt{75}}$ $25 \cdot 3$

$\frac{5 \cdot 25 \cdot \sqrt{3}}{15 \cdot \sqrt{3} \cdot \sqrt{3}}$

$\frac{5\sqrt{3}}{3}$

d) $\frac{5\sqrt{21}}{7\sqrt{70}} \cdot \frac{\sqrt{70}}{\sqrt{70}}$

$\frac{5\sqrt{3 \cdot 7 \cdot 7 \cdot 10}}{7(70)}$

$\frac{35\sqrt{30}}{490}$

$98 \quad 14$

e) $\frac{\sqrt{20}}{\sqrt{5}}$

$\sqrt{4}$

2

$\frac{\sqrt{30}}{14}$

8. Multiply using the distributive property. Simplify each expression.

a) $(3\sqrt{2}-5\sqrt{10})(\sqrt{6}+8\sqrt{12})$

$3(2)\sqrt{3} + 24(2)\sqrt{6} - 5(2)\sqrt{15} - 40(2)\sqrt{30}$

$6\sqrt{3} + 48\sqrt{6} - 10\sqrt{15} - 80\sqrt{30}$

b) $(3+4\sqrt{7})(5\sqrt{7}+2)$

$15\sqrt{7} + 6 + 20(7) + 8\sqrt{7}$

$23\sqrt{7} + 6 + 140$

$23\sqrt{7} + 146$

c) $(3\sqrt{15}+\sqrt{10})(2\sqrt{5}-7)$

$6(5)\sqrt{3} - 21\sqrt{15} + 2(5)\sqrt{2} - 7\sqrt{10}$

$30\sqrt{3} - 21\sqrt{15} + 10\sqrt{2} - 7\sqrt{10}$

d) $(4\sqrt{3}+3\sqrt{5})^2$

$(4\sqrt{3}+3\sqrt{5})(4\sqrt{3}+3\sqrt{5})$

$16(3) + 12\sqrt{15} + 12\sqrt{15} + 9(5)$

$48 + 24\sqrt{15} + 45$

$93 + 24\sqrt{15}$

9. Rationalize each denominator. Simplify.

a) $\frac{(\sqrt{3}-1)\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$

$\frac{3-\sqrt{3}}{3}$

b) $\frac{(6\sqrt{2}+2\sqrt{6})\cdot\sqrt{6}}{3\sqrt{6}\cdot\sqrt{6}}$

$\frac{6(2)\sqrt{3}+2(6)}{3(6)}$

$\frac{12\sqrt{3}+12}{18}$ (2)

$\frac{2\sqrt{3}+2}{3}$ or $\frac{2\sqrt{3}}{3}+\frac{2}{3}$

10. Rationalize each denominator. Simplify.

a) $\frac{4}{(\sqrt{3}+1)}\cdot\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$\frac{4\sqrt{3}-4}{3-1}$

$\frac{4\sqrt{3}-4}{2}$

$2\sqrt{3}-2$

b) $\frac{\sqrt{5}(\sqrt{5}+5)}{(\sqrt{5}-5)(\sqrt{5}+5)}$

$\frac{5+5\sqrt{5}}{5-25}$

$\frac{5+5\sqrt{5}}{-20}$

$\frac{1+\sqrt{5}}{-4} = \frac{-1-\sqrt{5}}{4}$

or

$\frac{-1}{4} - \frac{\sqrt{5}}{4}$

c) $\frac{\sqrt{15}(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})}$

$\frac{\sqrt{30}+3\sqrt{5}}{2-3}$

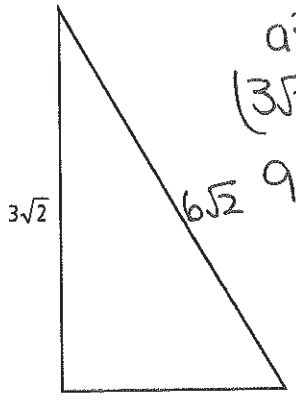
$\frac{\sqrt{30}+3\sqrt{5}}{-1} = -\sqrt{30}-3\sqrt{5}$

d) $\frac{(2\sqrt{2}-\sqrt{6})(2\sqrt{6}+\sqrt{5})}{(2\sqrt{6}-\sqrt{5})(2\sqrt{6}+\sqrt{5})}$

$\frac{4(2)\sqrt{3}+2\sqrt{10}-2(6)-\sqrt{30}}{4(6)-5}$

$\frac{8\sqrt{3}+2\sqrt{10}-12-\sqrt{30}}{19}$

11. What is the perimeter and area of the right triangle shown? State the answer as an exact value.



$a^2+b^2=c^2$

$(3\sqrt{2})^2+(3\sqrt{6})^2=c^2$

$9(2)+9(6)=c^2$

$\sqrt{72}=\sqrt{c^2}$

$9\cdot 8$

$4\cdot 2$

$3(2)\sqrt{2}$

$6\sqrt{2}$

per $= 3\sqrt{2}+3\sqrt{6}+6\sqrt{2}$

$= 9\sqrt{2}+3\sqrt{6}$

area $= \frac{1}{2}(3\sqrt{6})(3\sqrt{2})$

$\frac{9(2)\sqrt{3}}{2}$

$9\sqrt{3}$

Level 2

1. Solve for x in each equation.

a) $\sqrt{x+3}=7$

$x+3=49$ $\{46\}$

$x=46$

Ck $\sqrt{46+3}=7$

$\sqrt{49}=7$

$7=7$ ✓

b) $\sqrt{5x}=4$

$5x=16$

$x=\frac{16}{5}$ $\left\{\frac{16}{5}\right\}$

Ck $\sqrt{5\left(\frac{16}{5}\right)}=4$

$\sqrt{16}=4$ $4=4$ ✓

c) $\sqrt{-2x}=24$

$-2x=576$

$x=-288$ $\{-288\}$

Ck $\sqrt{-2(-288)}=24$

$\sqrt{576}=24$

2. Solve and verify.

a) $\sqrt{7x}+1=15$

$-1 \quad -1$

$\sqrt{7x}=14$

$7x=196$

$x=28$ $\{28\}$

Ck $\sqrt{7(28)}+1=15$

$\sqrt{196}+1=15$

$14+1=15$

$15=15$

b) $8-\sqrt{1+v}=5$

$-8 \quad -8$

$-\sqrt{1+v}=-3$ (-1)

$\sqrt{1+v}=3$

$1+v=9$ $\{8\}$

$-1 \quad -1$

$v=8$

Ck $8-\sqrt{8+1}=5$

$8-\sqrt{9}=5$

$8-3=5$

$5=5$ ✓

c) $\sqrt{3x+15}=3$

$3x+15=9$

$3x=-6$

$x=-2$ $\{-2\}$

Ck $\sqrt{3(-2)+15}=3$

$\sqrt{-6+15}=3$

$\sqrt{9}=3$

$3=3$ ✓

Level 3

3. Solve and verify.

a) $\sqrt{4-3m}=m$

$4-3m=m^2$

$0=m^2+3m-4$

$0=(m+4)(m-1)$

$m=-4 \quad m=1$

$\sqrt{4+12}=-4$ $\sqrt{4-3}=1$

$\sqrt{16}\neq -4$ $\sqrt{1}=1$ ✓

b) $\sqrt{x^2-1}=\left(2\sqrt{x+1}\right)$

$x^2-1=4(x+1)$

$x^2-1=4x+4$ $\{-1,5\}$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

$x=5 \quad x=-1$

$\sqrt{24}=2\sqrt{6}$

$\sqrt{1-1}=2\sqrt{0}$

$\sqrt{0}=2\sqrt{6}$

$0=0$ ✓

$2\sqrt{6}=2\sqrt{6}$ ✓

c) $n-\sqrt{n}=4$

$-\sqrt{n} \quad +\sqrt{n}$

$(n-4)^2=\sqrt{n}^2$

$n^2-8n+16=n$ $\left\{\frac{9+\sqrt{17}}{2}\right\}$

$n^2-9n+16=0$

$x=\frac{9\pm\sqrt{17}}{2}$

$\frac{9+\sqrt{17}}{2}\approx 6.56$

$\frac{9-\sqrt{17}}{2}\approx 2.44$

d) $\sqrt{3x^2+2}=(2x+1)$

$3x^2+2=4x^2+4x+1$

$0=x^2+4x-1$ $\{-2+\sqrt{5}\}$

$x=\frac{-4\pm\sqrt{20}}{2}$

$x=\frac{-4\pm 2\sqrt{5}}{2}$

$x=-2\pm\sqrt{5}$

$-2+\sqrt{5}\approx 0.236$

$1.47\ldots=1.47\ldots$ ✓

$-2-\sqrt{5}\approx -4.236$

$7.47\ldots\neq -7.47$

e) $\sqrt{y^2+1}-y=1$

$(\sqrt{y^2+1})^2=(1+y)^2$

$y^2+1=y^2+2y+1$

$1=2y+1$

$0=2y$

Chk. $\sqrt{1}-0=1$

$1=1$ ✓

$\{0\}$.

f) $\sqrt{3x+4}=\sqrt{x-2}$

$3x+4=x-2$

$2x=-6$

$x=-3$

Ch. $\sqrt{-9+4}=\sqrt{-3-2}$

$\sqrt{-5}=\sqrt{-5}$

Not allowed

$\varnothing$

g) $\sqrt{2p^2-3}=\sqrt{5p}$

$2p^2-3=5p$

$2p^2-5p-3=0$

$(2p+1)(p-3)$

$p=-\frac{1}{2}$ $p=3$

$\sqrt{-2\cdot-\frac{1}{2}-3}=\sqrt{-2.5}$ $\sqrt{18-3}=\sqrt{15}$

x $\sqrt{15}=\sqrt{15}$ ✓

$\{3\}$

h) $(\sqrt{w}+1)^2=(\sqrt{w+4})^2$

$w+2\sqrt{w}+1=w+4$

$(2\sqrt{w})^2=(3)^2$

$4w=9$

$w=\frac{9}{4}$

Ch. $\sqrt{\frac{9}{4}}+1=\sqrt{\frac{9}{4}+4}$

$\frac{3}{2}+1=\sqrt{\frac{25}{4}}$

$\frac{5}{2}=\frac{5}{2}$ ✓

$\{\frac{9}{4}\}$

i) $\sqrt{2x+4}-\sqrt{x}=2$ $+\sqrt{x}$ $+\sqrt{x}$

$(\sqrt{2x+4})^2=(2+\sqrt{x})^2$

$2x+4=x+4\sqrt{x}+4$

$(x)^2=(4\sqrt{x})^2$

$x^2=16x$

$x^2-16x=0$

$x=0$ $x=16$

Chk $\sqrt{4}-\sqrt{0}=2$

$2=2$ ✓

$\sqrt{36}-\sqrt{16}=2$

$6-4=2$ ✓

$\{0,16\}$.

j) $\sqrt{5y+1}-\sqrt{3y-5}=2$

$(\sqrt{5y+1})^2=(2+\sqrt{3y-5})^2$

$5y+1=3y-5+4\sqrt{3y-5}+4$

$2y+2=4\sqrt{3y-5}$

$(y+1)^2=(2\sqrt{3y-5})^2$

$y^2+2y+1=4(3y-5)$

$y^2+2y+1=12y-20$

$y^2-10y+21=0$

$(y-3)(y-7)=0$

$y=3$ $y=7$

$\sqrt{16}-\sqrt{4}=2$

$4-2=2$ ✓

$\sqrt{36}-\sqrt{16}=2$

$6-4=2$ ✓

$\{3,7\}$

k) $\sqrt{x-5}-\sqrt{x+10}=-3$

$(\sqrt{x-5})^2=(-3+\sqrt{x+10})^2$

$x-5=x+10-6\sqrt{x+10}+9$

$-24=-6\sqrt{x+10}$

$\frac{-24}{-6}=\frac{-6\sqrt{x+10}}{-6}$

$4^2=\sqrt{x+10}^2$

$16=x+10$

$6=x$ $\{6\}$

Ch $\sqrt{1}-\sqrt{16}=-3$

$1-4=-3$ ✓

Chapter 6a – Outcome 20.3a – Rational Expressions

Level 2

Determine the non-permissible value(s) for each rational expression.

a) $\frac{5}{x+3}$ $x \neq -3$

b) $\frac{x+3}{(x+4)(x-5)}$ $x \neq -4 \quad x \neq 5$

c) $\frac{1-x}{3x+5}$ $x \neq -\frac{5}{3}$

c) $\frac{2a}{a^2-3a}$ $a(a-3)$ $a \neq 0 \quad a \neq 3$

d) $\frac{m+1}{m^2+5m+6}$ $(m+2)(m+3)$ $m \neq -2, -3$

3. Simplify each rational expression. State any non-permissible values.

a) $\frac{3(x-5)}{(x-5)(x-5)}$ $\frac{3}{x-5}$ $x \neq 5$

b) $\frac{(x-7)(x+2)}{-5x(7-x)}$ $\frac{-1(x+2)}{5x}$ $\frac{x+2}{5x}$ $x \neq 0, 7$

c) $\frac{(x+3)^2}{3(x+3)(x-3)}$ $\frac{x+3}{3(x-3)}$ $x \neq \pm 3$

d) $\frac{3x(4x-1)}{(4x-1)(3x+1)}$ $\frac{3x}{3x+1}$ $x \neq \frac{1}{4}, -\frac{1}{3}$

e) $\frac{5\,25(x-5)(x+1)}{2\,10(2x+1)(x-5)}$ $\frac{5(x+1)}{2(2x+1)}$ $x \neq -\frac{1}{2}, 5$

f) $\frac{4xy(y-9)}{(y-9)(x+4)}$ $\frac{4xy}{x+4}$ $y \neq 9, \; x \neq -4$

Level 3

4. Simplify. State any non-permissible values.

a) $\frac{3\,6r^2st}{5\,10rs^2t^2}$ $\frac{3r}{5st}$ $r \neq 0, s \neq 0 \; t \neq 0$

b) $\frac{3x-6}{x^2-4}$ $x \neq \pm 2$ $\frac{3(x-2)}{(x+2)(x-2)}$ $\frac{3}{x+2}$

c) $\frac{cd}{cd+d}$ $\frac{cd}{d(c+1)}$ $\frac{c}{c+1}$ $d \neq 0 \quad c \neq -1$

d) $\frac{7m-c}{4c-28m}$ $\frac{7m-c}{-4(-c+7m)}$ $-\frac{1}{4}$ $c \neq 7m \quad m \neq \frac{c}{7}$

e) $\frac{x^2+x}{x^2-4x-5}$ $\frac{x(x+1)}{(x-5)(x+1)}$ $\frac{x}{x-5}$ $x \neq 5, -1$

f) $\frac{y^2-2y-3}{y^2-3y}$ $\frac{(y-3)(y+1)}{y(y-3)}$ $\frac{y+1}{y}$ $y \neq 0, 3$

5. Simplify. Identify any non-permissible values.

a) $\frac{3x^2+11x-4}{x^2+8x+16}$

$\frac{(3x-1)(x+4)}{(x+4)(x+4)}$

$\frac{3x-1}{x+4}$ $\quad x \neq -4$

b) $\frac{4+8a+4a^2}{16-16a^2}$

$\frac{4(a^2+2a+1)}{16(1-a^2)}$

$\frac{4(a+1)(a+1)}{16(1-a)(1+a)}$

$\frac{a+1}{4(1-a)}$ $\quad a \neq \pm 1$

c) $\frac{2x^2+5x+2}{5x^2-5x-30}$ $\quad 5(x^2-x-6)$

$\frac{(2x+1)(x+2)}{5(x-3)(x+2)}$

$\frac{2x+1}{5(x-3)}$

$x \neq 3, -2$

d) $\frac{12x^2+4x}{3x^2-5x-2}$

$\frac{4x(3x+1)}{(3x+1)(x-2)}$

$\frac{4x}{x-2}$

$x \neq -\frac{1}{3}, 2$

e) $\frac{4t^3-16t}{6+t-2t^2}$ $\quad t \neq \frac{-3}{2}, 2$

$\frac{4t(t^2-4)}{-(2t^2-t-6)}$

$\frac{4t(t+2)(t-2)}{-(2t+3)(t-2)}$

$\frac{-4t(t+2)}{2t+3}$

f) $\frac{15x^3+5x^2}{6x^2-13x-5}$

$\frac{5x^2(3x+1)}{(3x+1)(2x-5)}$

$\frac{5x^2}{2x-5}$

$x \neq -\frac{1}{3}, \frac{5}{2}$

Chapter 6b – Outcome 20.3b – Operations with Rational Expressions

1. Simplify each expression. List all non-permissible values

a) $\left(\frac{9x}{14y^2}\right)\left(\frac{7y^3}{3x^3}\right)$

$x \neq 0$, $y \neq 0$

$\frac{3y}{2x}$

b) $\left(\frac{x-2}{x^2-4}\right)\left(\frac{x^2-2x-8}{x+2}\right)$

$\frac{(x-2)(x-4)(x+2)}{(x-2)(x+2)(x+2)}$ $\quad \frac{x-4}{x+2}$

$x \neq -2, 2$

c) $\left(\frac{x+1}{3x+5}\right) \div \left(\frac{x+3}{3x+5}\right)$

$\frac{x+1}{3x+5} \times \frac{3x+5}{x+3}$ $\quad x \neq -\frac{5}{3}, -3$

d) $\frac{x^2+8x+7}{x^2-6x-7} \div \frac{x^2+7x+6}{x^2-x-42}$

$\frac{(x+7)(x+1)}{(x-7)(x+1)} \cdot \frac{(x+6)(x+1)}{(x-7)(x+6)}$

$\frac{(x+7)(x+1)(x-7)(x+6)}{(x-7)(x+1)(x+6)(x+1)}$

$x \neq 7, -1, -6$

e) $\frac{x+1}{3x} + \frac{x+3}{4x-5}$

$\frac{5x-4}{3x}$ $\quad x \neq 0$

f) $\frac{4x^2}{x+5} + \frac{x+1}{x+5} - \frac{x^2}{x+5}$

$\frac{3x^2+x+1}{x+5}$ $\quad x \neq -5$

g) $\frac{7x+6}{(x+2)(x-2)} - \frac{(3x-2)}{(x+2)(x-2)}$ $\quad x \neq \pm 2$

$7x+6-3x+8$

$4x+8$

$4(x+2)$

$(x+2)(x-2)$

$\frac{4}{x-2}$

Level 3

2. Simplify the following completely. List all non-permissible values

a) $\left(\frac{5y-5}{y^2+4y-5}\right)\left(\frac{y^2-25}{y^2-2y-15}\right)$ $\quad y \neq -5, 1, 5, -3$

$\frac{5(y-1)}{(y+5)(y-1)} \cdot \frac{(y+5)(y-5)}{(y-5)(y+3)}$

$\frac{5}{y+3}$

b) $\left(\frac{x^3-9x}{2x^2-x-15}\right)\left(\frac{2x^2+x-10}{x^2+x-6}\right)$ $\quad x \neq -\frac{5}{2}, 3, -3, 2$

$\frac{x(x+3)(x-3)}{(2x+5)(x-3)} \cdot \frac{(2x+5)(x-2)}{(x+3)(x-2)}$

x

c) $\frac{x^2+8x+16}{(x-3)(x+5)} \div \frac{3x^2-3}{(x+4)}$ $3(x^2-1)$

$x \neq 3, -5, -4, -1, 1$

$\frac{(x+4)(x+4)}{(x-3)(x+5)} \times \frac{x+4}{3(x+1)(x-1)}$

$\frac{(x+4)^3}{3(x-3)(x+5)(x+1)(x-1)}$

d) $\frac{16a^2b}{a^2-2a} \div \frac{4ab^2}{a^2b-4b}$ $a \neq 0, \pm 2$ $b(a^2-4)$ $b \neq 0$

$\frac{4 \cancel{16} \cancel{a^2} \cancel{b}}{\cancel{a}(a-2)} \times \frac{\cancel{b}(a+2)(a-2)}{4 \cancel{a} \cancel{b^2}}$

~~4at~~ $4(a+2)$

e) $\frac{9y^2-1}{y+3} \div \frac{3y^2-8y-3}{9-y^2}$ $y \neq -3, 3, \frac{-1}{3}$

$\frac{(3y+1)(3y-1)}{(y+3)} \times \frac{(3+y)(3-y)}{(3y+1)(y-3)}$ -1

$-(3y-1)$

$-3y+1$

f) $\frac{9}{x^2-36} + \frac{3x}{x-6}$ $(x+6)$ $(x+6)(x-6)$ $x \neq \pm 6$

$\frac{9+3x^2+18x}{(x+6)(x-6)}$

$\frac{3(x^2+6x+3)}{(x+6)(x-6)}$

g) $\frac{3(x+7)}{x-5} + \frac{2}{x+7}$ $(x-5)$ $(x+7)(x-5)$ $x \neq -7, 5$.

$\frac{3x+21+2x-10}{(x+7)(x-5)}$

$\frac{5x+11}{(x+7)(x-5)}$

h) $\frac{x^2-144}{12x^2} \div \left(\frac{x^2-x-6}{x^2-2x}\right)\left(\frac{x^2+4x+4}{x^2+10x-24}\right)$ $x \neq 0, 3, -2, 2, 12$

$\frac{(x+12)(x-12)}{12x^2} \times \frac{x(x-2)}{(x-3)(x+2)} \cdot \frac{(x+2)(x+2)}{(x-2)(x+6)(x+4)}$

$\frac{(x-12)(x+2)}{12x(x-3)}$

i) $\frac{(2a)}{(2a+6)}^2 \frac{(a^2+9)}{a^2-9}$ $(a-3)$ $a \neq \pm 3$

$(a-3)(a-3)$ $2(a+3)$ $(a+3)(a-3)(2)$

$\frac{2a^2-6a-2a^2-18}{2(a+3)(a-3)}$

$\frac{-6a-18}{2(a+3)(a-3)}$

$\frac{-6(a+3)}{2(a+3)(a-3)}$

$\frac{-3}{a-3}$

j) $\frac{3y}{y^2-4} + \frac{6y}{y^2+5y+6}$ $(y+3)$ $(y-2)$

$(y+2)(y-2)$ $(y+3)(y+2)(y-2)$

$3y^2+9y+6y^2-12y$

$9y^2-3y$

$3y(y-1)$

$(y+2)(y-2)(y+3)$

$y \neq 2, -2, -3$

k) $\frac{(x-6)(x-1)}{x^2-11x+28} - \frac{(x-5)(x-4)}{x^2-8x+7}$

$(x-1)(x-4)(x-7) \quad (x-7)(x-1)(x-4)$

$x^2-6x-x+6-(x^2-5x-4x+20)$

$x^2-7x+6-(x^2-9x+20)$

$x^2-7x+6-x^2+9x-20$

$2x-14$

$x \neq 1, 4, 7$

$\frac{2(x-7)}{(x-1)(x-4)(x-7)}$

$\frac{2}{(x-1)(x-4)}$

l) $\frac{8}{3x-18} - \frac{x+1}{x^2-5x-6}$

$\frac{(x+1)\,8}{(x+1)\,3(x-6)} - \frac{x+1(3)}{(x-6)(x+1)(3)}$

$8x+8-3x-3 \qquad x \neq -1, 6$

$5x+5$

$\frac{5(x+1)}{3(x+1)(x-6)} \qquad \frac{5}{3(x-6)}$

m) $\frac{x^2-49}{x^2-8x+7} + \frac{2-2x}{x^2-1}$ $\quad x \neq \pm 1, 7$

$\frac{(x+7)(x-7)}{(x-1)(x-7)} + \frac{2-2x}{(x+1)(x-1)}$

$\frac{(x+1)(x+7)}{(x+1)(x-1)} + \frac{2-2x}{(x+1)(x-1)}$

$x^2+x+7x+7+2-2x$

x^2+6x+9

$\frac{(x+3)(x+3)}{(x+1)(x-1)}$

$\frac{(x+3)^2}{(x+1)(x-1)}$

n) $\frac{(2x+8)(x-5)}{x^2+5x+6} - \frac{(x-9)(x+2)}{(x+3)(x-5)}$ $\quad (x+2)$

$(x-5)(x+2)(x+3)$

$2x^2+8x-10x-40-(x^2-9x+2x-18)$

$2x^2-2x-40-(x^2-7x-18)$

$2x^2-2x-40-x^2+7x+18$

$\frac{x^2+5x-22}{(x-5)(x+2)(x+3)}$

$x \neq 5, -2, -3.$

---

3. Simplify.

a) $\frac{\frac{1}{x}+1}{1-\frac{1}{x}}$

$\frac{1}{x} + 1 \div 1 - \frac{1}{x}$ $\quad \left(\frac{x}{x}\right) \quad \left(\frac{x}{x}\right)$

$\frac{1+x}{x} \div \frac{x-1}{x}$

$\frac{1+x}{x} \times \frac{x}{x-1}$

$\frac{1+x}{x-1}$ $\quad x \neq 0, 1$

b) $\frac{\frac{x}{x-3}}{4-\frac{x}{x-3}}$ $\quad x \neq 3, 0$ $\quad \left(\frac{x-3}{x-3}\right)$

$\frac{x}{x-3} \div \frac{4(x-3)}{x-3} - \frac{(x)}{x-3}$

$\frac{x}{x-3} \div \frac{4x-12-x}{x-3}$

$\frac{x}{x-3} \div \frac{3x-12}{x-3}$

$\frac{x}{x-3} \times \frac{x-3}{3(x-4)}$

$\frac{x}{3(x-4)}$

c) $\frac{\frac{1}{4+h}-\frac{1}{4}}{h}$

$\frac{1}{(4+h)}\frac{(4)}{(4)} - \frac{1}{4}\frac{(4+h)}{(4+h)} \div h$

$\frac{4}{4(4+h)} - \frac{4+h}{4(4+h)} \times \frac{1}{h}$

$\frac{-h}{4(4+h)} \times \frac{1}{h}$

$\frac{-1}{4(4+h)}$

$h \neq 0, -4$

Chapter 6c – Outcome 20.3 – Solving Rational Equations

Level 2

1. Solve each equation. Identify any non-permissible values

a) $\frac{2x}{3} = \frac{x}{4} + \frac{5}{6}$ | 12

$\frac{2x}{3}(12)^4 = \frac{x}{4}(12)^3 + \frac{5}{6}(12)^2$

$8x = 3x + 10$

$5x = 10$

$x = 2$

$\{2\}$

b) $\frac{5}{2a} = 3 - \frac{2}{a}$ | $2a$ | $2a$ | $2a$

$5 = 6a - 4$

$9 = 6a$

$\frac{9}{6} = a$

$\frac{3}{2} = a$

$\{\frac{3}{2}\}$

c) $\frac{x-3}{x^2} = \frac{11}{15x} - \frac{2}{5x}$ | $15x^2$ | $15x^2$ | $15x^2$

$15x^2 - 45 = 11x - 6x$

$15x - 45 = 5x$

$-45 = -10x$

$x = -4.5$

$\{-4.5\}$

d) $\frac{x^2}{3} + \frac{11x}{18} = \frac{1}{9}$ | 18 | 18 | 18

$6x^2 + 11x = 2$

$6x^2 + 11x - 2 = 0$

$(6x - 1)(x + 2) = 0$

$x = \frac{1}{6}$ $x = -2$

$\{-2, \frac{1}{6}\}$

2. Solve each equation. Identify any non-permissible values

a) $\frac{21}{5x+3} = -3$ | $5x+3$ | $5x+3$

$21 = -15x - 9$

$30 = -15x$

$-2 = x$

$\{-2\}$

b) $\frac{x+1}{x-3} = \frac{x}{x-5}$ | $(x-3)(x-5)$ | $(x-3)(x-5)$

$x^2 - 5x + 1x - 5 = x^2 - 3x$

$-5 = x$

$\{-5\}$

c) $\frac{x+4}{x-2} = \frac{x-4}{x+4}$ | $(x+4)(x-2)$ | $(x+4)(x-2)$

$x^2 + 8x + 16 = x^2 - 6x + 8$

$8 = -14x$

$-\frac{8}{14} = x$

$\{-\frac{4}{7}\}$

d) $\frac{x-2}{x} = \frac{2-x}{x+1}$ | $x(x+1)$ | $x(x+1)$

$x^2 - x - 2 = 2x - x^2$

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2)$

$x = -\frac{1}{2}$ $x = 2$

$\{-\frac{1}{2}, 2\}$

Level 3

3. Solve each rational equation. Identify any non-permissible values

a) $\frac{3}{x+3} = \frac{x+15}{x+3} - 5$ $(x+3)$ $x \neq -3$

$3 = x+15-5(x+3)$

$3 = x+15-5x-15$

$3 = -4x$

$-3 = -x$

$\frac{-3}{4}$ $\left\{\frac{-3}{4}\right\}$

b) $\frac{x}{x+1} - \frac{x+4}{x+1} = \frac{6}{x}$ $(x)(x+1)$ $x \neq -1, 0$

$x(x) - (x+4)(x) = 6(x+1)$

$x^2 - x^2 - 4x = 6x+6$

$-6x \quad -6x$

$-10x = 6$

$x = -0.6$ $\{-0.6\}$

4. Solve the following equations. Identify any non-permissible values

a) $\frac{x}{x-3} + \frac{x^2+9}{x^2-9} = \frac{2x}{x+3}$ $(x+3)(x-3)$ $x \neq \pm 3$

$x(x+3) + x^2 + 9 = 2x(x-3)$

$x^2+3x+x^2+9 = 2x^2-6x$

$-x^2 \quad -3x \quad -x^2 \qquad -x^2 \quad -3x$

$-x^2$

$9 = -9x$

$-1 = x$ $\{-1\}$

b) $\frac{5}{x+1} - \frac{1}{x^2-x-2} = \frac{3}{x-2}$ $(x-2)(x+1)$ $x \neq 2, -1$

$5(x-2) - 1 = 3(x+1)$

$5x-10-1 = 3x+3$

$-3x \quad +10 \quad +1 \quad -3x \quad +10$

$+1$

$2x = 14$

$x = 7$ $\{7\}$

c) $\frac{x+5}{2x+4} = \frac{x}{x-3} - \frac{2x+9}{x^2-x-6}$ $2(x+2)(x-3)$

$2(x+2) \quad (x-3)(x+2)$ $x \neq -2, 3$

$(x+5)(x-3) = 2x(x+2) - 2(2x+9)$

$x^2+2x-15 = 2x^2+4x \quad -4x-18$

$-x^2 \quad -2x \quad +15 \quad -x^2 \quad -2x \quad +15$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3 \quad x = -1$

$\{-1\}$

d) $\frac{3x}{2x+3} + \frac{20}{2x^2-x-6} = \frac{4}{x-2}$ $(2x+3)(x-2)$ $x \neq 2, -\frac{3}{2}$

$(2x+3)(x-2)$

$3x(x-2) + 20 = 4(2x+3)$

$3x^2 - 6x + 20 = 8x + 12$

$-8x \quad +12 \quad -8x \quad +12$

$3x^2 - 14x + 8 = 0$

$(3x-2)(x-4) = 0$

$x = \frac{2}{3} \quad x = 4$

$\left\{\frac{2}{3}, 4\right\}$

5. Solve each rational equation. Give exact and approximate answer. Round approximate answers to the nearest hundredth.

a) $\frac{x}{2x+3} + \frac{5}{8x+12} = \frac{x+1}{5}$

$4(2x+3)(5)$

$4(2x+3)$

$x \neq -\frac{3}{5}$

$4(5)(x) + 5(5) = 4(2x+3)(x+1)$

$20x + 25 = 4(2x^2 + 5x + 3)$

$20x + 25 = 8x^2 + 20x + 12$

$0 = 8x^2 - 13$

$13 = 8x^2$

$\pm\sqrt{\frac{13}{8}} = \sqrt{x^2}$

$\pm\frac{\sqrt{13}\cdot\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = x$

$\pm\frac{\sqrt{26}}{4} = x$

$\left\{\pm\frac{\sqrt{26}}{4}\right\}$

b) $\frac{x-1}{x} - \frac{(x+2)}{x^2} = \frac{x+1}{3x}$

$3x^2$

$x \neq 0$

$3x(x-1) - 3(x+2) = x(x+1)$

$3x^2 - 3x - 3x - 6 = x^2 + x$

$3x^2 - 6x - 6 = x^2 + x$

$-x^2 \quad -x \quad -x^2 \quad -x$

$2x^2 - 7x - 6 = 0$

$x = \frac{7 \pm \sqrt{97}}{4}$

$\left\{\frac{7 \pm \sqrt{97}}{4}\right\}$

c) $\frac{x-5}{2x+10} - \frac{8}{x^2-25} = \frac{x}{x-5}$

$2(x+5)(x-5)$

$2(x+5) \quad (x+5)(x-5)$

$(x-5)(x-5) - 8(2) = 2(x+5)(x)$

$x^2 - 10x + 25 - 16 = 2x^2 + 10x$

$-25 \quad +16$

$-x^2 + 10x + 9 = 2x^2 + 10x + 16$

$-x^2 + 10x + 16$

$0 = x^2 + 20x - 9$

$436 = 4 \cdot 109 = 4 \cdot 2 \cdot 2 \cdot 109$

$x = \frac{-20 \pm \sqrt{436}}{2}$

$x = \frac{-20 \pm 2\sqrt{109}}{2} = \{-10 \pm \sqrt{109}\}$

Chapter 7a – Outcome 20.1a – Absolute Value

Level 2

1 Evaluate.

a) $|-42|$ 42

b) $\left|\frac{-82}{3}\right|$ $\frac{82}{3}$

c) $|3.75|$ 3.75

2. Evaluate each expression.

a) $|-4-10|$
$|-14|$
14

b) $|3-5(7)|$
$|3-35|$
$|-32|$
32

c) $5(|-2|)+|-3|$
$5(2)+3$
13

Level 3

3. Evaluate the following

a) $|4-8|-|1-6|$
$|-4|-|-5|$
$4-5$
-1

c) $3|5-3|-3|6-9|$
$3(2)-3(3)$
$6-9$
-3

c) $-|-42|-4|-7|+5|4-6|$
$-42-4(7)+5(2)$
$-42-28+10$ -60

d) $-2(3-|4|+3|6-4|)$
$-2(3-4+3(2))$
$-2(5)$
-10

e) $|2^3-4|-4|5-8(2)|$
$|8-4|-4|-11|$
$4-4(11)$
$4-44$
-40

f) $4|3-3^2|-3+7|4-3|$
$4|3-9|-3+7(1)$
$4|-6|-3+7$
$4(6)-3+7$
$24-3+7$
28

Level 2

Given the table of values for $y = f(x)$, create a table of values for $y = |f(x)|$.

a)

x	y
0	1
2	0
4	−1
6	−2
8	−3

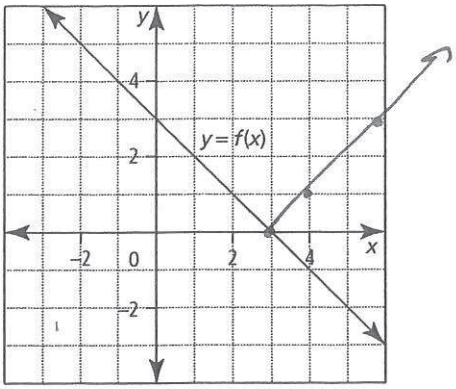
x	$\lvert f(x) \rvert$
0	1
2	0
4	1
6	2
8	3

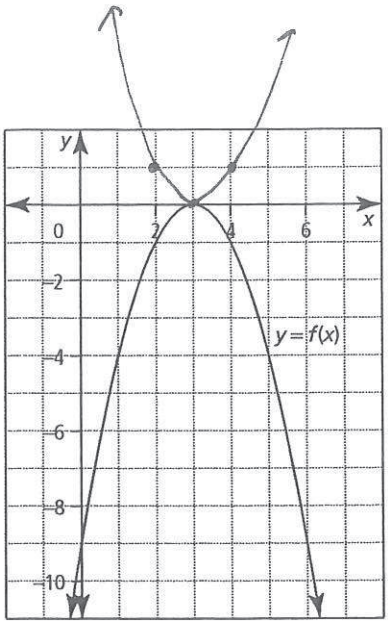
b)

x	y
−4	−8
−2	0
0	0
2	−8
4	−24

x	$\lvert f(x) \rvert$
−4	8
−2	0
0	0
2	8
4	24

2. Use the graph of $y = f(x)$ to sketch the graph of $y = |f(x)|$.





3. Solve each absolute value equation. Verify the solution.

a) $|x + 1| = 2$

$x+1 = 2$ $\quad x+1 = -2$

$x = 1$ ✓ $\quad x = -3$ ✓

$\{-3, 1\}$

b) $|x - 3| + 1 = 0$

$|x+3| = -1$

$\{\}$

c) $|2x| = 5$

$2x = 5$ $\quad 2x = -5$

$x = \frac{5}{2}$ ✓ $\quad x = \frac{-5}{2}$ ✓

$\{\pm \frac{5}{2}\}$

d) $2|x-5|=8$

$|x-5|=4$

$x-5=4 \quad x-5=-4$

$x=9 \quad x=1$

$\{1,9\}$

e) $|-2x-3|=5$

$-2x-3=5 \quad -2x-3=-5$

$-2x=8 \quad -2x=-2$

$x=-4 \quad x=1$

$\{-4,1\}$

f) $|3x-2|+6=12$

$-6 \quad -6$

$|3x-2|=6$

$3x-2=6 \quad 3x-2=-6$

$3x=8 \quad 3x=-4$

$x=\frac{8}{3} \quad x=-\frac{4}{3}$

$\{-\frac{4}{3}, \frac{8}{3}\}$

Level 3

4. Solve each absolute value equation algebraically.

a) $|x-5|=3x+4$

$x-5=3x+4 \quad x-5=-3x-4$

$-9=2x \quad 4x=1$

$-\frac{9}{2}=x \quad x=\frac{1}{4}$

$x \quad \checkmark$

$\{\frac{1}{4}\}$

b) $|3m+2|=m$

$3m+2=m$

$2=-2m$

$-1=m$

x

$\{\}$

c) $|x^2-3x|=4$

$x^2-3x=4 \quad x^2-3x=-4$

$x^2-3x-4=0 \quad x^2-3x+4=0$

$(x-4)(x+1)=0 \quad x=\frac{3\pm\sqrt{-7}}{2}$

$x=4 \quad x=-1$

$\checkmark \quad \checkmark \quad x$

$\{-1,4\}$

d) $3=|-4x^2+8x|$

$-4x^2+8x=3 \quad -4x^2+8x=-3$

$4x^2-8x+3=0 \quad 4x^2-8x-3=0$

$(2x-1)(2x-3)=0 \quad x=\frac{8\pm\sqrt{112}}{8}$

$x=\frac{1}{2} \quad x=\frac{3}{2} \quad \checkmark \quad x=\frac{8\pm4\sqrt{7}}{8}$

$x=\frac{2\pm\sqrt{7}}{2} \quad \checkmark$

$\{\frac{1}{2}, \frac{3}{2}, \frac{2\pm\sqrt{7}}{2}\}$

e) $|4x| = x^2 - 5$

$4x = x^2 - 5$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = 5 \quad x = -1$

$|20| = 20$ ✓ $\quad |-4| = 4$ ✗

$4x = -x^2 + 5$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5 \quad x = 1$

$|-20| = 20$ ✓ $\quad |4| = 4$ ✗

$\{-5, 5\}$

f) $|2(x-4)^2 - 5| = 3$

$2(x-4)^2 - 5 = 3$

$2(x^2 - 8x + 16) - 5 = 3$

$2x^2 - 16x + 32 - 5 = 3$

$2x^2 - 16x + 24 = 0$

$2(x^2 - 8x + 12) = 0$

$(x-6)(x-2) = 0$

$x = 6 \quad x = 2$

$|3| = 3$ ✓ $\quad |3| = 3$ ✓

$\{6, 2, 3, 5\}$

$2(x-4)^2 - 5 = -3$

$2(x^2 - 8x + 16) - 5 = -3$

$2x^2 - 16x + 32 - 5 = -3$

$2x^2 - 16x + 30 = 0$

$2(x^2 - 8x + 15) = 0$

$2(x-3)(x-5) = 0$

$x = 3 \quad x = 5$

✓ ✓

g) $0 = |x^2 - 2x - 3| - 4$

$4 = |x^2 - 2x - 3|$

$4 = x^2 - 2x - 3$

$0 = x^2 - 2x - 7$

$x = \frac{2 \pm \sqrt{32}}{2}$

$= \frac{2 \pm 4\sqrt{2}}{2}$

$= 1 \pm 2\sqrt{2}$ ✓

$-4 = x^2 - 2x - 3$

$0 = x^2 - 2x + 1$

$(x-1)(x-1)$

$x = 1$ ✓

$\{1, 1 \pm 2\sqrt{2}\}$

h) $x + 1 = |x^2 - 1|$

$x + 1 = x^2 - 1$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2 \quad x = -1$

$|3| = 3$ $\quad 0 = |0|$ ✓

$-x - 1 = x^2 - 1$

$0 = x^2 + x$

$0 = x(x+1)$

$x = 0 \quad x = -1$

$\{-1, 0, 2\}$

i) $|x^2 - 3x| = 3x - 8$

$x^2 - 3x = 3x - 8$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2 \quad x = 4$

$|-2| = -2$ ✗ $\quad |4| = 4$ ✓

$x^2 - 3x = -3x + 8$

$\sqrt{x^2} = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

$2\sqrt{2}$

$|-.48| = .48$ ✓

$-2\sqrt{2}$

$|16.48| = -16.48$ ✗

$\{4, 2\sqrt{2}\}$

5. For each equation, state the x and y intercepts, and the domain and range.

a) $f(x) = |2x + 1|$

y int $(0, 1)$

x int $(-\frac{1}{2}, 0)$

D $(-\infty, \infty)$

R $[0, \infty)$

b) $g(x) = |-x - 4|$

$g(0) = |-4| = 4$

$-x - 4 = 0$

$x = -4$

y int $(0, 4)$

x int $(-4, 0)$

D $(-\infty, \infty)$

R $[0, \infty)$

c) $y = |x^2 + 6x + 5|$

$y = |0 + 0 + 5| = 5$ y int $(0, 5)$

$0 = x^2 + 6x + 5 = 0$

$(x+5)(x+1)$

$x = -5, -1$

$(-5, 0)$ $(-1, 0)$ x int

D $(-\infty, \infty)$

R $[0, \infty)$

d) $f(x) = |(2x + 1)(x - 3)|$

$f(0) = |(1)(-3)| = 3$

y int $(0, 3)$

x int $(-\frac{1}{2}, 0)$ $(3, 0)$

D $(-\infty, +\infty)$

R $[0, \infty)$

Chapter 7c – Reciprocal Functions – Outcome 20.11

Level 2

1. For each function,
   - **i)** write the reciprocal function
   - **ii)** state the non-permissible values

a) $f(x) = 3 + x$

$y = \frac{1}{3+x}$

$x \neq -3$

b) $g(x) = 2x - 1$

$y = \frac{1}{2x-1}$

$x \neq \frac{1}{2}$

c) $h(x) = (x+2)(x-3)$ $y = \frac{1}{(x+2)(x-3)}$ $x \neq -2, 3$

d) $j(x) = -2x^2 - 12x - 10$ $x \neq -1, -5$

$y = \frac{1}{-2x^2-12x-10}$

$-2(x+6x+5)$

$-2(x+5)(x+1)$

2. State the equation(s) of the vertical asymptote(s) for each function.

a) $f(x) = \frac{1}{5-x}$ $x = 5$

b) $g(x) = \frac{1}{7x-2}$ $x = 2/7$

c) $h(x) = \frac{1}{(x+1)(2x+1)}$ $x = -1$ $x = -\frac{1}{2}$

d) $h(x) = \frac{1}{2x^2+2x-24}$

$2(x^2+x-12)$ $x = -4$

$(x+4)(x-3)$ $x = 3$

3. What are the x-intercepts and y-intercepts of each function?

a) $y = \frac{1}{2x+5}$

y int $(0, \frac{1}{5})$

no x ints → V.a. @ $x = -\frac{5}{2}$

b) $y = \frac{1}{3-2x}$

y int $(0, \frac{1}{3})$

no x int

V.A. @ $x = 3/2$

c) $f(x) = \frac{1}{(2x+3)(x-1)}$

$2x^2+5x-3$

y int $(0, -\frac{1}{3})$

no x int : V.A. @ $x = \frac{-3}{2}, x = 1$

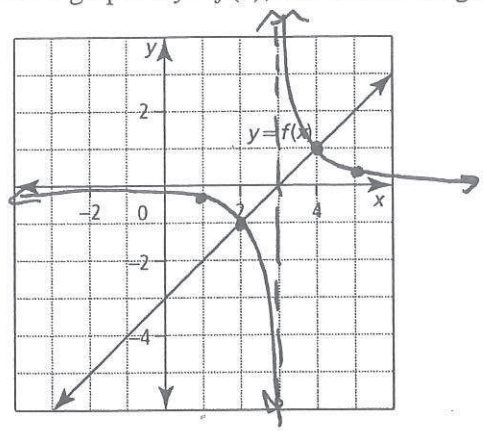
d) $g(x) = \frac{1}{x^2+7x+12}$ $(x+3)(x+4)$

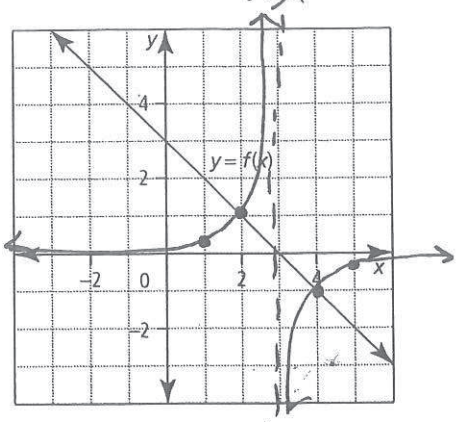
y int $(0, \frac{1}{12})$

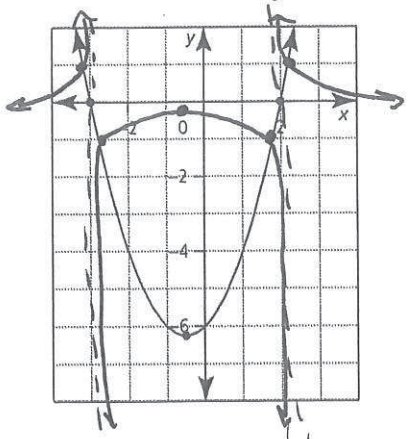
no x int

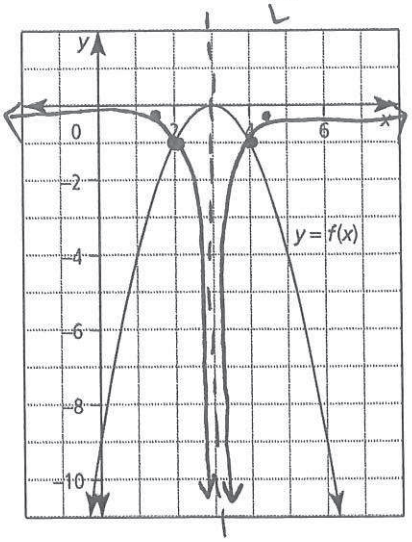
V.A @ $x = -3$ $x = -4$

4. Given graph of $y = f(x)$, and sketch the graph of the reciprocal function, $y = \frac{1}{f(x)}$.









Level 3

5. Sketch the graph of $y = f(x)$ and the graph of $y = \frac{1}{f(x)}$ on the same set of axes. Label the asymptotes, and the intercepts.

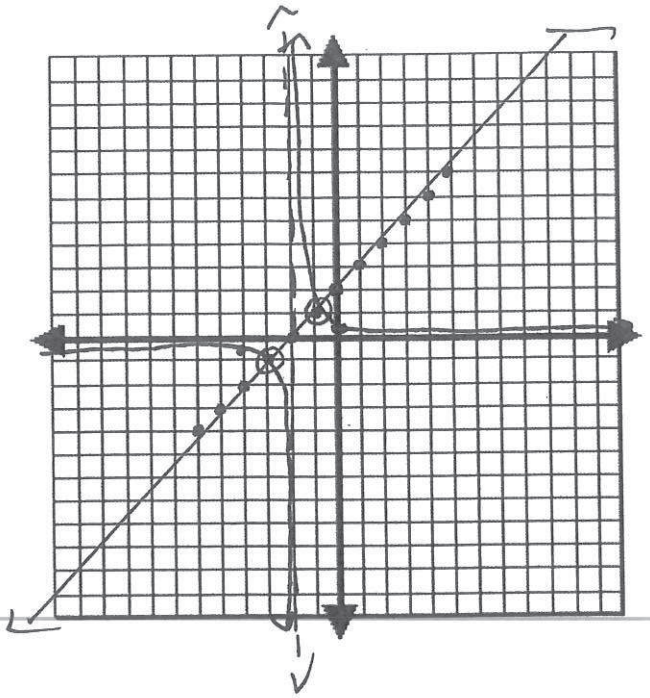
a) $f(x) = x + 2$

$f(x) = \frac{1}{x+2}$

no x int

y int $(0, \frac{1}{2})$

V.A. @ $x = -2$



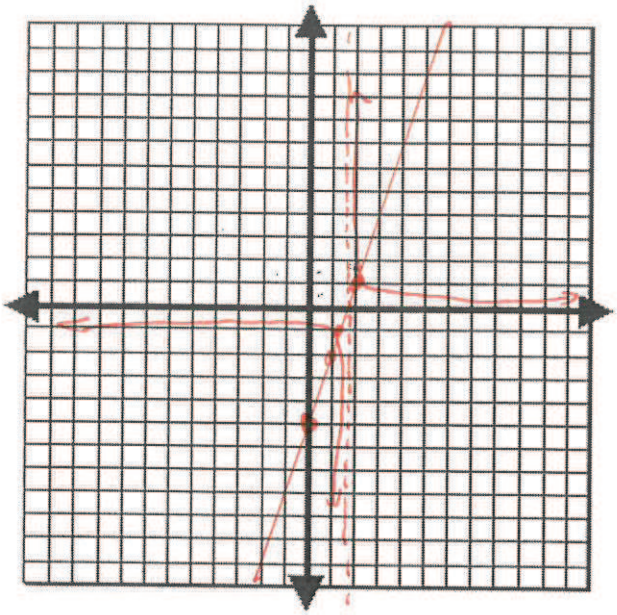
b) $f(x) = 3x - 5$

$f(x) = \frac{1}{3x-5}$

y int $(0, -\frac{1}{5})$

NO x int

VA @ $x = \frac{5}{3}$



c) $f(x) = (x-3)(x+3)$

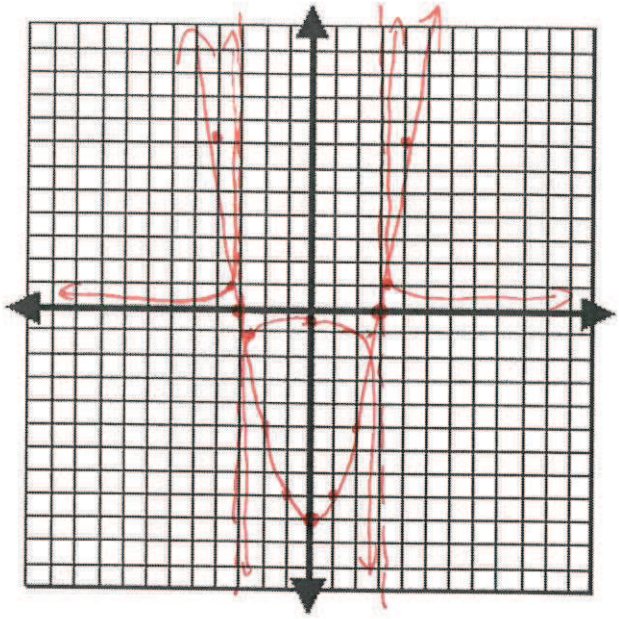
$\frac{1}{2}$ way $x = 0$

$f(0) = (-3)(3) = -9$ $V(0,9)$

VA $x = 3$ $x = -3$

NO x int

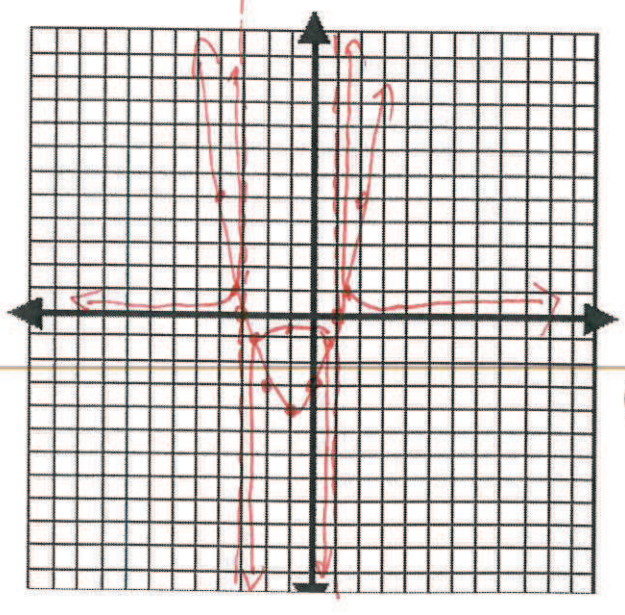
y $(0, -\frac{1}{9})$



d) $f(x) = (x+1)^2 - 4$

$V(-1,-4)$

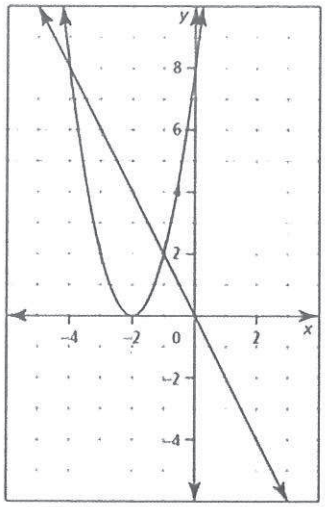
VA $x = -3, x = 1$



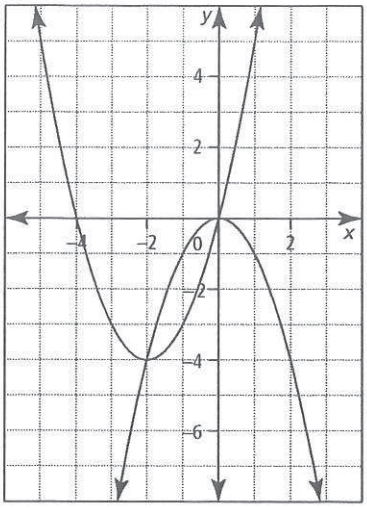
Chapter 8 – Outcome 20.8a – Solving Systems of Equations

Level 2

1. Solve each system of equations below



(-1, 2) (-4, 8)



(-2, -4) (0, 0)

2. Solve each linear-quadratic system below

a) ① $2x^2 - 5x - y = -1$

② $7x + y = 1$

$-7x \quad -7x$

② $y = -7x + 1$

② → ①

$2x^2 - 5x - (-7x + 1) = -1$

$2x^2 - 5x + 7x - 1 = -1$

$+1 \quad +1$

$2x^2 + 2x = 0$

$2x(x + 1) = 0$

$x = 0 \quad x = -1$

$y = -7(0) + 1$

(0, 1)

$y = -7(-1) + 1$

$= 7 + 1$

$= 8$

(-1, 8)

b) ① $0 = x^2 - 9x - y + 14$

② $y + 7 = x$

$-7 \quad -7$

② $y = x - 7$

② → ①

$0 = x^2 - 9x - (x - 7) + 14$

$0 = x^2 - 9x - x + 7 + 14$

$0 = x^2 - 10x + 21$

$0 = (x - 3)(x - 7)$

$x = 3 \quad x = 7$

$y = 3 - 7$

(3, -4)

$y = 7 - 7$

(7, 0)

c) ① $-2x$ $-2x$
$2x + y = 9$

② $2x^2 - 4x - y = -5$

① $y = 9 - 2x$

① → ②

$2x^2 - 4x - (9 - 2x) = -5$

$2x^2 - 4x - 9 + 2x = -5$

$+5$

$2x^2 - 2x - 4 = 0$

$2(x^2 - x - 2) = 0$

$2(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$y = 9 - 2(2)$ $\quad y = 9 - 2(-1)$

$y = 11$

$(2, 5)$ $\quad (-1, 11)$

d) ① $y = 2x + 1$

② $y = x^2 - 5x + 13$

① = ②

$2x + 1 = x^2 - 5x + 13$

$0 = x^2 - 7x + 12$

$0 = (x-3)(x-4)$

$x = 3 \quad x = 4$

$y = 2(3) + 1$ $\quad y = 2(4) + 1$

$(3, 7)$ $\quad (4, 9)$

e) $3x + y - 4 = 0$ $\quad -3x \quad +4$

② $2x^2 - 4x - y - 2 = 0$

① $y = -3x + 4$

① → ②

$2x^2 - 4x - (-3x + 4) - 2 = 0$

$2x^2 - 4x + 3x - 4 - 2 = 0$

$2x^2 - x - 6 = 0$

$(2x + 3)(x - 2) = 0$

$x = -\frac{3}{2} \quad x = 2$

$y = -3(-\frac{3}{2}) + 4$ $\quad y = -3(2) + 4$

$(-\frac{3}{2}, +\frac{17}{2})$ $\quad (2, -2)$

f) $3x^2 + x - 3y = -8$

$x + 3y = 9$

$+ \quad 3x^2 + x - 3y = -8$

$\quad\quad 0x^2 + x + 3y = 9$

$3x^2 + 2x = +1$

$3x^2 + 2x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$x = \frac{1}{3} \quad x = -1$

$-1 + 3y = 9$

$3y = 10$

$y = \frac{10}{3}$

$(-1, \frac{10}{3})$

$\frac{1}{3} + 3y = 9 \quad | \cdot 3$

$1 + 9y = 27$

$-1 \quad\quad -1$

$9y = 26$

$y = \frac{26}{9}$

$(\frac{1}{3}, \frac{26}{9})$

Level 3

3. Solve the following quadratic-quadratic systems

a) $y = x^2 + 2x - 3$

$y = -x^2 - 2x - 5$

$x^2 + 2x - 3 = -x^2 - 2x - 5$

$2x^2 + 4x + 2 = 0$

$2(x^2 + 2x + 1) = 0$

$2(x+1)(x+1) = 0$

$x = -1$

$y = (-1)^2 + 2(-1) - 3$

$= 1 - 2 - 3$

$= -4$

$(-1, -4)$

b) $y = -x^2 - 3x + 14$

$y = 3x^2 + 5x - 18$

$-x^2 - 3x + 14 = 3x^2 + 5x - 18$

$0 = 4x^2 + 8x - 32$

$0 = 4(x^2 + 2x - 8)$

$0 = 4(x+4)(x-2)$

$x = -4 \quad x = 2$

$y = -(-4)^2 - 3(-4) + 14$

$(-4, 10)$

$y = -(2)^2 - 3(2) + 14$

$(2, 4)$

c) $4x + y + 5 = x^2$ ① $-4x - 5$

$x^2 = 5x + 2y$ ②

① $y = x^2 - 4x - 5$

① → ②

$x^2 = 5x + 2(x^2 - 4x - 5)$

$x^2 = 5x + 2x^2 - 8x - 10$

$-x^2 \quad\quad -x^2$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5 \quad x = -2$

$y = x^2 - 4x - 5$

$y = 5^2 - 4(5) - 5$

$(5, 0)$

$y = (-2)^2 - 4(-2) - 5$

$(-2, 7)$

d) $y = 2x^2 - x + 1$
$2y = 2x^2 - x - 1$

$2(2x^2 - x + 1) = 2x^2 - x - 1$

$4x^2 - 2x + 2 - 2x^2 - x - 1$

$-2x^2 + x + 1 \quad -2x^2 + x + 1$

$2x^2 - x + 3 = 0$

$x = \frac{1 \pm \sqrt{-23}}{4}$

$-\ y = 2x^2 - x + 1$

$2y = 2x^2 - x - 1$

$-y = 2$

$y = -2$

$-2 = 2x^2 - x + 1$

$0 = 2x^2 - x + 3$

$\{\}$ No solution

e) $x^2 + y = 4x + 5$

② $\left(5x + \frac{1}{3}y = x^2\right)3$

② $15x + y = 3x^2$

$x^2 + y - 4x - 5 = 0$

$-3x^2 + y + 15x + 0 = 0$

$4x^2 - 19x - 5 = 0$

$(4x + 1)(x - 5)$

$x = -\frac{1}{4} \quad x = 5$

(5, 0)

(-0.25, -3.9375)

f) $x^2 + 8x - y = -12$

② $x^2 - y = 8$ -8

$-8 \quad +y \quad +y$

② $y = x^2 - 8$

② → ①

$x^2 + 8x - (x^2 - 8) = -12$

$x^2 + 8x - x^2 + 8 = -12$

$-8 \quad -8$

$8x = -20$

$x = \frac{-20}{8} = \frac{-5}{2}$

$y = \left(\frac{-5}{2}\right)^2 - 8$

$\frac{25}{4} - \frac{32}{4}$

$-\frac{7}{4}$

$\left(-\frac{5}{2}, -\frac{7}{4}\right)$

Chapter 9a – Graphing Inequalities – Outcome 20.9a

Level 2

1. Which ordered pairs are solutions to each given inequality?

a) $x - 6 \le y$

A (1, 6) **B** (6, 1) **C** (–1, 6) **D** (–1, –6)

$1-6 \le 6$ $6-6 \le 1$ $-1-6 \le 6$ $-1-6 \le -6$

$-5 \le 6$ ✓ $0 \le 1$ ✓ $-7 \le 6$ ✓ $-7 \le -6$ ✓

b) $y \le 3(x-1)^2 + 4$

A (1, 5) **B** (0, 7) **C** (–1, 9) **D** (3, 18)

$5 \le 3(0)^2+4$ $7 \le 3(-1)^2+4$ $9 \le 3(-2)^2+4$ $18 \le 3(2)^2+4$

$5 \le 4$ $7 \le 7$ ✓ $9 \le 16$ ✓ $18 \le 16$

X X

2. Graph each inequality

a) $2x - 7y \ge 14$

$2x = 14$ $-7y = 14$ $(0,0)$

$x = 7$ $y = -2$ $0 \ge 14$ X

b) $5 - x + 3y < 0$

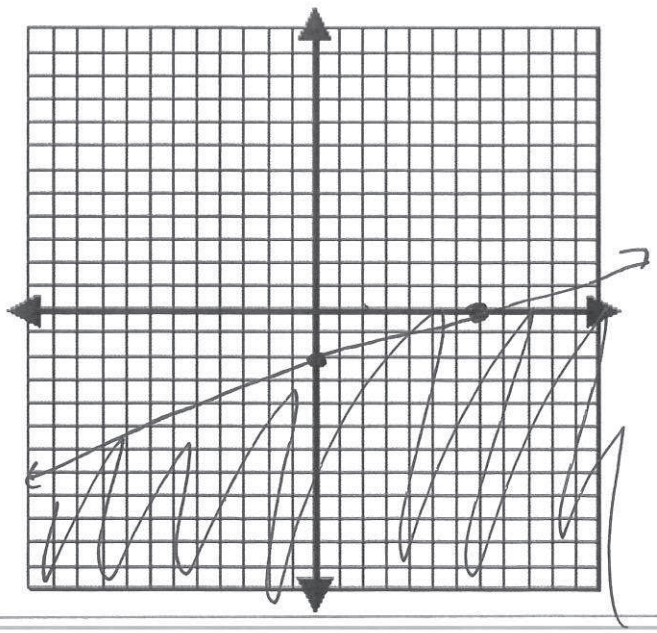
$5 - 0 + 0 < 0$

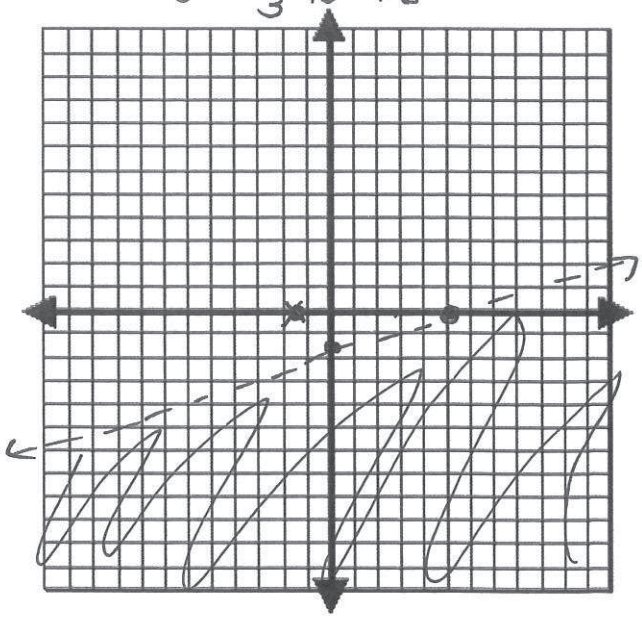
$5 < 0$ X

$5 + 3y = 0$ $5 - x = 0$

$3y = -5$ $x = 5$

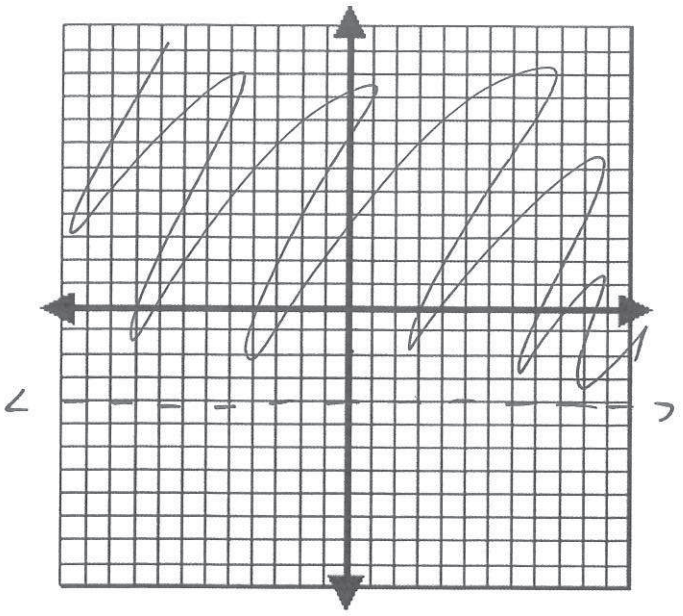
$y = -\frac{5}{3} \approx -1.\overline{6}$





c) $y + 4 > 0$

$y > -4$



d) $5x + 2y \leq 4$

$0 + 0 \leq 4$

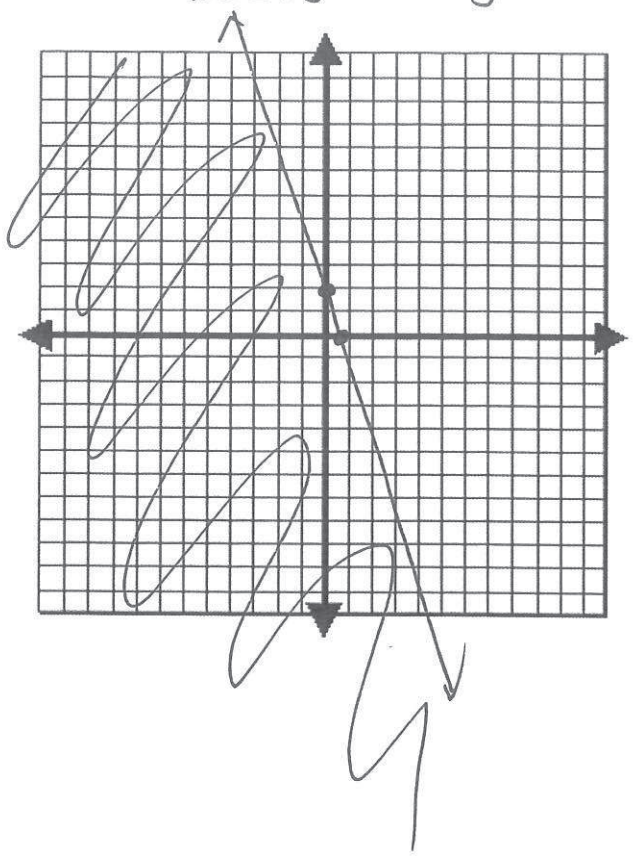
$0 \leq 4$ ✓

$5x = 4$

$x = 0.8$

$2y = 4$

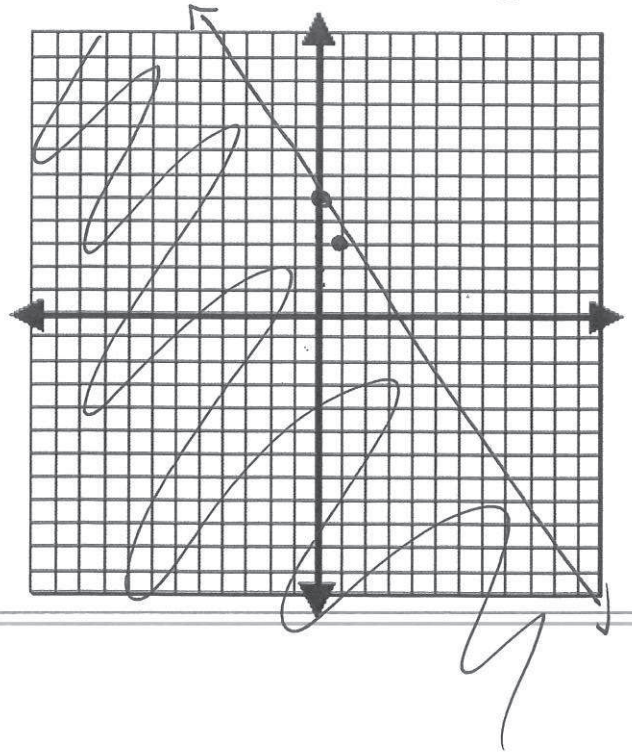
$y = 2$



e) $y \leq -2x + 5$

$0 \leq 0 + 5$

$0 \leq 5$ ✓

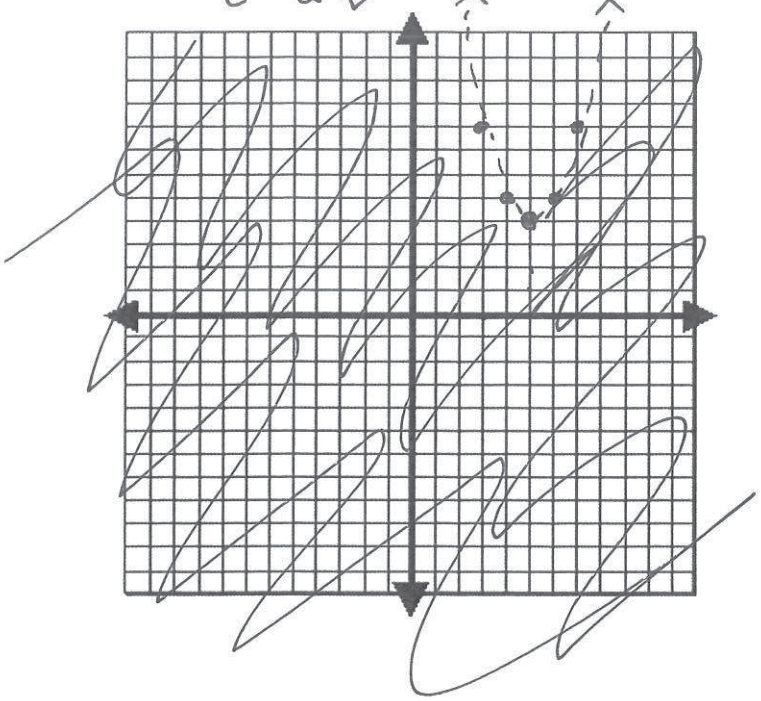


Level 3

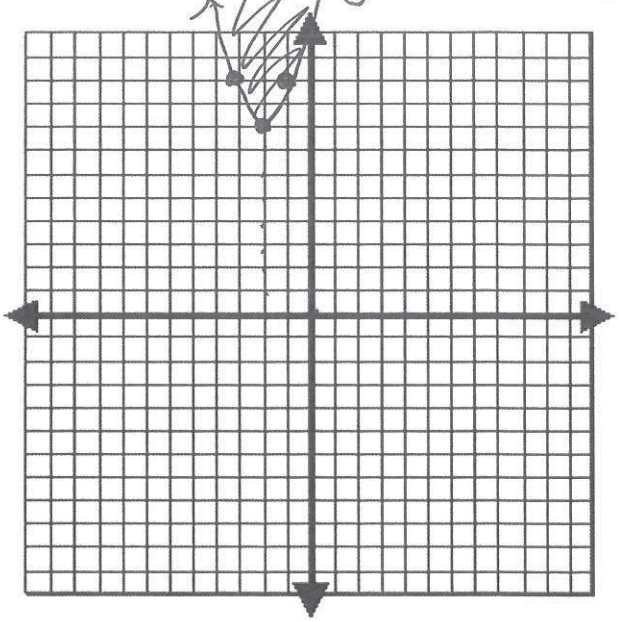
3. Graph each inequaltity

Graph each quadratic inequality.

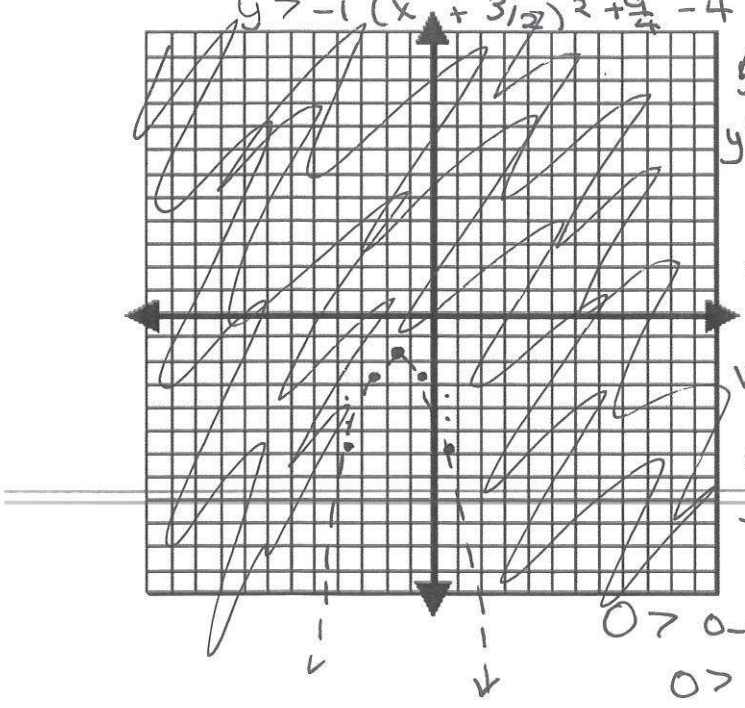
a) $y < (x-5)^2 + 4$



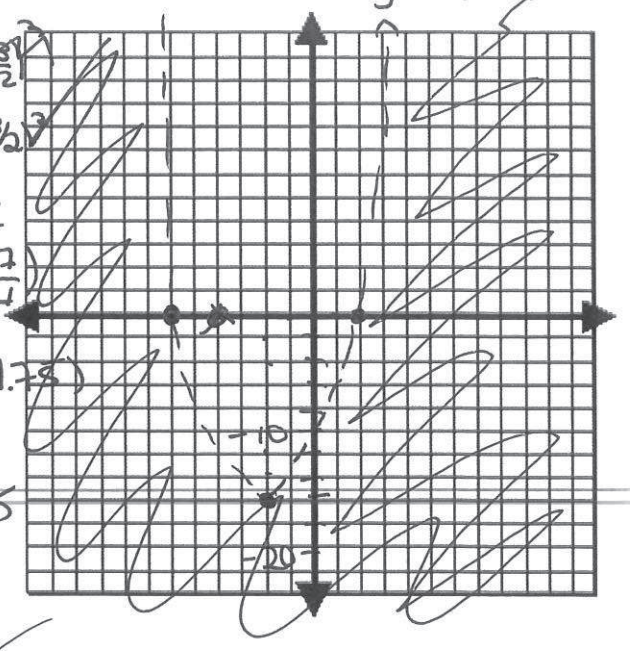
b) $y - 8 \geq 2(x+2)^2$



c) $y > -x^2 - 3x - 4$



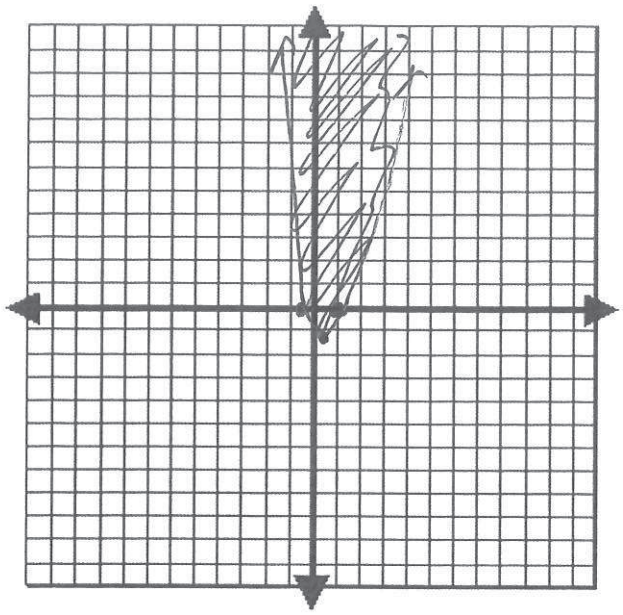
d) $y < (x-2)(x+6)$



$0 \geq 0 - 0 - 1$

$0 \geq -1$ ✓

e) $y \geq 2x^2 - x - 1$



$y \geq (2x + 1)(x - 1)$

$x = \frac{-1}{2} \qquad x = 1$

halfway $= \frac{1}{4}$

$y = 2(\frac{1}{4})^2 - \frac{1}{4} - 1$

$V(0.25, -1.125)$

4. Pierre wants to take his extended family to a movie at an IMAX theatre. He has a budget of $150 to spend on tickets. Tickets for children cost $9.50, and tickets for adults cost $13.95.

a) Write an inequality that represents the number of tickets that Pierre can afford.

b) Graph the solution region.

c) Interpret the solution set in reference to the number of tickets.

let c = child ticket $\quad c \in W$

a = adult ticket $\quad a \in W$

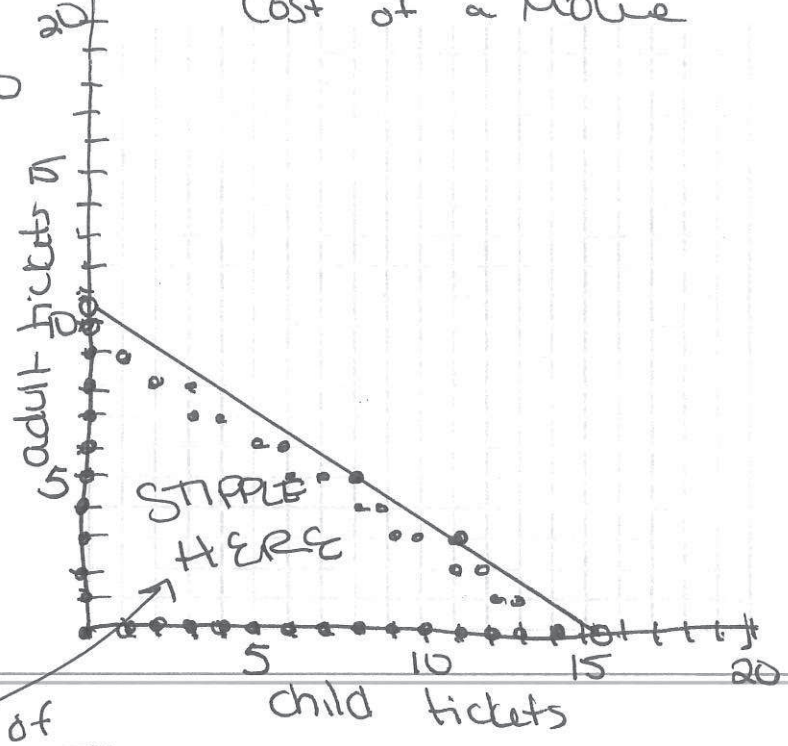
Cost of a Movie

$9.50c + 13.95a \leq 150$

$c = 15.78\ldots$

$a = 10.75\ldots$

$0 + 0 \leq 150$ ✓



any pair of coordinates in this region

Chapter 9b – Outcome 20.9b – Solving Quadratic inequalities in one variable

Level 2

1. Is the value of x a solution to the given inequality? Show your work.

a) $x^2 + 3x > -5, x = 0$

~~$x^2 + 3x + 5 > 0$~~

$0^2 + 0 > -5$

$0 > -5$ Yes!

b) $(x - 4)(x + 3) \leq 7, x = 0$

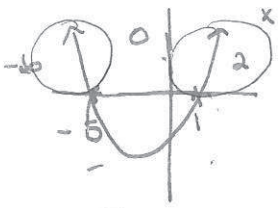
$(-4)(3) \leq 7$

$-12 < 7$ ✓

Yes

2. Determine the solution to each inequality.

a) $(x - 1)(x + 5) > 0$



$x = -6 \quad (-7)(-1) > 0$

$7 > 0$ ✓

$x = 0 \quad (-1)(5) > 0$

$-5 > 0$ ✗

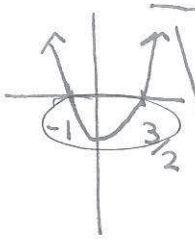
$x = 2 \quad (1)(7) > 0$

$7 > 0$ ✓

$\{x \mid x < -5 \cup x > 1, x \in \mathbb{R}\}$

$(-\infty, -5) \cup (1, \infty)$

b) $3(x + 1)(2x - 3) \leq 0$



$\{x \mid -1 \leq x \leq \frac{3}{2}, x \in \mathbb{R}\}$

$[-1, \frac{3}{2}]$

3. Solve each inequality. You may choose any of the methods we have covered during class. Graph paper is provided if you want it.

a) $4x^2 + 18 > 17x$

$4x^2 - 17x + 18 > 0$

~~$(2x \quad)(2x$~~

$(4x - 9)(x - 2) > 0$

$x = \frac{9}{4} = 2.25 \quad x = 2$

$\{x \mid x < 2 \cup x > 2.25, x \in \mathbb{R}\}$

$(-\infty, 2) \cup (2.25, \infty)$

b) $-8x^2 + 2x + 15 \geq 0$

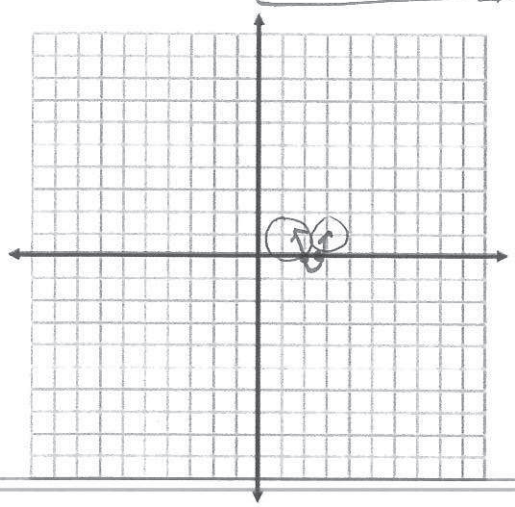
$-(8x^2 - 2x - 15)$

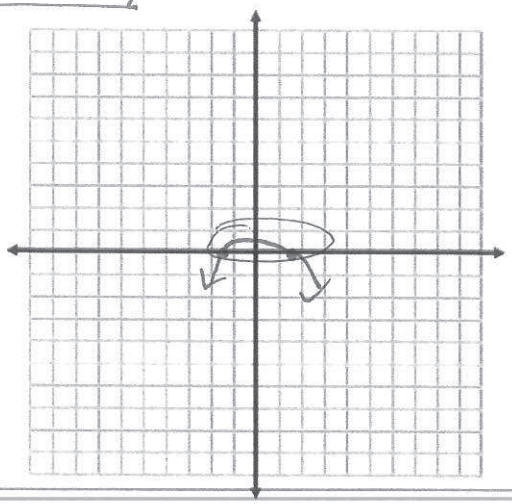
$-(2x - 3)(4x + 5) \geq 0$

$x = \frac{3}{2} = 1.5 \quad x = -\frac{5}{4} = -1.25$

$\{x \mid -\frac{5}{4} \leq x \leq \frac{3}{2}, x \in \mathbb{R}\}$

$[-\frac{5}{4}, \frac{3}{2}]$





c) $x^2 - x + 2 \leq 0$

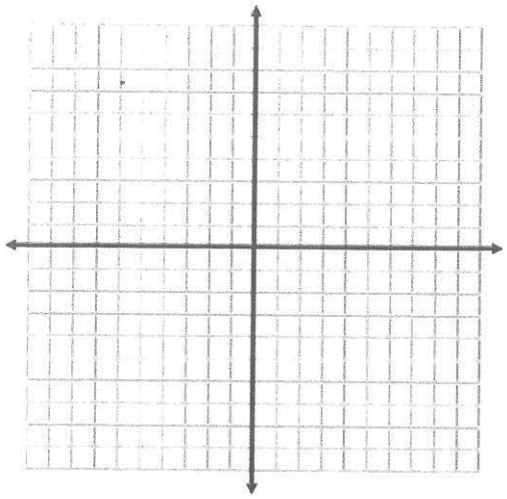
$a = 1$
$b = -1$
$c = 2$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-7}}{2}$$

no roots
concave up
→ never ≤ 0

$\{\ \}$



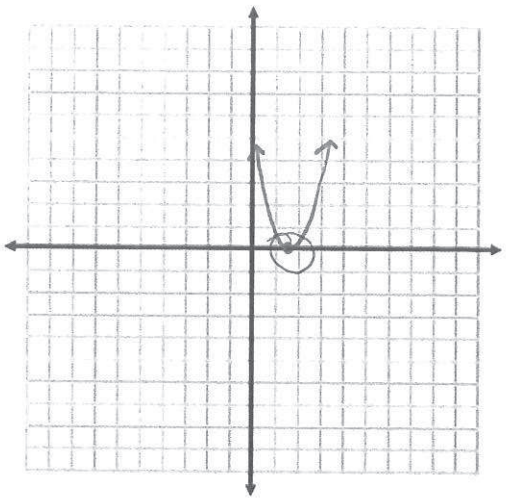
d) $4x^2 - 12x + 9 \leq 0$

$(2x - 3)(2x - 3) \leq 0$

$x = \frac{3}{2} = 1.5$

$\{x \mid x = \frac{3}{2}\}$

$[\frac{3}{2}]$



4. Given the function $f(x) = x^2 + 6x$,

a) determine the zeros of the function

b) solve the inequality $f(x) > 0$

c) solve the inequality $f(x) \leq -5$

a) $f(x) = x(x+6)$

$0 = x(x+6)$

$x = 0$ $\quad x = -6$

$(0,0)$ $\quad (-6,0)$

b) $(-\infty, -6) \cup (0, \infty)$

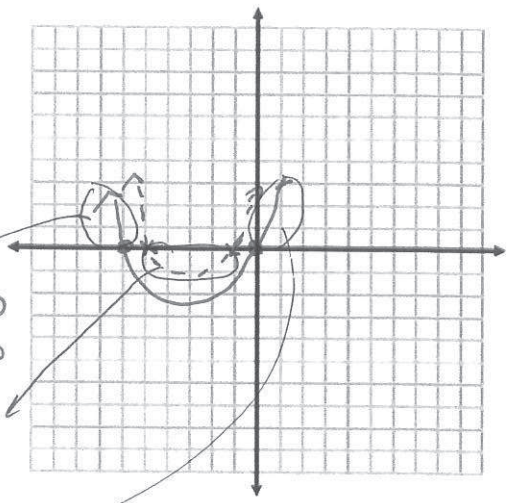
c) $x^2 + 6x \leq -5$

$x^2 + 6x + 5 \leq 0$

$(x+1)(x+5) \leq 0$

$x = -1 \quad x = -5$

$[-1, -5]$



5. The royalties received by an author depend on the number of books sold and the price of each book. For a particular book, the royalties, R, in dollars, depend on the price, P, in dollars, according to the equation $R = 0.02P(20\,000 - 200P)$. For what range of prices would the author receive more than \$8400 in royalties?

$$R = 0.02P(20000 - 200P)$$

$$8400 < 0.02P(20000 - 200P)$$

$$-8400 \qquad\qquad -8400$$

$$0 < 400P - 4P^2 - 8400$$

$$0 < -4P^2 + 400P - 8400$$

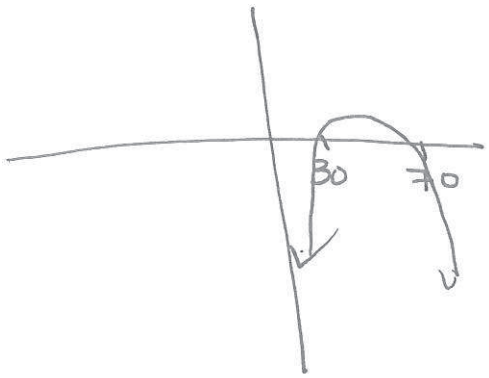
$$0 < -4(P^2 - 100P + 2100)$$

$$x = \frac{100 \pm \sqrt{1600}}{2}$$

$$x = \frac{100 \pm 40}{2}$$

$$x = \frac{140}{2} \qquad x = \frac{60}{2}$$

$$x = 70 \qquad x = 30$$



(30, 70)